Class 12 Maths NCERT Solutions Chapter - 4

Determinants Exercise 4.1

Q 1:

Evaluate the determinants in Exercises 1 and 2.

$$\begin{vmatrix} 2 & 4 \\ -5 & -1 \end{vmatrix}$$

Answer::

$$\begin{vmatrix} 2 & 4 \\ -5 & -1 \end{vmatrix} = 2(-1) - 4(-5) = -2 + 20 = 18$$

Q 2:

Evaluate the determinants in Exercises 1 and 2.

(i) $\begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$ (ii) $\begin{vmatrix} x^2 - x + 1 & x - 1 \\ x + 1 & x + 1 \end{vmatrix}$

Answer::

(i) $\begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix} = (\cos\theta)(\cos\theta) - (-\sin\theta)(\sin\theta) = \cos^2\theta + \sin^2\theta = 1$

(ii) $\begin{vmatrix} x^2 - x + 1 & x - 1 \\ x + 1 & x + 1 \end{vmatrix}$

$= (x^2 - x + 1)(x + 1) - (x - 1)(x + 1)$

$= x^3 - x^2 + x + x^2 - x + 1 - (x^2 - 1)$

$= x^3 + 1 - x^2 + 1$

$= x^3 - x^2 + 2$

Q 3:

If $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$, then show that $|2A| = 4|A|$

Answer::

The given matrix is $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$

$$\therefore 2A = 2\begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 8 & 4 \end{bmatrix}$$

$$\therefore \text{L.H.S.} = |2A| = \begin{vmatrix} 2 & 4 \\ 8 & 4 \end{vmatrix} = 2 \times 4 - 4 \times 8 = 8 - 32 = -24$$

$$\text{Now, } |A| = \begin{vmatrix} 1 & 2 \\ 4 & 2 \end{vmatrix} = 1 \times 2 - 2 \times 4 = 2 - 8 = -6$$

$$\therefore \text{R.H.S.} = 4|A| = 4 \times (-6) = -24$$

$$\therefore \text{L.H.S.} = \text{R.H.S.}$$

Q 4:

If $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$, then show that $|3A| = 27|A|$.

Answer::

The given matrix is $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$.

It can be observed that in the first column, two entries are zero. Thus, we expand along the first column (C_1) for easier calculation.

$$|A| = 1\begin{vmatrix} 1 & 2 \\ 0 & 4 \end{vmatrix} - 0\begin{vmatrix} 0 & 1 \\ 0 & 4 \end{vmatrix} + 0\begin{vmatrix} 0 & 1 \\ 1 & 2 \end{vmatrix} = 1(4-0) - 0 + 0 = 4$$

$$\therefore 27|A| = 27(4) = 108 \qquad \ldots(i)$$

$$\text{Now, } 3A = 3\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 3 \\ 0 & 3 & 6 \\ 0 & 0 & 12 \end{bmatrix}$$

$$\therefore |3A| = 3\begin{vmatrix} 3 & 6 \\ 0 & 12 \end{vmatrix} - 0\begin{vmatrix} 0 & 3 \\ 0 & 12 \end{vmatrix} + 0\begin{vmatrix} 0 & 3 \\ 3 & 6 \end{vmatrix}$$

$$= 3(36-0) = 3(36) = 108 \qquad \ldots(ii)$$

From equations (i) and (ii), we have:

$$|3A| = 27|A|$$

Hence, the given result is proved.

Q 5:

Evaluate the determinants

(i) $\begin{vmatrix} 3 & -1 & -2 \\ 0 & 0 & -1 \\ 3 & -5 & 0 \end{vmatrix}$ (iii) $\begin{vmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix}$

(ii) $\begin{vmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{vmatrix}$ (iv) $\begin{bmatrix} 2 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{bmatrix}$

Answer::

(i) Let $A = \begin{vmatrix} 3 & -1 & -2 \\ 0 & 0 & -1 \\ 3 & -5 & 0 \end{vmatrix}$.

It can be observed that in the second row, two entries are zero. Thus, we expand along the second row for easier calculation.

$$|A| = -0\begin{vmatrix} -1 & -2 \\ -5 & 0 \end{vmatrix} + 0\begin{vmatrix} 3 & -2 \\ 3 & 0 \end{vmatrix} - (-1)\begin{vmatrix} 3 & -1 \\ 3 & -5 \end{vmatrix} = (-15+3) = -12$$

(ii) Let $A = \begin{bmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{bmatrix}$.

By expanding along the first row, we have:

$$|A| = 3\begin{vmatrix} 1 & -2 \\ 3 & 1 \end{vmatrix} + 4\begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix} + 5\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix}$$
$$= 3(1+6) + 4(1+4) + 5(3-2)$$
$$= 3(7) + 4(5) + 5(1)$$
$$= 21 + 20 + 5 = 46$$

(iii) Let $A = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{bmatrix}$.

By expanding along the first row, we have:

$|A| = 0 \begin{vmatrix} 0 & -3 \\ 3 & 0 \end{vmatrix} - 1 \begin{vmatrix} -1 & -3 \\ -2 & 0 \end{vmatrix} + 2 \begin{vmatrix} -1 & 0 \\ -2 & 3 \end{vmatrix}$

$= 0 - 1(0 - 6) + 2(-3 - 0)$

$= -1(-6) + 2(-3)$

$= 6 - 6 = 0$

(iv) Let $A = \begin{bmatrix} 2 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{bmatrix}$.

By expanding along the first column, we have:

$|A| = 2 \begin{vmatrix} 2 & -1 \\ -5 & 0 \end{vmatrix} - 0 \begin{vmatrix} -1 & -2 \\ -5 & 0 \end{vmatrix} + 3 \begin{vmatrix} -1 & -2 \\ 2 & -1 \end{vmatrix}$

$= 2(0 - 5) - 0 + 3(1 + 4)$

$= -10 + 15 = 5$

Q 6:

If $A = \begin{bmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{bmatrix}$, find $|A|$.

Answer::

Let $A = \begin{bmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{bmatrix}$.

By expanding along the first row, we have:

$|A| = 1 \begin{vmatrix} 1 & -3 \\ 4 & -9 \end{vmatrix} - 1 \begin{vmatrix} 2 & -3 \\ 5 & -9 \end{vmatrix} - 2 \begin{vmatrix} 2 & 1 \\ 5 & 4 \end{vmatrix}$

$= 1(-9 + 12) - 1(-18 + 15) - 2(8 - 5)$

$= 1(3) - 1(-3) - 2(3)$

$= 3 + 3 - 6$

$= 6 - 6$

$= 0$

Q 7:

Find values of x, if

(i) $\begin{vmatrix} 2 & 4 \\ 2 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$ (ii) $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$

Answer::

(i) $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$

$\Rightarrow 2 \times 1 - 5 \times 4 = 2x \times x - 6 \times 4$

$\Rightarrow 2 - 20 = 2x^2 - 24$

$\Rightarrow 2x^2 = 6$

$\Rightarrow x^2 = 3$

$\Rightarrow x = \pm\sqrt{3}$

(ii) $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$

$\Rightarrow 2 \times 5 - 3 \times 4 = x \times 5 - 3 \times 2x$

$\Rightarrow 10 - 12 = 5x - 6x$

$\Rightarrow -2 = -x$

$\Rightarrow x = 2$

Q 8:

If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$, then x is equal to

(A) 6 (B) ±6 (C) −6 (D) 0

Answer::

Answer:: B

$\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$

$\Rightarrow x^2 - 36 = 36 - 36$

$\Rightarrow x^2 - 36 = 0$

$\Rightarrow x^2 = 36$

$\Rightarrow x = \pm 6$

Hence, the correct Answer: is B.

Exercise 4.2

Q 1:

Using the property of determinants and without expanding, prove that:

$$\begin{vmatrix} x & a & x+a \\ y & b & y+b \\ z & c & z+c \end{vmatrix} = 0$$

Answer::

$$\begin{vmatrix} x & a & x+a \\ y & b & y+b \\ z & c & z+c \end{vmatrix} = \begin{vmatrix} x & a & x \\ y & b & y \\ z & c & z \end{vmatrix} + \begin{vmatrix} x & a & a \\ y & b & b \\ z & c & c \end{vmatrix} = 0 + 0 = 0$$

[Here, the two columns of the determinants are identical]

Q 2:

Using the property of determinants and without expanding, prove that:

$$\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix} = 0$$

Answer::

$$\Delta = \begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$$

Applying $R_1 \rightarrow R_1 + R_2$, we have:

$$\Delta = \begin{vmatrix} a-c & b-a & c-b \\ b-c & c-a & a-b \\ -(a-c) & -(b-a) & -(c-b) \end{vmatrix}$$

$$= - \begin{vmatrix} a-c & b-a & c-b \\ b-c & c-a & a-b \\ a-c & b-a & c-b \end{vmatrix}$$

Here, the two rows R_1 and R_3 are identical.

$\therefore \Delta = 0$.

Q 3:

Using the property of determinants and without expanding, prove that:

$$\begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix} = 0$$

Answer::

$$\begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix} = \begin{vmatrix} 2 & 7 & 63+2 \\ 3 & 8 & 72+3 \\ 5 & 9 & 81+5 \end{vmatrix}$$

$$= \begin{vmatrix} 2 & 7 & 63 \\ 3 & 8 & 72 \\ 5 & 9 & 81 \end{vmatrix} + \begin{vmatrix} 2 & 7 & 2 \\ 3 & 8 & 3 \\ 5 & 9 & 5 \end{vmatrix}$$

$$= \begin{vmatrix} 2 & 7 & 9(7) \\ 3 & 8 & 9(8) \\ 5 & 9 & 9(9) \end{vmatrix} + 0 \qquad \left[\text{Two columns are identical} \right]$$

$$= 9\begin{vmatrix} 2 & 7 & 7 \\ 3 & 8 & 8 \\ 5 & 9 & 9 \end{vmatrix}$$

$$= 0 \qquad \left[\text{Two columns are identical} \right]$$

Q 4:

Using the property of determinants and without expanding, prove that:

$$\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix} = 0$$

Answer::

$$\Delta = \begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix}$$

By applying $C_3 \rightarrow C_3 + C_2$, we have:

$$\Delta = \begin{vmatrix} 1 & bc & ab+bc+ca \\ 1 & ca & ab+bc+ca \\ 1 & ab & ab+bc+ca \end{vmatrix}$$

Here, two columns C_1 and C_3 are proportional.

