# NTSE STAGE - I 02 - A/2017 - 18 (For Class - X) MENTAL ABILITY TEST (MAT) **HINTS & SOLUTIONS**

1. 
$$\frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \frac{1}{\sqrt{4}+\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{6}} + \frac{1}{\sqrt{6}+\sqrt{7}} + \frac{1}{\sqrt{7}+\sqrt{8}} + \frac{1}{\sqrt{8}+\sqrt{9}}$$

$$\left[ \left( \frac{1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} \right) + \left( \frac{1}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} \right) + \left( \frac{1}{\sqrt{4}+\sqrt{3}} \times \frac{\sqrt{4}-\sqrt{3}}{\sqrt{4}-\sqrt{3}} \right) + \dots + \left( \frac{1}{\sqrt{9}+\sqrt{8}} \times \frac{\sqrt{9}-\sqrt{8}}{\sqrt{9}-\sqrt{8}} \right) \right]$$

$$\Rightarrow \left[ \left( \sqrt{2}-1 \right) + \left( \sqrt{3}-\sqrt{2} \right) + \left( \sqrt{4}-\sqrt{3} \right) + \dots + \left( \sqrt{9}-\sqrt{8} \right) \right]$$

$$\Rightarrow \left( -1+\sqrt{9} \right) = -1+3=2$$

#### No option correct. 2.

3. 
$$\frac{n(n-3)}{2} = 35$$

$$n^2 - 3n - 70 = 0$$

$$n = 10$$

$$\frac{(n-2) \times 180}{n} = \frac{8 \times 180}{10} = 144^0$$

4. 
$$15-2 \div 900 + 90 \times 100 = ?$$
  
 $\Rightarrow 15 \times 2 + 900 \div 90 - 100 = 15 \times 2 + 10 - 100$   
 $\Rightarrow 30 + 10 - 100 = 40 - 100 = -60$ 

5. 
$$(k+1)x^2 - 5x + 2k = 0$$
  
Product of roots =  $\alpha \cdot \frac{1}{\alpha} = \frac{2k}{k+1}$   
 $\Rightarrow k+1 = 2k \Rightarrow k=1$ 

Let each side of cube = 2a : diameter of sphere = 2a

$$\therefore \frac{\text{Vol. of cube}}{\text{Vol of sphere}} = \frac{(2a)^3}{\frac{4}{3}\pi(a)^3} = \frac{8a^3}{\frac{4}{3}\pi a^3} = \frac{6}{\pi}$$

7. 
$$2x^{2} - 5x + 16 = 0$$

$$\left(\frac{\alpha^{2}}{\beta}\right)^{\frac{1}{3}} + \left(\frac{\beta^{2}}{\alpha}\right)^{\frac{1}{3}}$$

$$= \frac{\alpha^{2/3} \cdot \alpha^{1/3} + \beta^{2/3} \cdot \beta^{1/3}}{(\alpha \beta)^{1/3}} = \frac{\alpha + \beta}{(\alpha \beta)^{1/3}} = \frac{5/2}{2} = \frac{5}{4}$$

- 8.
- 8. Divided = Divisor x Quotient + Rem

Quotien = 10

∴ divisor = 100

& remainder = 10

 $\therefore$  divided = 100 x 10 + 10 = 1010

9. 2

9. 
$$\left[ (0.111)^3 + (0.222)^3 - (0.333)^3 + (0.333)^2 (0.222) \right]^2$$
Taking common (0.111) 
$$(0.111)^3 + (0.111)^3 (2)^3 - (0.111)^3 (3)^3 + (0.111)^2 (3)^2 (0.111)(2)$$

$$= (0.111)^3 (1 + 8 - 27 + 18)$$

$$= 0$$

- 10.
- 10.  $9^{2n} 4^{2n}$  is of form  $a^n b^n$  (where n is even) Therefore  $a^n - b^n$  is always divisible by (a - b) & (a + b)So it is divisible by (9 + 4) and (9 - 4) i.e., 5 and 13
- 11.