$\therefore \Delta = 0$.

Q 5:

Using the property of determinants and without expanding, prove that:

$$\begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix} = 2 \begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$$

Answer::

$$\Delta = \begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix}$$

$$= \begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a & p & x \end{vmatrix} + \begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ b & q & y \end{vmatrix}$$

$$= \Delta_1 + \Delta_2 \ (\text{say}) \qquad \qquad \dots(1)$$

Now, $\Delta_1 = \begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a & p & x \end{vmatrix}$

Applying $R_2 \rightarrow R_2 - R_3$, we have:

$$\Delta_1 = \begin{vmatrix} b+c & q+r & y+z \\ c & r & z \\ a & p & x \end{vmatrix}$$

Applying $R_1 \rightarrow R_1 - R_2$, we have:

$$\Delta_1 = \begin{vmatrix} b & q & y \\ c & r & z \\ a & p & x \end{vmatrix}$$

Applying $R_1 \leftrightarrow R_3$ and $R_2 \leftrightarrow R_3$, we have:

$$\Delta_1 = (-1)^2 \begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix} = \begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix} \qquad \ldots(2)$$

$$\Delta_2 = \begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ b & q & y \end{vmatrix}$$

Applying $R_1 \rightarrow R_1 - R_3$, we have:

$$\Delta_2 = \begin{vmatrix} c & r & z \\ c+a & r+p & z+x \\ b & q & y \end{vmatrix}$$

Applying $R_2 \rightarrow R_2 - R_1$, we have:

$$\Delta_2 = \begin{vmatrix} c & r & z \\ a & p & x \\ b & q & y \end{vmatrix}$$

Applying $R_1 \leftrightarrow R_2$ and $R_2 \leftrightarrow R_3$, we have:

$$\Delta_2 = (-1)^2 \begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix} = \begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix} \qquad \ldots(3)$$

From (1), (2), and (3), we have:

$$\Delta = 2 \begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$$

Hence, the given result is proved.

Q 6:

By using properties of determinants, show that:

$$\begin{vmatrix} 0 & a & -b \\ -a & 0 & -c \\ b & c & 0 \end{vmatrix} = 0$$

Answer::

We have,

$$\Delta = \begin{vmatrix} 0 & a & -b \\ -a & 0 & -c \\ b & c & 0 \end{vmatrix}$$

Applying $R_1 \to cR_1$, we have:

$$\Delta = \frac{1}{c}\begin{vmatrix} 0 & ac & -bc \\ -a & 0 & -c \\ b & c & 0 \end{vmatrix}$$

Applying $R_1 \to R_1 - bR_2$, we have:

$$\Delta = \frac{1}{c}\begin{vmatrix} ab & ac & 0 \\ -a & 0 & -c \\ b & c & 0 \end{vmatrix}$$

$$= \frac{a}{c}\begin{vmatrix} b & c & 0 \\ -a & 0 & -c \\ b & c & 0 \end{vmatrix}$$

Here, the two rows R_1 and R_3 are identical.

$\therefore \Delta = 0.$

Q 7:

By using properties of determinants, show that:

$$\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix} = 4a^2b^2c^2$$

:

Answer:

$$\Delta = \begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix}$$

$$= abc \begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix} \qquad \left[\text{Taking out factors } a, b, c \text{ from } R_1, R_2, \text{ and } R_3\right]$$

$$= a^2 b^2 c^2 \begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix} \qquad \left[\text{Taking out factors } a, b, c \text{ from } C_1, C_2, \text{ and } C_3\right]$$

Applying $R_2 \rightarrow R_2 + R_1$ and $R_3 \rightarrow R_3 + R_1$, we have:

$$\Delta = a^2 b^2 c^2 \begin{vmatrix} -1 & 1 & 1 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{vmatrix}$$

$$= a^2 b^2 c^2 (-1) \begin{vmatrix} 0 & 2 \\ 2 & 0 \end{vmatrix}$$

$$= -a^2 b^2 c^2 (0-4) = 4a^2 b^2 c^2$$

Q 8:

By using properties of determinants, show that:

(i) $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

(ii) $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$

Answer::

(i) Let $\Delta = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$.

Applying $R_1 \rightarrow R_1 - R_3$ and $R_2 \rightarrow R_2 - R_3$, we have:

$$\Delta = \begin{vmatrix} 0 & a-c & a^2-c^2 \\ 0 & b-c & b^2-c^2 \\ 1 & c & c^2 \end{vmatrix}$$

$$= (c-a)(b-c)\begin{vmatrix} 0 & -1 & -a-c \\ 0 & 1 & b+c \\ 1 & c & c^2 \end{vmatrix}$$

Applying $R_1 \rightarrow R_1 + R_2$, we have:

$$\Delta = (b-c)(c-a)\begin{vmatrix} 0 & 0 & -a+b \\ 0 & 1 & b+c \\ 1 & c & c^2 \end{vmatrix}$$

$$= (a-b)(b-c)(c-a)\begin{vmatrix} 0 & 0 & -1 \\ 0 & 1 & b+c \\ 1 & c & c^2 \end{vmatrix}$$

Expanding along C_1, we have:

$$\Delta = (a-b)(b-c)(c-a)\begin{vmatrix} 0 & -1 \\ 1 & b+c \end{vmatrix} = (a-b)(b-c)(c-a)$$

Hence, the given result is proved.

(ii) Let $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix}$.

Applying $C_1 \rightarrow C_1 - C_3$ and $C_2 \rightarrow C_2 - C_3$, we have:

$$\Delta = \begin{vmatrix} 0 & 0 & 1 \\ a-c & b-c & c \\ a^3-c^3 & b^3-c^3 & c^3 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 0 & 1 \\ a-c & b-c & c \\ (a-c)(a^2+ac+c^2) & (b-c)(b^2+bc+c^2) & c^3 \end{vmatrix}$$

$$= (c-a)(b-c)\begin{vmatrix} 0 & 0 & 1 \\ -1 & 1 & c \\ -(a^2+ac+c^2) & (b^2+bc+c^2) & c^3 \end{vmatrix}$$

Applying $C_1 \to C_1 + C_2$, we have:

$$\Delta = (c-a)(b-c)\begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & c \\ (b^2-a^2)+(bc-ac) & (b^2+bc+c^2) & c^3 \end{vmatrix}$$

$$= (b-c)(c-a)(a-b)\begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & c \\ -(a+b+c) & (b^2+bc+c^2) & c^3 \end{vmatrix}$$

$$= (a-b)(b-c)(c-a)(a+b+c)\begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & c \\ -1 & (b^2+bc+c^2) & c^3 \end{vmatrix}$$

Expanding along C_1, we have:

$$\Delta = (a-b)(b-c)(c-a)(a+b+c)(-1)\begin{vmatrix} 0 & 1 \\ 1 & c \end{vmatrix}$$

$$= (a-b)(b-c)(c-a)(a+b+c)$$

Hence, the given result is proved.

Q 9:

By using properties of determinants, show that:

$$\begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx)$$

Answer::

Let $\Delta = \begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix}$.

Applying $R_2 \rightarrow R_2 - R_1$ and $R_3 \rightarrow R_3 - R_1$, we have:

$$\Delta = \begin{vmatrix} x & x^2 & yz \\ y-x & y^2-x^2 & zx-yz \\ z-x & z^2-x^2 & xy-yz \end{vmatrix}$$

$$= \begin{vmatrix} x & x^2 & yz \\ -(x-y) & -(x-y)(x+y) & z(x-y) \\ (z-x) & (z-x)(z+x) & -y(z-x) \end{vmatrix}$$

$$= (x-y)(z-x)\begin{vmatrix} x & x^2 & yz \\ -1 & -x-y & z \\ 1 & z+x & -y \end{vmatrix}$$

Applying $R_3 \rightarrow R_3 + R_2$, we have:

$$\Delta = (x-y)(z-x)\begin{vmatrix} x & x^2 & yz \\ -1 & -x-y & z \\ 0 & z-y & z-y \end{vmatrix}$$

$$= (x-y)(z-x)(z-y)\begin{vmatrix} x & x^2 & yz \\ -1 & -x-y & z \\ 0 & 1 & 1 \end{vmatrix}$$

Expanding along R_3, we have:

$$\Delta = \left[(x-y)(z-x)(z-y)\right]\left[(-1)\begin{vmatrix} x & yz \\ -1 & z \end{vmatrix} + 1\begin{vmatrix} x & x^2 \\ -1 & -x-y \end{vmatrix}\right]$$

$$= (x-y)(z-x)(z-y)\left[(-xz-yz)+(-x^2-xy+x^2)\right]$$

$$= -(x-y)(z-x)(z-y)(xy+yz+zx)$$

$$= (x-y)(y-z)(z-x)(xy+yz+zx)$$

Hence, the given result is proved.

Q 10:

By using properties of determinants, show that:

(i) $\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5x+4)(4-x)^2$

(ii) $\begin{vmatrix} y+k & y & y \\ y & y+k & y \\ y & y & y+k \end{vmatrix} = k^2(3y+k)$

Answer::

(i) $\Delta = \begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$

Applying $R_1 \rightarrow R_1 + R_2 + R_3$, we have:

$$\Delta = \begin{vmatrix} 5x+4 & 5x+4 & 5x+4 \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$$

$$= (5x+4)\begin{vmatrix} 1 & 1 & 1 \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$$

Applying $C_2 \rightarrow C_2 - C_1$, $C_3 \rightarrow C_3 - C_1$, we have:

$$\Delta = (5x+4)\begin{vmatrix} 1 & 0 & 0 \\ 2x & -x+4 & 0 \\ 2x & 0 & -x+4 \end{vmatrix}$$

$$= (5x+4)(4-x)(4-x)\begin{vmatrix} 1 & 0 & 0 \\ 2x & 1 & 0 \\ 2x & 0 & 1 \end{vmatrix}$$

Expanding along C_3, we have:

$$\Delta = (5x+4)(4-x)^2\begin{vmatrix} 1 & 0 \\ 2x & 1 \end{vmatrix}$$

$$= (5x+4)(4-x)^2$$

Hence, the given result is proved.

(ii) $\Delta = \begin{vmatrix} y+k & y & y \\ y & y+k & y \\ y & y & y+k \end{vmatrix}$

Applying $R_1 \rightarrow R_1 + R_2 + R_3$, we have:

$$\Delta = \begin{vmatrix} 3y+k & 3y+k & 3y+k \\ y & y+k & y \\ y & y & y+k \end{vmatrix}$$

$$= (3y+k)\begin{vmatrix} 1 & 1 & 1 \\ y & y+k & y \\ y & y & y+k \end{vmatrix}$$

Applying $C_2 \rightarrow C_2 - C_1$ and $C_3 \rightarrow C_3 - C_1$, we have:

$$\Delta = (3y+k)\begin{vmatrix} 1 & 0 & 0 \\ y & k & 0 \\ y & 0 & k \end{vmatrix}$$

$$= k^2(3y+k)\begin{vmatrix} 1 & 0 & 0 \\ y & 1 & 0 \\ y & 0 & 1 \end{vmatrix}$$

Expanding along C_3, we have:

$$\Delta = k^2 (3y+k) \begin{vmatrix} 1 & 0 \\ y & 1 \end{vmatrix} = k^2 (3y+k)$$

Hence, the given result is proved.

Q 11:

By using properties of determinants, show that:

(i) $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$

(ii) $\begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix} = 2(x+y+z)^3$

Answer::

(i) $\Delta = \begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$

Applying $R_1 \rightarrow R_1 + R_2 + R_3$, we have:

$$\Delta = \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$$= (a+b+c) \begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

Applying $C_2 \rightarrow C_2 - C_1$, $C_3 \rightarrow C_3 - C_1$, we have:

$$\Delta = (a+b+c) \begin{vmatrix} 1 & 0 & 0 \\ 2b & -(a+b+c) & 0 \\ 2c & 0 & -(a+b+c) \end{vmatrix}$$

$$= (a+b+c)^3 \begin{vmatrix} 1 & 0 & 0 \\ 2b & -1 & 0 \\ 2c & 0 & -1 \end{vmatrix}$$

Expanding along C_3, we have:

$$\Delta = (a+b+c)^3 (-1)(-1) = (a+b+c)^3$$

Hence, the given result is proved.

(ii) $\Delta = \begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix}$

Applying $C_1 \to C_1 + C_2 + C_3$, we have:

$$\Delta = \begin{vmatrix} 2(x+y+z) & x & y \\ 2(x+y+z) & y+z+2x & y \\ 2(x+y+z) & x & z+x+2y \end{vmatrix}$$

$$= 2(x+y+z) \begin{vmatrix} 1 & x & y \\ 1 & y+z+2x & y \\ 1 & x & z+x+2y \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, we have:

$$\Delta = 2(x+y+z) \begin{vmatrix} 1 & x & y \\ 0 & x+y+z & 0 \\ 0 & 0 & x+y+z \end{vmatrix}$$

$$= 2(x+y+z)^3 \begin{vmatrix} 1 & x & y \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Expanding along R_3, we have:

$$\Delta = 2(x+y+z)^3 (1)(1-0) = 2(x+y+z)^3$$

Hence, the given result is proved.

Q 12:

By using properties of determinants, show that:

$$\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (1-x^3)^2$$

Answer::

$$\Delta = \begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix}$$

Applying $R_1 \rightarrow R_1 + R_2 + R_3$, we have:

$$\Delta = \begin{vmatrix} 1+x+x^2 & 1+x+x^2 & 1+x+x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix}$$

$$= (1+x+x^2) \begin{vmatrix} 1 & 1 & 1 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix}$$

Applying $C_2 \rightarrow C_2 - C_1$ and $C_3 \rightarrow C_3 - C_1$, we have:

$$\Delta = (1+x+x^2) \begin{vmatrix} 1 & 0 & 0 \\ x^2 & 1-x^2 & x-x^2 \\ x & x^2-x & 1-x \end{vmatrix}$$

$$= (1+x+x^2)(1-x)(1-x) \begin{vmatrix} 1 & 0 & 0 \\ x^2 & 1+x & x \\ x & -x & 1 \end{vmatrix}$$

$$= (1-x^3)(1-x) \begin{vmatrix} 1 & 0 & 0 \\ x^2 & 1+x & x \\ x & -x & 1 \end{vmatrix}$$

Expanding along R_1, we have:

$$\Delta = (1-x^3)(1-x)(1) \begin{vmatrix} 1+x & x \\ -x & 1 \end{vmatrix}$$

$$= (1-x^3)(1-x)(1+x+x^2)$$

$$= (1-x^3)(1-x^3)$$

$$= (1-x^3)^2$$

Hence, the given result is proved.

Q 13:

By using properties of determinants, show that:

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = \left(1+a^2+b^2\right)^3$$

Answer::

$$\Delta = \begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$$

Applying $R_1 \to R_1 + bR_3$ and $R_2 \to R_2 - aR_3$, we have:

$$\Delta = \begin{vmatrix} 1+a^2+b^2 & 0 & -b\left(1+a^2+b^2\right) \\ 0 & 1+a^2+b^2 & a\left(1+a^2+b^2\right) \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$$

$$= \left(1+a^2+b^2\right)^2 \begin{vmatrix} 1 & 0 & -b \\ 0 & 1 & a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$$

Expanding along R_1, we have:

$$\Delta = \left(1+a^2+b^2\right)^2 \left[(1)\begin{vmatrix} 1 & a \\ -2a & 1-a^2-b^2 \end{vmatrix} - b\begin{vmatrix} 0 & 1 \\ 2b & -2a \end{vmatrix} \right]$$

$$= \left(1+a^2+b^2\right)^2 \left[1-a^2-b^2 + 2a^2 - b(-2b) \right]$$

$$= \left(1+a^2+b^2\right)^2 \left(1+a^2+b^2\right)$$

$$= \left(1+a^2+b^2\right)^3$$

Q 14:

By using properties of determinants, show that:

$$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$$

Answer::

$$\Delta = \begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix}$$

Taking out common factors a, b, and c from R_1, R_2, and R_3 respectively, we have:

$$\Delta = abc \begin{vmatrix} a+\dfrac{1}{a} & b & c \\ a & b+\dfrac{1}{b} & c \\ a & b & c+\dfrac{1}{c} \end{vmatrix}$$

Applying $R_2 \rightarrow R_2 - R_1$ and $R_3 \rightarrow R_3 - R_1$, we have:

$$\Delta = abc \begin{vmatrix} a+\dfrac{1}{a} & b & c \\ -\dfrac{1}{a} & \dfrac{1}{b} & 0 \\ -\dfrac{1}{a} & 0 & \dfrac{1}{c} \end{vmatrix}$$

Applying $C_1 \rightarrow aC_1$, $C_2 \rightarrow bC_2$, and $C_3 \rightarrow cC_3$, we have:

$$\Delta = abc \times \dfrac{1}{abc} \begin{vmatrix} a^2+1 & b^2 & c^2 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} a^2+1 & b^2 & c^2 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix}$$

Expanding along R_3, we have:

$$\Delta = -1 \begin{vmatrix} b^2 & c^2 \\ 1 & 0 \end{vmatrix} + 1 \begin{vmatrix} a^2+1 & b^2 \\ -1 & 1 \end{vmatrix}$$

$$= -1(-c^2) + (a^2+1+b^2) = 1 + a^2 + b^2 + c^2$$

Hence, the given result is proved.

Q 15:

Choose the correct
Answer:.
Let A be a square matrix of order 3 × 3, then $|kA|$ is equal to

A. $k|A|$ **B.** $k^2|A|$ **C.** $k^3|A|$ **D.** $3k|A|$

Answer:**Answer:: C**

A is a square matrix of order 3 × 3.

$$\text{Let } A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}.$$

$$\text{Then, } kA = \begin{bmatrix} ka_1 & kb_1 & kc_1 \\ ka_2 & kb_2 & kc_2 \\ ka_3 & kb_3 & kc_3 \end{bmatrix}.$$

$$\therefore |kA| = \begin{vmatrix} ka_1 & kb_1 & kc_1 \\ ka_2 & kb_2 & kc_2 \\ ka_3 & kb_3 & kc_3 \end{vmatrix}$$

$$= k^3 \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \qquad \text{(Taking out common factors } k \text{ from each row)}$$

$$= k^3 |A|$$

$$\therefore |kA| = k^3 |A|$$

Hence, the correct Answer: is C.

Q 16:

Which of the following is correct?

A. Determinant is a square matrix.

B. Determinant is a number associated to a matrix.

C. Determinant is a number associated to a square matrix.

D. None of these

Answer:

Answer::

C

We know that to every square matrix, $A = \left[aij \right]$ of order n. We can associate a number

called the determinant of square matrix A, where $aij = \left(i, j \right)^{th}$ element of A.