- 12. *′*
- 12. Speed of train =  $\frac{\text{length of train} + \text{length of platform}}{\text{time taken by train}}$  $\Rightarrow 120 \times \frac{5}{18} = \frac{320 + x}{24} \text{ (Let length of platform be x m)}$   $\therefore x = 480 \text{ m}$

∴ Speed of man = 
$$\frac{\text{length of platform}}{\text{time taken by man}}$$
  
=  $\frac{480}{4 \times 60}$  = 2m / sec

13. 
$$\frac{a^{n+1} + b^{n+1}}{a^n + b^n} = \frac{a+b}{2} \text{ when (n = 0)}$$

$$\Rightarrow \text{ value of n = 0}$$

∴ women = 
$$\frac{1}{3}x$$

$$\therefore$$
 men =  $\frac{2}{3}$ x

Women with children = 
$$\frac{1}{3} \times \frac{1}{2} \times \frac{1}{3} x = \frac{x}{18}$$

Men with children = 
$$\frac{3}{4} \times \frac{2}{3} \times \frac{2x}{3} = \frac{x}{3}$$

Worker with children = 
$$\frac{x}{18} + \frac{x}{3} = \frac{7x}{18}$$

Worker without children = 
$$x - \frac{7x}{18} = \frac{11}{18}$$
 of x

Then Alok earns = 
$$x + \frac{75x}{100} = \frac{175x}{100} = \frac{7x}{4}$$

$$=\frac{\frac{7x}{4}-x}{\frac{7x}{4}}\times100$$

$$= \frac{\frac{7x}{4}}{\frac{7x}{4}} \times 100$$

$$=\frac{3}{7}\times100=42.85\%$$

∴ Speed of spider = 
$$\frac{5x}{15}$$
 m / min

$$\therefore \text{ Time taken to climb the highest one} = \frac{7x}{\frac{5x}{15}}$$

$$= \frac{7x \times 15}{5x}$$
$$= 21 \text{ min}$$

17. Centroid = 
$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$
  

$$(0, 1) = \left(\frac{4 + 2 + x_3}{3}, \frac{-2 - 6 + y_3}{3}\right)$$

$$\therefore (x_3, y_3) = (-6, 11)$$

18. 
$$\cos^{2} \alpha = \sin \alpha . \tan \alpha$$

$$\cos^{3} \alpha = \sin^{2} \alpha$$

$$\frac{\cos^{2} \alpha}{\sin^{2} \alpha} = \frac{1}{\cos \alpha} \Rightarrow \cot^{2} \alpha = \frac{1}{\cos \alpha}$$

$$\cot^{6} \alpha - \cot^{2} \alpha = \cot^{2} \alpha \left(\cot^{4} \alpha - 1\right)$$

$$= \frac{1}{\cos \alpha} \left(\frac{1}{\cos^{2} \alpha} - 1\right)$$

$$= \frac{1}{\cos \alpha} \left(\frac{\sin^{2} \alpha}{\cos^{2} \alpha}\right) = \frac{\sin^{2} \alpha}{\cos^{3} \alpha} = 1$$

20. We can choose vertices out of 6 in  ${}^6C_3 = 20$  ways

Chosen vertices can form equilateral triangle in just 2 ways

Required probability =  $\frac{2}{20} = \frac{1}{10}$ 

21. A = 630 males teacher in F = 100   
B = 720 female teacher in C = 28% of 
$$4500 - (600)$$
   
C = 1260 = 660   
D = 675   
E = 945 female in B = 16% of  $1500 - 400$    
F = 270 = 320   
 $\Rightarrow 100 + 660 + 320 = 1080$ 

$$22. \qquad \frac{x}{100} \times 630 = 575 \Longrightarrow x = 90\%$$

Female teachers in F = 170 Difference = 945 - 170 = 775

- 25. 3
- 25. Male in  $C \rightarrow 600$ Female in B  $\rightarrow$  320  $C_{M}: B_{F} = 600: 320 = 15:8$
- 26.
- 26.  $25 \times 10 + 5 = 255$ ,  $255 \times 10 - 5 = 2545$ ,  $2545 \times 10 + 5$ ,  $25455 \times 10 - 5 = 254545$
- 27.
- 27.  $4 \times 3 = 12 \rightarrow Alphabet position of L$ 1 x 17 = 17  $\rightarrow$  Alphabet position of Q  $5 \times 4 = 20 \rightarrow Alphabet position of T$
- 28.
- 28. After removing even numbers 15971535975913513 Answer  $\rightarrow$  9
- 29.
- 29. Total hours = 89 hours

Faulty time 23 hours & 44 min =  $23 + \frac{44}{60}$  i.e,  $\frac{356}{15}$  hr

 $\frac{356}{15}$  hr of incorrect watch = 24 hr of correct watch

∴ 89 hours of incorrect watch =  $\frac{24 \times 15}{356} \times 89 = 90$  hrs

.. Actually watch will be 1 hr faster than faulty watch. i.e., 11 pm

- 30.
- 1 + 2 + 3 .... 5 6 30.