Thus, the determinant is a number associated to a square matrix.

Hence, the correct Answer: is C.

Exercise 4.3

Q 1:

Find area of the triangle with vertices at the point given in each of the following:

(i) (1, 0), (6, 0), (4, 3)   (ii) (2, 7), (1, 1), (10, 8)

(iii) (−2, −3), (3, 2), (−1, −8)

Answer:

(i) The area of the triangle with vertices (1, 0), (6, 0), (4, 3) is given by the relation,

$$\Delta = \frac{1}{2}\begin{vmatrix} 1 & 0 & 1 \\ 6 & 0 & 1 \\ 4 & 3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[1(0-3)-0(6-4)+1(18-0)\right]$$

$$= \frac{1}{2}[-3+18] = \frac{15}{2} \text{ square units}$$

(ii) The area of the triangle with vertices (2, 7), (1, 1), (10, 8) is given by the relation,

$$\Delta = \frac{1}{2}\begin{vmatrix} 2 & 7 & 1 \\ 1 & 1 & 1 \\ 10 & 8 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[2(1-8)-7(1-10)+1(8-10)\right]$$

$$= \frac{1}{2}\left[2(-7)-7(-9)+1(-2)\right]$$

$$= \frac{1}{2}[-14+63-2] = \frac{1}{2}[-16+63]$$

$$= \frac{47}{2} \text{ square units}$$

(iii) The area of the triangle with vertices (−2, −3), (3, 2), (−1, −8) is given by the relation,

$$\Delta = \frac{1}{2} \begin{vmatrix} -2 & -3 & 1 \\ 3 & 2 & 1 \\ -1 & -8 & 1 \end{vmatrix}$$

$$= \frac{1}{2} \left[-2(2+8) + 3(3+1) + 1(-24+2) \right]$$

$$= \frac{1}{2} \left[-2(10) + 3(4) + 1(-22) \right]$$

$$= \frac{1}{2} \left[-20 + 12 - 22 \right]$$

$$= -\frac{30}{2} = -15$$

Hence, the area of the triangle is $|-15| = 15$ square units.

Q 2:

Show that points

$A(a, b+c), B(b, c+a), C(c, a+b)$ are collinear

Answer::

Area of ΔABC is given by the relation,

$$\Delta = \frac{1}{2} \begin{vmatrix} a & b+c & 1 \\ b & c+a & 1 \\ c & a+b & 1 \end{vmatrix}$$

$$= \frac{1}{2} \begin{vmatrix} a & b+c & 1 \\ b-a & a-b & 0 \\ c-a & a-c & 0 \end{vmatrix} \quad (\text{Applying } R_2 \to R_2 - R_1 \text{ and } R_3 \to R_3 - R_1)$$

$$= \frac{1}{2}(a-b)(c-a) \begin{vmatrix} a & b+c & 1 \\ -1 & 1 & 0 \\ 1 & -1 & 0 \end{vmatrix}$$

$$= \frac{1}{2}(a-b)(c-a) \begin{vmatrix} a & b+c & 1 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{vmatrix} \quad (\text{Applying } R_3 \to R_3 + R_2)$$

$$= 0 \qquad (\text{All elements of } R_3 \text{ are } 0)$$

Thus, the area of the triangle formed by points A, B, and C is zero.

Hence, the points A, B, and C are collinear.

Q 3:

Find values of k if area of triangle is 4 square units and vertices are

(i) $(k, 0)$, $(4, 0)$, $(0, 2)$ (ii) $(-2, 0)$, $(0, 4)$, $(0, k)$

Answer:

We know that the area of a triangle whose vertices are (x_1, y_1), (x_2, y_2), and (x_3, y_3) is the absolute value of the determinant (Δ), where

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

It is given that the area of triangle is 4 square units.

$\therefore \Delta = \pm 4$.

(i) The area of the triangle with vertices $(k, 0)$, $(4, 0)$, $(0, 2)$ is given by the relation,

$$\Delta = \frac{1}{2}\begin{vmatrix} k & 0 & 1 \\ 4 & 0 & 1 \\ 0 & 2 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[k(0-2)-0(4-0)+1(8-0)\right]$$

$$= \frac{1}{2}\left[-2k+8\right] = -k+4$$

$\therefore -k + 4 = \pm 4$

When $-k + 4 = -4$, $k = 8$.

When $-k + 4 = 4$, $k = 0$.

Hence, $k = 0, 8$.

(ii) The area of the triangle with vertices (−2, 0), (0, 4), (0, *k*) is given by the relation,

$$\Delta = \frac{1}{2} \begin{vmatrix} -2 & 0 & 1 \\ 0 & 4 & 1 \\ 0 & k & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[-2(4-k)\right]$$

$$= k - 4$$

∴*k* − 4 = ± 4

When *k* − 4 = − 4, *k* = 0.

When *k* − 4 = 4, *k* = 8.

Hence, *k* = 0, 8.

Q 4:

(i) Find equation of line joining (1, 2) and (3, 6) using determinants(ii) Find equation of line joining (3, 1) and (9, 3) using determinants

Answer::

(i) Let P (*x*, *y*) be any point on the line joining points A (1, 2) and B (3, 6). Then, the points A, B, and P are collinear. Therefore, the area of triangle ABP will be zero.

$$\therefore \frac{1}{2} \begin{vmatrix} 1 & 2 & 1 \\ 3 & 6 & 1 \\ x & y & 1 \end{vmatrix} = 0$$

$$\Rightarrow \frac{1}{2}\left[1(6-y)-2(3-x)+1(3y-6x)\right]=0$$

$$\Rightarrow 6-y-6+2x+3y-6x=0$$

$$\Rightarrow 2y-4x=0$$

$$\Rightarrow y=2x$$

Hence, the equation of the line joining the given points is *y* = 2*x*.

(ii) Let P (*x*, *y*) be any point on the line joining points A (3, 1) and

B (9, 3). Then, the points A, B, and P are collinear. Therefore, the area of triangle ABP will be zero.

$$\therefore \frac{1}{2}\begin{vmatrix} 3 & 1 & 1 \\ 9 & 3 & 1 \\ x & y & 1 \end{vmatrix} = 0$$

$$\Rightarrow \frac{1}{2}\left[3(3-y)-1(9-x)+1(9y-3x)\right]=0$$

$$\Rightarrow 9-3y-9+x+9y-3x=0$$

$$\Rightarrow 6y-2x=0$$

$$\Rightarrow x-3y=0$$

Hence, the equation of the line joining the given points is $x - 3y = 0$.

Q 5:

If area of triangle is 35 square units with vertices (2, −6), (5, 4), and (k, 4). Then k is

A. 12 **B.** −2 **C.** −12, −2 **D.** 12, −2

Answer::

Answer:: D

The area of the triangle with vertices (2, −6), (5, 4), and (k, 4) is given by the relation,

$$\Delta = \frac{1}{2}\begin{vmatrix} 2 & -6 & 1 \\ 5 & 4 & 1 \\ k & 4 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[2(4-4)+6(5-k)+1(20-4k)\right]$$

$$= \frac{1}{2}\left[30-6k+20-4k\right]$$

$$= \frac{1}{2}\left[50-10k\right]$$

$$= 25-5k$$

It is given that the area of the triangle is ±35.

Therefore, we have:

$$\Rightarrow 25-5k=\pm 35$$

$$\Rightarrow 5(5-k)=\pm 35$$

$$\Rightarrow 5-k=\pm 7$$

When $5 - k = -7$, $k = 5 + 7 = 12$.

When $5 - k = 7$, $k = 5 - 7 = -2$.

Hence, $k = 12, -2$.

The correct Answer: is D.

Exercise 4.4

Q 1:

Write Minors and Cofactors of the elements of following determinants:

(i) $\begin{vmatrix} 2 & -4 \\ 0 & 3 \end{vmatrix}$ (ii) $\begin{vmatrix} a & c \\ b & d \end{vmatrix}$

Answer::

(i) The given determinant is $\begin{vmatrix} 2 & -4 \\ 0 & 3 \end{vmatrix}$.

Minor of element a_{ij} is M_{ij}.

$\therefore M_{11}$ = minor of element a_{11} = 3

M_{12} = minor of element a_{12} = 0

M_{21} = minor of element a_{21} = -4

M_{22} = minor of element a_{22} = 2

Cofactor of a_{ij} is $A_{ij} = (-1)^{i+j} M_{ij}$.

$\therefore A_{11} = (-1)^{1+1} M_{11} = (-1)^2 (3) = 3$

$A_{12} = (-1)^{1+2} M_{12} = (-1)^3 (0) = 0$

$A_{21} = (-1)^{2+1} M_{21} = (-1)^3 (-4) = 4$

$A_{22} = (-1)^{2+2} M_{22} = (-1)^4 (2) = 2$

(ii) The given determinant is $\begin{vmatrix} a & c \\ b & d \end{vmatrix}$.

Minor of element a_{ij} is M_{ij}.

∴M_{11} = minor of element a_{11} = d

M_{12} = minor of element a_{12} = b

M_{21} = minor of element a_{21} = c

M_{22} = minor of element a_{22} = a

Cofactor of a_{ij} is A_{ij} = $(-1)^{i+j} M_{ij}$.

∴A_{11} = $(-1)^{1+1} M_{11}$ = $(-1)^2 (d)$ = d

A_{12} = $(-1)^{1+2} M_{12}$ = $(-1)^3 (b)$ = $-b$

A_{21} = $(-1)^{2+1} M_{21}$ = $(-1)^3 (c)$ = $-c$

A_{22} = $(-1)^{2+2} M_{22}$ = $(-1)^4 (a)$ = a

Q 2:

(i) $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$ (ii) $\begin{vmatrix} 1 & 0 & 4 \\ 3 & 5 & -1 \\ 0 & 1 & 2 \end{vmatrix}$

Answer::

(i) The given determinant is $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$.