Sum = 
$$\frac{n(n+1)}{2} = \frac{56 \times 57}{2} = 28 \times 57 = 1596$$

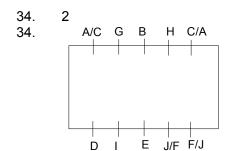
- 31.
- Total white box =  $\frac{64}{2}$  = 32 31.

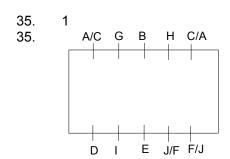
Now odd in white box =  $\frac{32}{2}$  = 16

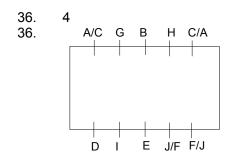
- 32.
- 4 32.  $H^{-}$ 
  - + means male member
  - means female members

# means couple means brother / daughter means son / daughter

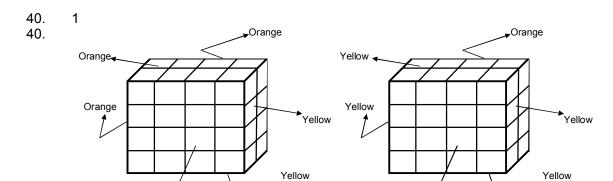








38. 2  
38. 
$$52 < 4 \land 5 > 8 \lor 2$$
  
 $52 - 4 \times 5 + 8 \div 2$   
 $52 - 4 \times 5 + 4$   
 $56 - 20 = 36$ 

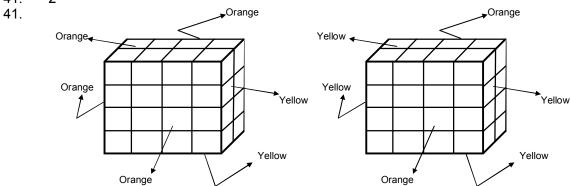


Orange

All the smaller cubes have colour on their faces,

Orange



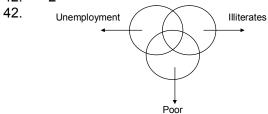


The larger faces will have one face coloured cube which is four on each face. So, in total there are four larger faces.

$$4 \times 4 = 16$$

16 cubes will have only one face colour.

42. 2



- 43. 1
- 43. By observation
- 44. 4
- 44. From one row to another the contents of each cell moves one place towards right.
- 45. 3
- 45. By observation
- 46.
- 46. By observation.

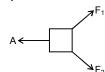
47. Blue 
$$\rightarrow$$
 2  
Sky  $\rightarrow$  1  
Was  $\rightarrow$  3  
People  $\rightarrow$  8  
Like  $\rightarrow$  0  
In  $\rightarrow$  6  
Birds  $\rightarrow$  9  
'People like birds'  $\rightarrow$  809

50. 
$$\sqrt{64} + \sqrt{36} + \sqrt{49} = 8 + 6 + 7 = 21$$
$$\sqrt{121} + \sqrt{81} + \sqrt{100} = 11 + 9 + 10 = 30$$

# NTSE STAGE – I SCHOLASTIC APTITUDE TEST (SAT) HINTS & SOLUTIONS

#### **PHYSICS**

101. 101.



Minimum number of forces required is 3.

102.

102. Resistance of the heater be R. New resistance of heater is R/2.

Initial power = 
$$\frac{V^2}{R}$$
  
Final power =  $\frac{V^2}{R/2} = 2\frac{V^2}{R}$ 

:. Heat generated is doubled.

103.

103.



Force acting at different part of the magnet is different. So both torque and force will be acting.

104. 2

104. 
$$P = 2 \times 10^3 \text{ watt}$$
  
Energy in 1 min =  $2 \times 10^3 \times 60 \text{ J.}$   
 $\Rightarrow 2 \times 10^3 \times 60 = \text{m} \times 10 \times 10$   
 $\Rightarrow \text{m} = 1200 \text{ kg}$ ; Volume = 1200 litre.