By the definition of minors and cofactors, we have:

M_{11} = minor of a_{11}= $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$ = 1

M_{12} = minor of a_{12}= $\begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix}$ = 0

M_{13} = minor of a_{13} = $\begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} = 0$

M_{21} = minor of a_{21} = $\begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0$

M_{22} = minor of a_{22} = $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$

M_{23} = minor of a_{23} = $\begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 0$

M_{31} = minor of a_{31} = $\begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} = 0$

M_{32} = minor of a_{32} = $\begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 0$

M_{33} = minor of a_{33} = $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$

A_{11} = cofactor of a_{11} = $(-1)^{1+1} M_{11}$ = 1

A_{12} = cofactor of a_{12} = $(-1)^{1+2} M_{12}$ = 0

A_{13} = cofactor of a_{13} = $(-1)^{1+3} M_{13}$ = 0

A_{21} = cofactor of a_{21} = $(-1)^{2+1} M_{21}$ = 0

A_{22} = cofactor of a_{22} = $(-1)^{2+2} M_{22}$ = 1

A_{23} = cofactor of a_{23} = $(-1)^{2+3} M_{23}$ = 0

A_{31} = cofactor of a_{31} = $(-1)^{3+1} M_{31}$ = 0

A_{32} = cofactor of a_{32} = $(-1)^{3+2} M_{32}$ = 0

A_{33} = cofactor of a_{33} = $(-1)^{3+3} M_{33}$ = 1

(ii) The given determinant is $\begin{vmatrix} 1 & 0 & 4 \\ 3 & 5 & -1 \\ 0 & 1 & 2 \end{vmatrix}$.

By definition of minors and cofactors, we have:

M_{11} = minor of a_{11} = $\begin{vmatrix} 5 & -1 \\ 1 & 2 \end{vmatrix} = 10 + 1 = 11$

M_{12} = minor of a_{12} = $\begin{vmatrix} 3 & -1 \\ 0 & 2 \end{vmatrix} = 6 - 0 = 6$

M_{13} = minor of a_{13} = $\begin{vmatrix} 3 & 5 \\ 0 & 1 \end{vmatrix} = 3 - 0 = 3$

M_{21} = minor of a_{21} = $\begin{vmatrix} 0 & 4 \\ 1 & 2 \end{vmatrix} = 0 - 4 = -4$

M_{22} = minor of a_{22} = $\begin{vmatrix} 1 & 4 \\ 0 & 2 \end{vmatrix} = 2 - 0 = 2$

M_{23} = minor of a_{23} = $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1$

M_{31} = minor of a_{31} = $\begin{vmatrix} 0 & 4 \\ 5 & -1 \end{vmatrix} = 0 - 20 = -20$

M_{32} = minor of a_{32} = $\begin{vmatrix} 1 & 4 \\ 3 & -1 \end{vmatrix} = -1 - 12 = -13$

M_{33} = minor of a_{33} = $\begin{vmatrix} 1 & 0 \\ 3 & 5 \end{vmatrix} = 5 - 0 = 5$

A_{11} = cofactor of a_{11} = $(-1)^{1+1} M_{11}$ = 11

A_{12} = cofactor of a_{12} = $(-1)^{1+2} M_{12}$ = -6

A_{13} = cofactor of a_{13} = $(-1)^{1+3} M_{13}$ = 3

A_{21} = cofactor of a_{21} = $(-1)^{2+1} M_{21}$ = 4

A_{22} = cofactor of a_{22} = $(-1)^{2+2} M_{22}$ = 2

A_{23} = cofactor of a_{23} = $(-1)^{2+3} M_{23}$ = -1

A_{31} = cofactor of a_{31} = $(-1)^{3+1} M_{31}$ = -20

A_{32} = cofactor of a_{32} = $(-1)^{3+2} M_{32}$ = 13

A_{33} = cofactor of a_{33} = $(-1)^{3+3} M_{33}$ = 5

Q 3:

Using Cofactors of elements of second row, evaluate $\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$.

Answer:

The given determinant is $\begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$.

We have:

$$M_{21} = \begin{vmatrix} 3 & 8 \\ 2 & 3 \end{vmatrix} = 9 - 16 = -7$$

$\therefore A_{21}$ = cofactor of $a_{21} = (-1)^{2+1} M_{21} = 7$

$$M_{22} = \begin{vmatrix} 5 & 8 \\ 1 & 3 \end{vmatrix} = 15 - 8 = 7$$

$\therefore A_{22}$ = cofactor of $a_{22} = (-1)^{2+2} M_{22} = 7$

$$M_{23} = \begin{vmatrix} 5 & 3 \\ 1 & 2 \end{vmatrix} = 10 - 3 = 7$$

$\therefore A_{23}$ = cofactor of $a_{23} = (-1)^{2+3} M_{23} = -7$

We know that Δ is equal to the sum of the product of the elements of the second row with their corresponding cofactors.

$\therefore \Delta = a_{21}A_{21} + a_{22}A_{22} + a_{23}A_{23} = 2(7) + 0(7) + 1(-7) = 14 - 7 = 7$

Q 4:

Using Cofactors of elements of third column, evaluate $\Delta = \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix}$

Answer::

The given determinant is $\begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix}$.

We have:

$$M_{13} = \begin{vmatrix} 1 & y \\ 1 & z \end{vmatrix} = z - y$$

$$M_{23} = \begin{vmatrix} 1 & x \\ 1 & z \end{vmatrix} = z - x$$

$$M_{33} = \begin{vmatrix} 1 & x \\ 1 & y \end{vmatrix} = y - x$$

$\therefore A_{13} = $ cofactor of $a_{13} = (-1)^{1+3} M_{13} = (z - y)$

$A_{23} = $ cofactor of $a_{23} = (-1)^{2+3} M_{23} = -(z - x) = (x - z)$

$A_{33} = $ cofactor of $a_{33} = (-1)^{3+3} M_{33} = (y - x)$

We know that Δ is equal to the sum of the product of the elements of the second row with their corresponding cofactors.

$$\therefore \Delta = a_{13}A_{13} + a_{23}A_{23} + a_{33}A_{33}$$
$$= yz(z-y) + zx(x-z) + xy(y-x)$$
$$= yz^2 - y^2z + x^2z - xz^2 + xy^2 - x^2y$$
$$= (x^2z - y^2z) + (yz^2 - xz^2) + (xy^2 - x^2y)$$
$$= z(x^2 - y^2) + z^2(y - x) + xy(y - x)$$
$$= z(x-y)(x+y) + z^2(y-x) + xy(y-x)$$
$$= (x-y)\left[zx + zy - z^2 - xy\right]$$
$$= (x-y)\left[z(x-z) + y(z-x)\right]$$
$$= (x-y)(z-x)[-z+y]$$
$$= (x-y)(y-z)(z-x)$$

Hence, $\Delta = (x-y)(y-z)(z-x)$.

Q 5:

For the matrices A and B, verify that $(AB)' = B'A'$ where

(i) $A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}$

(ii) $A = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 5 & 7 \end{bmatrix}$

Answer::

(i) $AB = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix} \begin{bmatrix} -1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 2 & 1 \\ 4 & -8 & -4 \\ -3 & 6 & 3 \end{bmatrix}$

$\therefore (AB)' = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$

Now, $A' = \begin{bmatrix} 1 & -4 & 3 \end{bmatrix}$, $B' = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$

$\therefore B'A' = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & -4 & 3 \end{bmatrix} = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$

Hence, we have verified that $(AB)' = B'A'$.

(ii) $AB = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & 5 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 5 & 7 \\ 2 & 10 & 14 \end{bmatrix}$

$\therefore (AB)' = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 5 & 10 \\ 0 & 7 & 14 \end{bmatrix}$

Now, $A' = \begin{bmatrix} 0 & 1 & 2 \end{bmatrix}$, $B' = \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix}$

$\therefore B'A' = \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 5 & 10 \\ 0 & 7 & 14 \end{bmatrix}$

Hence, we have verified that $(AB)' = B'A'$.

Exercise 4.5

Q 1:

Find adjoint of each of the matrices.

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Answer::

Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

We have,

$A_{11} = 4,\ A_{12} = -3,\ A_{21} = -2,\ A_{22} = 1$

$\therefore adjA = \begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

Q 2:

Find adjoint of each of the matrices.

$$\begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 5 \\ -2 & 0 & 1 \end{bmatrix}$$

Answer::

Let $A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 5 \\ -2 & 0 & 1 \end{bmatrix}$.

We have,

$A_{11} = \begin{vmatrix} 3 & 5 \\ 0 & 1 \end{vmatrix} = 3 - 0 = 3$

$A_{12} = -\begin{vmatrix} 2 & 5 \\ -2 & 1 \end{vmatrix} = -(2 + 10) = -12$

$A_{13} = \begin{vmatrix} 2 & 3 \\ -2 & 0 \end{vmatrix} = 0 + 6 = 6$

$$A_{21} = -\begin{vmatrix} -1 & 2 \\ 0 & 1 \end{vmatrix} = -(-1-0) = 1$$

$$A_{22} = \begin{vmatrix} 1 & 2 \\ -2 & 1 \end{vmatrix} = 1+4 = 5$$

$$A_{23} = -\begin{vmatrix} 1 & -1 \\ -2 & 0 \end{vmatrix} = -(0-2) = 2$$

$$A_{31} = \begin{vmatrix} -1 & 2 \\ 3 & 5 \end{vmatrix} = -5-6 = -11$$

$$A_{32} = -\begin{vmatrix} 1 & 2 \\ 2 & 5 \end{vmatrix} = -(5-4) = -1$$

$$A_{33} = \begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix} = 3+2 = 5$$

Hence, $adj A = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \begin{bmatrix} 3 & 1 & -11 \\ -12 & 5 & -1 \\ 6 & 2 & 5 \end{bmatrix}.$

Q 3:

Verify $A (adj\ A) = (adj\ A) A = |A| I$.

$$\begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$$

Answer::

$$A = \begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$$

we have,

$$|A| = -12 - (-12) = -12 + 12 = 0$$

$$\therefore |A| I = 0 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Now,

$$A_{11} = -6, A_{12} = 4, A_{21} = -3, A_{22} = 2$$

$$\therefore adjA = \begin{bmatrix} -6 & -3 \\ 4 & 2 \end{bmatrix}$$

Now,

$$A(adjA) = \begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix} \begin{bmatrix} -6 & -3 \\ 4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -12+12 & -6+6 \\ 24-24 & 12-12 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Also, $(adjA) A = \begin{bmatrix} -6 & -3 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$

$$= \begin{bmatrix} -12+12 & -18+18 \\ 8-8 & 12-12 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Hence, $A(adjA) = (adjA) A = |A| I$.

Q 4:

Verify $A\ (adj\ A) = (adj\ A)\ A = |A|\,I$.

$$\begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$$

Answer::

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$$

$$|A| = 1(0-0) + 1(9+2) + 2(0-0) = 11$$

$$\therefore |A|\,I = 11\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

Now,

$$A_{11} = 0,\ A_{12} = -(9+2) = -11,\ A_{13} = 0$$

$$A_{21} = -(-3-0) = 3,\ A_{22} = 3-2 = 1,\ A_{23} = -(0+1) = -1$$

$$A_{31} = 2-0 = 2,\ A_{32} = -(-2-6) = 8,\ A_{33} = 0+3 = 3$$

$$\therefore adjA = \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix}$$

Now,

$$A(adjA) = \begin{vmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{vmatrix} \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0+11+0 & 3-1-2 & 2-8+6 \\ 0+0+0 & 9+0+2 & 6+0-6 \\ 0+0+0 & 3+0-3 & 2+0+9 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

Also,

$$(adjA) \cdot A = \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0+9+2 & 0+0+0 & 0-6+6 \\ -11+3+8 & 11+0+0 & -22-2+24 \\ 0-3+3 & 0+0+0 & 0+2+9 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

Hence, $A(adjA) = (adjA)A = |A|I$.

Q 6:

Find the inverse of each of the matrices (if it exists).

$$\begin{bmatrix} -1 & 5 \\ -3 & 2 \end{bmatrix}$$

Answer:

Let $A = \begin{bmatrix} -1 & 5 \\ -3 & 2 \end{bmatrix}$.

we have,

$|A| = -2 + 15 = 13$

Now,

$A_{11} = 2, A_{12} = 3, A_{21} = -5, A_{22} = -1$

$\therefore adjA = \begin{bmatrix} 2 & -5 \\ 3 & -1 \end{bmatrix}$

$\therefore A^{-1} = \dfrac{1}{|A|} adjA = \dfrac{1}{13} \begin{bmatrix} 2 & -5 \\ 3 & -1 \end{bmatrix}$

Q 7:

Find the inverse of each of the matrices (if it exists).

$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix}$

Answer:

Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix}$.

We have,

$|A| = 1(10 - 0) - 2(0 - 0) + 3(0 - 0) = 10$

Now,

$A_{11} = 10 - 0 = 10, A_{12} = -(0 - 0) = 0, A_{13} = 0 - 0 = 0$

$A_{21} = -(10 - 0) = -10, A_{22} = 5 - 0 = 5, A_{23} = -(0 - 0) = 0$

$A_{31} = 8 - 6 = 2, A_{32} = -(4 - 0) = -4, A_{33} = 2 - 0 = 2$

$$\therefore adjA = \begin{bmatrix} 10 & -10 & 2 \\ 0 & 5 & -4 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} adjA = \frac{1}{10} \begin{bmatrix} 10 & -10 & 2 \\ 0 & 5 & -4 \\ 0 & 0 & 2 \end{bmatrix}$$

Q 8:

Find the inverse of each of the matrices (if it exists).

$$\begin{bmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{bmatrix}$$

Answer:

$$Let\ A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{bmatrix}.$$

We have,

$$|A| = 1(-3-0)-0+0 = -3$$

Now,

$$A_{11} = -3-0 = -3, A_{12} = -(-3-0) = 3, A_{13} = 6-15 = -9$$

$$A_{21} = -(0-0) = 0, A_{22} = -1-0 = -1, A_{23} = -(2-0) = -2$$

$$A_{31} = 0-0 = 0, A_{32} = -(0-0) = 0, A_{33} = 3-0 = 3$$

$$\therefore adjA = \begin{bmatrix} -3 & 0 & 0 \\ 3 & -1 & 0 \\ -9 & -2 & 3 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} adjA = -\frac{1}{3} \begin{bmatrix} -3 & 0 & 0 \\ 3 & -1 & 0 \\ -9 & -2 & 3 \end{bmatrix}$$

Q 9:

Find the inverse of each of the matrices (if it exists).

$$\begin{bmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ -7 & 2 & 1 \end{bmatrix}$$

Answer:

Let $A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ -7 & 2 & 1 \end{bmatrix}$.

We have,

$$|A| = 2(-1-0) - 1(4-0) + 3(8-7)$$
$$= 2(-1) - 1(4) + 3(1)$$
$$= -2 - 4 + 3$$
$$= -3$$

Now,

$$A_{11} = -1 - 0 = -1, \ A_{12} = -(4-0) = -4, \ A_{13} = 8 - 7 = 1$$
$$A_{21} = -(1-6) = 5, \ A_{22} = 2 + 21 = 23, \ A_{23} = -(4+7) = -11$$
$$A_{31} = 0 + 3 = 3, \ A_{32} = -(0-12) = 12, \ A_{33} = -2 - 4 = -6$$

$$\therefore adjA = \begin{bmatrix} -1 & 5 & 3 \\ -4 & 23 & 12 \\ 1 & -11 & -6 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} adjA = -\frac{1}{3} \begin{bmatrix} -1 & 5 & 3 \\ -4 & 23 & 12 \\ 1 & -11 & -6 \end{bmatrix}$$

Q 10:

Find the inverse of each of the matrices (if it exists).

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}.$$

Answer:

Let $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$.

By expanding along C_1, we have:

$$|A| = 1(8-6) - 0 + 3(3-4) = 2 - 3 = -1$$

Now,

$$A_{11} = 8 - 6 = 2, A_{12} = -(0+9) = -9, A_{13} = 0 - 6 = -6$$

$$A_{21} = -(-4+4) = 0, A_{22} = 4 - 6 = -2, A_{23} = -(-2+3) = -1$$

$$A_{31} = 3 - 4 = -1, A_{32} = -(-3-0) = 3, A_{33} = 2 - 0 = 2$$

$$\therefore adjA = \begin{bmatrix} 2 & 0 & -1 \\ -9 & -2 & 3 \\ -6 & -1 & 2 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} adjA = -\begin{bmatrix} 2 & 0 & -1 \\ -9 & -2 & 3 \\ -6 & -1 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$$

Q 11:

Find the inverse of each of the matrices (if it exists).

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & \sin\alpha & -\cos\alpha \end{bmatrix}$$

Answer:

Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & \sin\alpha & -\cos\alpha \end{bmatrix}$.

We have,

$$|A| = 1\left(-\cos^2\alpha - \sin^2\alpha\right) = -\left(\cos^2\alpha + \sin^2\alpha\right) = -1$$

Now,

$$A_{11} = -\cos^2\alpha - \sin^2\alpha = -1,\ A_{12} = 0,\ A_{13} = 0$$
$$A_{21} = 0,\ A_{22} = -\cos\alpha,\ A_{23} = -\sin\alpha$$
$$A_{31} = 0,\ A_{32} = -\sin\alpha,\ A_{33} = \cos\alpha$$

$$\therefore adjA = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos\alpha & -\sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} \cdot adjA = -\begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos\alpha & -\sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & \sin\alpha & -\cos\alpha \end{bmatrix}$$

Q 12:

Let $A = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$. Verify that $(AB)^{-1} = B^{-1}A^{-1}$

Answer:

Let $A = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$.

We have,

$|A| = 15 - 14 = 1$

Now,

$A_{11} = 5, A_{12} = -2, A_{21} = -7, A_{22} = 3$

$\therefore adjA = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$

$\therefore A^{-1} = \dfrac{1}{|A|} \cdot adjA = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$

Now, let $B = \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$.

We have,

$|B| = 54 - 56 = -2$

$\therefore adjB = \begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix}$

$\therefore B^{-1} = \dfrac{1}{|B|} adjB = -\dfrac{1}{2}\begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix} = \begin{bmatrix} -\dfrac{9}{2} & 4 \\ \dfrac{7}{2} & -3 \end{bmatrix}$

Now,

$B^{-1}A^{-1} = \begin{bmatrix} -\dfrac{9}{2} & 4 \\ \dfrac{7}{2} & -3 \end{bmatrix}\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$

$= \begin{bmatrix} -\dfrac{45}{2} - 8 & \dfrac{63}{2} + 12 \\ \dfrac{35}{2} + 6 & -\dfrac{49}{2} - 9 \end{bmatrix} = \begin{bmatrix} -\dfrac{61}{2} & \dfrac{87}{2} \\ \dfrac{47}{2} & -\dfrac{67}{2} \end{bmatrix}$ ...(1)

Then,

$$AB = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 18+49 & 24+63 \\ 12+35 & 16+45 \end{bmatrix}$$

$$= \begin{bmatrix} 67 & 87 \\ 47 & 61 \end{bmatrix}$$

Therefore, we have $|AB| = 67 \times 61 - 87 \times 47 = 4087 - 4089 = -2$.