105. 3

105. Initial velocity = V  
Final velocity = V'  

$$\frac{1}{2}mv^2 \times 4 = \frac{1}{2}m(v')^2$$

$$V' = 2V.$$

Initial momentum = mv Final momentum = 2mv.

.. Momentum is doubled.

106. 3

106. Total work done by gravity is zero.

107. 2

107. 
$$\Rightarrow \frac{GM_m \times 80}{(2R_m)^2} = 9.8$$
$$\Rightarrow \frac{GMm}{Rm^2} = \frac{9.8}{40} = 0.49 \text{ m/s}^2$$

108. 2  
108. 
$$P = P_1 + P_2$$
  
 $\Rightarrow P = \frac{1}{f_1} + \frac{1}{f_2}$   
 $\Rightarrow P = \frac{f_1 + f_2}{f_1 f_2}$ 

109. 1  
109. 
$$\frac{\sin i}{\sin r} = \frac{V_1}{V_2}$$

$$\Rightarrow \frac{\sin 30}{\sin 60} = \frac{V}{V'}$$

$$\Rightarrow V' = \sqrt{3} V$$

110. 2

110. 
$$\frac{1}{2}g(t)^2 - \frac{1}{2}g(t-2)^2 = 40$$
 $\Rightarrow 5t^2 - 5(t-2)^2 = 40$ 
 $\Rightarrow t^2 - (t-2)^2 = 8$ 
 $\Rightarrow t = 3$ 
 $\therefore \text{ height } = \frac{1}{2}gt^2 = \frac{1}{2} \times 10 \times 9 = 45 \text{ m.}$ 

111. 4
111. 
$$R = \frac{\rho \ell}{A}$$

New area = nA

∴ New length = 
$$\frac{\ell}{n}$$
  
⇒ R' =  $\frac{\rho \ell}{n^2 A} = \frac{R}{n^2}$ 

- 112. 3
- 112. Ammeter has low resistance and due to its resistance current in the circuit decreases.
- 113.
- 113. As the rays are diverging so the optical device is convex mirror.
- 114.
- 114. Work is zero only when force and displacement are perpendicular to each other. So, work will be down in all the cases.

#### **CHEMISTRY**

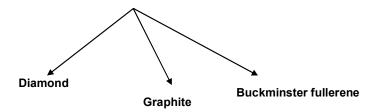
- 115. 2
- 115. Silver is a good conductor of electricity.
- 116.
- 116.  $Cr = [Ar] 4s^1 3d^5$  Half filled configuration of d subshell is more symmetrical.
- 117. 3
- 117. Amphoteric.

Al<sub>2</sub>O<sub>3</sub> can react with acid and base both.

- 118. 3
- 118. As Solution is acidic so pH < 7
- 119. 3
- 119. In basic solution phenolphthalein shows pink colour.
- 120. 1
- 120. 4s, 4p, 4d, 4f

$$4s = one orbital$$
,  $4d = five orbital$  Total = 7 + 5 + 3 + 1

- 121. 1
- 121. Addition of NaCl results precipitation of soap to separate out soap from the solution. (as in question paper sodium is given which is considered wrong. It should be NaCl)
- 122. 3
- 122. There are 3 allotropes of carbon.



- 123. 2
- 123. Due to Ca(NO<sub>3</sub>)<sub>2</sub> & CaSO<sub>4</sub>.

As oxides of nitrogen & sulphur reacts with limestone on Taj Mahal to form nitrates and sulphates of calcium.

- 124. 1
- 124. CO<sub>2</sub>, CH<sub>4</sub>, N<sub>2</sub>O, and O<sub>3</sub> Act as green house gases.
- 125. 1
- 125. s, p, d, f subshells are present in an atom.
- 126. 1
- 126. Boron and cadmium used in atomic reactors to control speed of neutron.
- 127. 2

127. Moles = 
$$\frac{1000}{108}$$
 = 9.25

No. of atoms = 
$$9.25 \times 6.023 \times 10^{23}$$
  
=  $5.571 \times 10^{24}$  atoms

#### **BIOLOGY**

- 128. 2
- 128. Chromosomes carry genes, which are the hereditary units, which carry hereditary characters to the offspring.
- 129. 3
- 129. Mitochondria is called the power house of the cell.
- 130. 4
- 130. Plasma membrane is made up of both protein and lipid.
- 131. 2
- 131. Oviduct is the site of fertilization in humans.
- 132. 1
- 132. Heart never takes rest as it has cardiac muscles which never gets fatigue.
- 133. 1
- 133. Lacteal present in the villi of the small intestine help to absorb fatty acids and glycerol.
- 134. 1
- 134. The experiment 'origin of primitive life on Earth' was performed by Urey and Miller.
- 135. 2
- 135. Bicuspid valve is present in the human heart in between left atrium and left ventricle.
- 136. 3
- 136. During the light reaction NADPH and ATP are synthesized which is utilized in dark reaction.
- 137. 4
- 137. Grafting in monocot plants is not possible because they have scattered Vascular Bundles.
- 138. ·
- 138. Haemophilia disease is linked with sex chromosome.
- 139. 3
- 139. The primary building blocks of DNA Nitrogenous base, phosphorus and deoxyribose.
- 140. 1
- 140. Islets of Langerhans helps in formation of insulin

#### **MATHEMATICS**

141. 
$$x^4 + 4x^3 + nx^2 + 4x + 1 = \left(ax^2 + bx + c\right)^2$$
 
$$x^4 + 4x^3 + nx^2 + 4x + 1 = a^2x^4 + b^2x^2 + c^2$$
 
$$+ 2abx^3 + 2bcx + 2acx^2$$
 
$$= a^2x^4 + 2abx^3 + \left(b^2 + 2ac\right)x^2 + 2bcx + c^2$$
 Comparing coefficients, 
$$a^2 = 1$$

$$c^{2} = 1$$

$$2ab = 4$$

$$b^2 + 2ac = n$$
$$2bc = 4$$

Solving, we get 
$$\frac{a}{c} = 1 \Rightarrow a = c$$

$$b = \pm 2$$
,  $a = \pm 1$ ,  $c = \pm 1$ 

$$b^2 + 2ac = 4 + 2 = 6$$

$$\therefore \text{ Required percent of increase } = \frac{10}{90} \times 100$$

$$=11\frac{1}{9}\%$$

143. Let 
$$\sqrt{x} = a, \sqrt{y} = b$$

Given equation 
$$a^3 + b^3 = 183$$
 and  $a^2b + ab^2 = 182$ 

$$\Rightarrow$$
  $(a+b)^3 - 3ab(a+b) = 183$  and  $ab(a+b) = 182$ 

$$\Rightarrow$$
  $(a+b)^3 = 183 + 3 \times 182$ 

$$\Rightarrow$$
 a + b = 9 and ab =  $\frac{182}{9}$ 

Now 
$$\sqrt{x} + \sqrt{y} = 9$$
 and  $\sqrt{xy} = \frac{182}{9}$ 

So, 
$$x + y = 81 - 2\sqrt{xy} = \frac{365}{9}$$

$$\Rightarrow \frac{18}{5}(x+y) = 146$$

145. Let 
$$x = a$$
,  $y = 2b$ ,  $z = 4c$ 

then 
$$(a)^2 + (2b)^2 + (4c)^2 = 48$$
  $\Rightarrow x^2 + y^2 + z^2 = 48$ 

and 
$$2ab + 8bc + 4ca = 48$$
  $\Rightarrow xy + yz + zx = 48$ 

Now, 
$$(x-y)^2 + (y-z)^2 + (z-x)^2 = 0$$

$$x = y = z$$

$$a = 2b = 4c$$

$$\frac{a}{4} = \frac{b}{2} = \frac{c}{1} = \lambda$$

So, 
$$ab + 4bc + 2ca = 4\lambda . 2\lambda + 8\lambda . \lambda + 2\lambda . 4\lambda = 24$$

$$\lambda^2 \left(8+8+8\right)=24$$

$$\lambda = \pm 1$$

$$\Rightarrow a^2+b^2+c^2=21\lambda^2=21$$

146. By angle sum property of  $\triangle ACE$  and  $\triangle DBF$ 

## 147. No option is correct

147. 
$$\sin^4 x = \cos^2 x$$

$$\Rightarrow (1-\cos^2 x)^2 = \cos^2 x$$

$$\Rightarrow \cos^4 x - 3\cos^2 x + 1 = 0$$

$$\Rightarrow \cos^2 x = \frac{3 - \sqrt{5}}{2}$$

Now, 
$$\cos^4 x + \cos^2 x = \left(\frac{3 - \sqrt{5}}{2}\right)^2 + \left(\frac{3 - \sqrt{5}}{2}\right)$$

$$=5-2\sqrt{5}$$

## 148. No option is correct

148. The roots are 1, 2, 3 and k

$$1 + 2 + 3 + k = 0$$

$$k = -6$$

$$c=1{\times}\,2{\times}\,3{\times}{-}6$$

$$c = -36$$

149.