Also,

$$adj(AB) = \begin{bmatrix} 61 & -87 \\ -47 & 67 \end{bmatrix}$$

$$\therefore (AB)^{-1} = \frac{1}{|AB|}adj(AB) = -\frac{1}{2}\begin{bmatrix} 61 & -87 \\ -47 & 67 \end{bmatrix}$$

$$= \begin{bmatrix} -\dfrac{61}{2} & \dfrac{87}{2} \\ \dfrac{47}{2} & -\dfrac{67}{2} \end{bmatrix} \quad ...(2)$$

From (1) and (2), we have:

$(AB)^{-1} = B^{-1}A^{-1}$

Hence, the given result is proved.

Q 13:

If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = O$. Hence find A^{-1}.

Answer:

$A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$A^2 = A \cdot A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$

$\therefore A^2 - 5A + 7I$

$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$= \begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Hence, $A^2 - 5A + 7I = O$.

$\therefore A \cdot A - 5A = -7I$

$\Rightarrow A \cdot A\left(A^{-1}\right) - 5AA^{-1} = -7IA^{-1}$ $\left[\text{Post-multiplying by } A^{-1} \text{ as } |A| \neq 0\right]$

$\Rightarrow A\left(AA^{-1}\right) - 5I = -7A^{-1}$

$\Rightarrow AI - 5I = -7A^{-1}$

$\Rightarrow A^{-1} = -\dfrac{1}{7}\left(A - 5I\right)$

$\Rightarrow A^{-1} = \dfrac{1}{7}\left(5I - A\right)$

$= \dfrac{1}{7}\left(\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\right) = \dfrac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$

$\therefore A^{-1} = \dfrac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$

Q 14:

For the matrix $A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$, find the numbers a and b such that $A^2 + aA + bI = O$.

Answer:

$$A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$$

$$\therefore A^2 = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 9+2 & 6+2 \\ 3+1 & 2+1 \end{bmatrix} = \begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix}$$

Now,

$$A^2 + aA + bI = O$$

$$\Rightarrow (AA)A^{-1} + aAA^{-1} + bIA^{-1} = O \qquad \left[\text{Post-multiplying by } A^{-1} \text{ as } |A| \neq 0 \right]$$

$$\Rightarrow A\left(AA^{-1}\right) + aI + b\left(IA^{-1}\right) = O$$

$$\Rightarrow AI + aI + bA^{-1} = O$$

$$\Rightarrow A + aI = -bA^{-1}$$

$$\Rightarrow A^{-1} = -\frac{1}{b}(A + aI)$$

Now,

$$A^{-1} = \frac{1}{|A|} adjA = \frac{1}{1} \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$$

We have:

$$\begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix} = -\frac{1}{b}\left(\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix} \right) = -\frac{1}{b} \begin{bmatrix} 3+a & 2 \\ 1 & 1+a \end{bmatrix} = \begin{bmatrix} \dfrac{-3-a}{b} & -\dfrac{2}{b} \\ -\dfrac{1}{b} & \dfrac{-1-a}{b} \end{bmatrix}$$

Comparing the corresponding elements of the two matrices, we have:

$$-\frac{1}{b} = -1 \Rightarrow b = 1$$

$$\frac{-3-a}{b} = 1 \Rightarrow -3-a = 1 \Rightarrow a = -4$$

Hence, −4 and 1 are the required values of a and b respectively.

Q 15:

For the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$ show that $A^3 - 6A^2 + 5A + 11\,I = O$. Hence, find A^{-1}.

Answer:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1+2 & 1+2-1 & 1-3+3 \\ 1+2-6 & 1+4+3 & 1-6-9 \\ 2-1+6 & 2-2-3 & 2+3+9 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}$$

$$A^3 = A^2 \cdot A = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 4+2+2 & 4+4-1 & 4-6+3 \\ -3+8-28 & -3+16+14 & -3-24-42 \\ 7-3+28 & 7-6-14 & 7+9+42 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix}$$

$\therefore A^3 - 6A^2 + 5A + 11I$

$$= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix} - 6\begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix} + 5\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} + 11\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix} - \begin{bmatrix} 24 & 12 & 6 \\ -18 & 48 & -84 \\ 42 & -18 & 84 \end{bmatrix} + \begin{bmatrix} 5 & 5 & 5 \\ 5 & 10 & -15 \\ 10 & -5 & 15 \end{bmatrix} + \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

$$= \begin{bmatrix} 24 & 12 & 6 \\ -18 & 48 & -84 \\ 42 & -18 & 84 \end{bmatrix} - \begin{bmatrix} 24 & 12 & 6 \\ -18 & 48 & -84 \\ 42 & -18 & 84 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O$$

Thus, $A^3 - 6A^2 + 5A + 11I = O$.

Now,

$A^3 - 6A^2 + 5A + 11I = O$

$\Rightarrow (AAA)A^{-1} - 6(AA)A^{-1} + 5AA^{-1} + 11IA^{-1} = 0$ $\qquad$ [Post-multiplying by A^{-1} as $|A| \neq 0$]

$\Rightarrow AA(AA^{-1}) - 6A(AA^{-1}) + 5(AA^{-1}) = -11(IA^{-1})$

$\Rightarrow A^2 - 6A + 5I = -11A^{-1}$

$\Rightarrow A^{-1} = -\dfrac{1}{11}(A^2 - 6A + 5I)$ $\qquad$ ...(1)

Now,

$A^2 - 6A + 5I$

$$= \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix} - 6\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} + 5\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix} - \begin{bmatrix} 6 & 6 & 6 \\ 6 & 12 & -18 \\ 12 & -6 & 18 \end{bmatrix} + \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 2 & 1 \\ -3 & 13 & -14 \\ 7 & -3 & 19 \end{bmatrix} - \begin{bmatrix} 6 & 6 & 6 \\ 6 & 12 & -18 \\ 12 & -6 & 18 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -4 & -5 \\ -9 & 1 & 4 \\ -5 & 3 & 1 \end{bmatrix}$$

From equation (1), we have:

$$A^{-1} = -\frac{1}{11}\begin{bmatrix} 3 & -4 & -5 \\ -9 & 1 & 4 \\ -5 & 3 & 1 \end{bmatrix} = \frac{1}{11}\begin{bmatrix} -3 & 4 & 5 \\ 9 & -1 & -4 \\ 5 & -3 & -1 \end{bmatrix}$$

Q 16:

If $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ verify that $A^3 - 6A^2 + 9A - 4I = O$ and hence find A^{-1}

Answer:

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4+1+1 & -2-2-1 & 2+1+2 \\ -2-2-1 & 1+4+1 & -1-2-2 \\ 2+1+2 & -1-2-2 & 1+1+4 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}$$

$$A^3 = A^2 A = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 12+5+5 & -6-10-5 & 6+5+10 \\ -10-6-5 & 5+12+5 & -5-6-10 \\ 10+5+6 & -5-10-6 & 5+5+12 \end{bmatrix}$$

$$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix}$$

Now,

$A^3 - 6A^2 + 9A - 4I$

$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} - 6\begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} + 9\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} - 4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} - \begin{bmatrix} 36 & -30 & 30 \\ -30 & 36 & -30 \\ 30 & -30 & 36 \end{bmatrix} + \begin{bmatrix} 18 & -9 & 9 \\ -9 & 18 & -9 \\ 9 & -9 & 18 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

$= \begin{bmatrix} 40 & -30 & 30 \\ -30 & 40 & -30 \\ 30 & -30 & 40 \end{bmatrix} - \begin{bmatrix} 40 & -30 & 30 \\ -30 & 40 & -30 \\ 30 & -30 & 40 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$\therefore A^3 - 6A^2 + 9A - 4I = O$

Now,

$A^3 - 6A^2 + 9A - 4I = O$

$\Rightarrow (AAA)A^{-1} - 6(AA)A^{-1} + 9AA^{-1} - 4IA^{-1} = O$ $\qquad$ $\left[\text{Post-multiplying by } A^{-1} \text{ as } |A| \neq 0\right]$

$\Rightarrow AA(AA^{-1}) - 6A(AA^{-1}) + 9(AA^{-1}) = 4(IA^{-1})$

$\Rightarrow AAI - 6AI + 9I = 4A^{-1}$

$\Rightarrow A^2 - 6A + 9I = 4A^{-1}$

$\Rightarrow A^{-1} = \dfrac{1}{4}\left(A^2 - 6A + 9I\right)$ $\qquad$ ...(1)

$A^2 - 6A + 9I$

$= \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} - 6\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} + 9\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$= \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} - \begin{bmatrix} 12 & -6 & 6 \\ -6 & 12 & -6 \\ 6 & -6 & 12 \end{bmatrix} + \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$

$= \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$

From equation (1), we have:

$$A^{-1} = \frac{1}{4}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

Q 17:

Let A be a nonsingular square matrix of order 3 × 3. Then $|adjA|$ is equal to

A. $|A|$ **B.** $|A|^2$ **C.** $|A|^3$ **D.** $3|A|$

Answer: **B**

We know that,

$$(adjA)\,A = |A|\,I = \begin{bmatrix} |A| & 0 & 0 \\ 0 & |A| & 0 \\ 0 & 0 & |A| \end{bmatrix}$$

$$\Rightarrow |(adjA)\,A| = \begin{vmatrix} |A| & 0 & 0 \\ 0 & |A| & 0 \\ 0 & 0 & |A| \end{vmatrix}$$

$$\Rightarrow |adjA||A| = |A|^3 \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = |A|^3(I)$$

$$\therefore |adjA| = |A|^2$$

Hence, the correct Answer: is B.