149. 
$$x = 4 + \sqrt{15}$$

$$y=4-\sqrt{15}$$

$$x + y = 8$$

$$xy = 1$$

$$x^{3} + y^{3} = (x + y)((x + y)^{2} - 3xy)$$

$$=8(64-3)$$

$$= 8(61)$$

- 150. 4
- 150. AD = 10 cm, BE = 12 cm, CF = 15 cm

$$AC = b cm$$

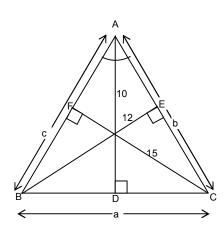
$$AB = c cm$$

Area of 
$$\Delta = \frac{1}{2} \times 10a = \frac{1}{2} \times 12b = \frac{1}{2} \times 15c$$

$$\Rightarrow$$
 a:b:c=6:5:4

let sides are 6x, 5x and 4x

$$\Rightarrow \frac{1}{2} \times 10 \times 6x = \sqrt{\frac{15x}{2} \left(\frac{15x}{2} - 6x\right) \left(\frac{15x}{2} - 5x\right) \left(\frac{15x}{2} - 4x\right)}$$



$$\Rightarrow x = \frac{8}{\sqrt{7}}$$

So, semi perimeter = 
$$\frac{60}{\sqrt{7}}$$
 cm

151. 
$$12\cot^2\theta - 31\cos ec \theta + 32 = 0$$

$$12 \left(\cos ec^2\theta - 1\right) - 31 cos ec \theta + 32 = 0$$

$$12\cos ec^2\theta - 31\cos ec\theta + 20 = 0$$

$$12\cos ec^2\theta - 16\cos ec\theta - 15\cos ec\theta + 20 = 0$$

$$4\cos ec \theta(3\cos ec \theta-4)-5(3\cos ec \theta-4)=0$$

$$\csc\theta = \frac{5}{4} \text{ or } \frac{4}{3} \Rightarrow \sin\theta = \frac{4}{5} \text{ or } \frac{3}{4}$$

152. 
$$\frac{1}{2}$$
ac = 16

$$ac = 32$$

$$ed = 18$$

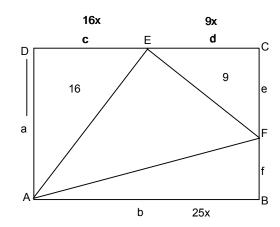
$$ab = 50$$

$$\frac{c}{b} = \frac{32}{50} = \frac{16}{25}$$

$$e.\frac{9}{25}b = 18$$

$$eb = \frac{18 \times 25}{9} = 50$$

$$e = a f = 0$$



$$S \rightarrow S^3$$

$$2S \rightarrow 8S^3$$

$$\therefore Increase percent = \frac{75^3}{5^3} \times 100$$
$$= 700\%$$

154. 
$$\frac{V_1}{V_2} = \frac{\pi (2x)^2 .5y}{\pi (3x)^2 .3y}$$
$$= \frac{20}{\pi (3x)^2 .3y}$$

$$\frac{1}{3}\pi r^{2}: r: \pi r^{2}r: \frac{2}{3}\pi r^{3}$$

$$= \frac{1}{3}: 1: \frac{2}{3}$$

$$= 1: 3: 2$$

157. 
$$a_1 + a_2 + \dots + a_7 + a_8 = 160$$
  
 $a_1 + a_2 = 31$   
 $a_3 + a_4 + a_5 = 64$   
 $a_7 - a_6 = 4$   
 $a_8 - a_6 = 7$   
Solving we get

$$a_6 = 18$$

$$a_7 = 22$$

$$a_8 = 25$$

158. 
$$R = \frac{2}{3} \times \text{ altitude}$$
$$= \frac{2}{3} \times \frac{\sqrt{3}}{2} \times 3\sqrt{3}$$
$$= 3 \text{ cm}$$

## 160. No option is correct

160. 
$$x - 10 = \frac{40}{100} \times x$$
$$\Rightarrow \overline{x} = \frac{50}{3}$$

If each observation is increased by 5 then new mean  $=\frac{65}{3}$