Q 18:

If A is an invertible matrix of order 2, then det (A^{-1}) is equal to

A. d et (A) **B.** $\dfrac{1}{\det(A)}$ **C.** 1 **D.** 0

Answer:

Since A is an invertible matrix, A^{-1} exists and $A^{-1} = \dfrac{1}{|A|} adjA.$

As matrix A is of order 2, let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}.$

Then, $|A| = ad - bc$ and $adjA = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$

Now,

$$A^{-1} = \frac{1}{|A|} adjA = \begin{bmatrix} \dfrac{d}{|A|} & \dfrac{-b}{|A|} \\ \dfrac{-c}{|A|} & \dfrac{a}{|A|} \end{bmatrix}$$

$$\therefore |A^{-1}| = \begin{vmatrix} \dfrac{d}{|A|} & \dfrac{-b}{|A|} \\ \dfrac{-c}{|A|} & \dfrac{a}{|A|} \end{vmatrix} = \frac{1}{|A|^2} \begin{vmatrix} d & -b \\ -c & a \end{vmatrix} = \frac{1}{|A|^2}(ad - bc) = \frac{1}{|A|^2}.|A| = \frac{1}{|A|}$$

$$\therefore \det(A^{-1}) = \frac{1}{\det(A)}$$

Hence, the correct Answer: is B.

Exercise 4.6

Q 1:
Examine the consistency of the system of equations.

$x + 2y = 2$

$2x + 3y = 3$

Answer:

The given system of equations is:

$x + 2y = 2$

$2x + 3y = 3$

The given system of equations can be written in the form of $AX = B$, where

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 \\ 3 \end{bmatrix}.$$

Now,

$$|A| = 1(3) - 2(2) = 3 - 4 = -1 \neq 0$$

∴ *A* is non-singular.

Therefore, A^{-1} exists.

Hence, the given system of equations is consistent.

Q 2:
Examine the consistency of the system of equations.

$2x - y = 5$

$x + y = 4$

Answer:

The given system of equations is:

$2x - y = 5$

$x + y = 4$

The given system of equations can be written in the form of $AX = B$, where

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}, \ X = \begin{bmatrix} x \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 \\ 4 \end{bmatrix}.$$

Now,

$$|A| = 2(1) - (-1)(1) = 2 + 1 = 3 \neq 0$$

$\therefore A$ is non-singular.

Therefore, A^{-1} exists.

Hence, the given system of equations is consistent.

Q 3:

Examine the consistency of the system of equations.

$x + 3y = 5$

$2x + 6y = 8$

Answer:

The given system of equations is:

$x + 3y = 5$

$2x + 6y = 8$

The given system of equations can be written in the form of $AX = B$, where

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}, \ X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 \\ 8 \end{bmatrix}.$$

Now,

$$|A| = 1(6) - 3(2) = 6 - 6 = 0$$

$\therefore A$ is a singular matrix.

Now,

$$(adjA) = \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$$

$$(adjA)B = \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 5 \\ 8 \end{bmatrix} = \begin{bmatrix} 30-24 \\ -10+8 \end{bmatrix} = \begin{bmatrix} 6 \\ -2 \end{bmatrix} \neq O$$

Thus, the solution of the given system of equations does not exist. Hence, the system of equations is inconsistent.

Q 4:

Examine the consistency of the system of equations.

x + y + z = 1

2x + 3y + 2z = 2

ax + ay + 2az = 4

Answer:

The given system of equations is:

x + y + z = 1

2x + 3y + 2z = 2

ax + ay + 2az = 4

This system of equations can be written in the form AX = B, where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ a & a & 2a \end{bmatrix}, \; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}.$$

Now,

$$|A| = 1(6a-2a) - 1(4a-2a) + 1(2a-3a)$$
$$= 4a-2a-a = 4a-3a = a \neq 0$$

∴ A is non-singular.

Therefore, A^{-1} exists.

Hence, the given system of equations is consistent.

Q 5:

Examine the consistency of the system of equations.

$3x - y - 2z = 2$

$2y - z = -1$

$3x - 5y = 3$

Answer:

The given system of equations is:

$3x - y - 2z = 2$

$2y - z = -1$

$3x - 5y = 3$

This system of equations can be written in the form of $AX = B$, where

$$A = \begin{bmatrix} 3 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}.$$

Now,

$$|A| = 3(0-5) - 0 + 3(1+4) = -15 + 15 = 0$$

∴ A is a singular matrix.

Now,

$$(adjA) = \begin{bmatrix} -5 & 10 & 5 \\ -3 & 6 & 3 \\ -6 & 12 & 6 \end{bmatrix}$$

$$\therefore (adjA)B = \begin{bmatrix} -5 & 10 & 5 \\ -3 & 6 & 3 \\ -6 & 12 & 6 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} -10-10+15 \\ -6-6+9 \\ -12-12+18 \end{bmatrix} = \begin{bmatrix} -5 \\ -3 \\ -6 \end{bmatrix} \neq O$$

Thus, the solution of the given system of equations does not exist. Hence, the system of equations is inconsistent.

Q 6:

Examine the consistency of the system of equations.

$5x - y + 4z = 5$

$2x + 3y + 5z = 2$

$5x - 2y + 6z = -1$

Answer:

The given system of equations is:

$5x - y + 4z = 5$

$2x + 3y + 5z = 2$

$5x - 2y + 6z = -1$

This system of equations can be written in the form of $AX = B$, where

$$A = \begin{bmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix}.$$

Now,

$$|A| = 5(18+10) + 1(12-25) + 4(-4-15)$$
$$= 5(28) + 1(-13) + 4(-19)$$
$$= 140 - 13 - 76$$
$$= 51 \neq 0$$

∴ A is non-singular.

Therefore, A^{-1} exists.

Hence, the given system of equations is consistent.

Q 7:

Solve system of linear equations, using matrix method.

$5x + 2y = 4$

$7x + 3y = 5$

Answer:

The given system of equations can be written in the form of $AX = B$, where

$$A = \begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} 4 \\ 5 \end{bmatrix}.$$

Now, $|A| = 15 - 14 = 1 \neq 0$.

Thus, A is non-singular. Therefore, its inverse exists.

Now,

$$A^{-1} = \frac{1}{|A|}(adjA)$$

$$\therefore A^{-1} = \begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix}$$

$$\therefore X = A^{-1}B = \begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix}\begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 - 10 \\ -28 + 25 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$

Hence, $x = 2$ and $y = -3$.

Q 8:

Solve system of linear equations, using matrix method.

$2x - y = -2$

$3x + 4y = 3$

Answer:

The given system of equations can be written in the form of $AX = B$, where

$$A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} -2 \\ 3 \end{bmatrix}.$$

Now,

$$|A| = 8 + 3 = 11 \neq 0$$

Thus, A is non-singular. Therefore, its inverse exists.

Now,

$$A^{-1} = \frac{1}{|A|} adjA = \frac{1}{11}\begin{bmatrix} 4 & 1 \\ -3 & 2 \end{bmatrix}$$

$$\therefore X = A^{-1}B = \frac{1}{11}\begin{bmatrix} 4 & 1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{11}\begin{bmatrix} -8+3 \\ 6+6 \end{bmatrix} = \frac{1}{11}\begin{bmatrix} -5 \\ 12 \end{bmatrix} = \begin{bmatrix} -\dfrac{5}{11} \\ \dfrac{12}{11} \end{bmatrix}$$

Hence, $x = \dfrac{-5}{11}$ and $y = \dfrac{12}{11}$.

Q 9:

Solve system of linear equations, using matrix method.

$$4x - 3y = 3$$

$$3x - 5y = 7$$

Answer:

The given system of equations can be written in the form of $AX = B$, where

$$A = \begin{bmatrix} 4 & -3 \\ 3 & -5 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{and } B = \begin{bmatrix} 3 \\ 7 \end{bmatrix}.$$

Now,

$$|A| = -20 + 9 = -11 \neq 0$$

Thus, A is non-singular. Therefore, its inverse exists.

Now,

$$A^{-1} = \frac{1}{|A|}(adjA) = -\frac{1}{11}\begin{bmatrix} -5 & 3 \\ -3 & 4 \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 5 & -3 \\ 3 & -4 \end{bmatrix}$$

$$\therefore X = A^{-1}B = \frac{1}{11}\begin{bmatrix} 5 & -3 \\ 3 & -4 \end{bmatrix}\begin{bmatrix} 3 \\ 7 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 5 & -3 \\ 3 & -4 \end{bmatrix}\begin{bmatrix} 3 \\ 7 \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 15-21 \\ 9-28 \end{bmatrix} = \frac{1}{11}\begin{bmatrix} -6 \\ -19 \end{bmatrix} = \begin{bmatrix} -\frac{6}{11} \\ -\frac{19}{11} \end{bmatrix}$$

Hence, $x = \frac{-6}{11}$ and $y = \frac{-19}{11}$.

Q 10:

Solve system of linear equations, using matrix method.

5x + 2y = 3

3x + 2y = 5

Answer:

The given system of equations can be written in the form of AX = B, where

$$A = \begin{bmatrix} 5 & 2 \\ 3 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} 3 \\ 5 \end{bmatrix}.$$

Now,

$$|A| = 10 - 6 = 4 \neq 0$$

Thus, A is non-singular. Therefore, its inverse exists.

Q 11:

Solve system of linear equations, using matrix method.

$$2x + y + z = 1$$

$$x - 2y - z = \frac{3}{2}$$

$$3y - 5z = 9$$

Answer:

The given system of equations can be written in the form of $AX = B$, where

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 0 & 3 & -5 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 \\ \frac{3}{2} \\ 9 \end{bmatrix}.$$

Now,

$$|A| = 2(10+3) - 1(-5-3) + 0 = 2(13) - 1(-8) = 26 + 8 = 34 \neq 0$$

Thus, A is non-singular. Therefore, its inverse exists.

Now, $A_{11} = 13, A_{12} = 5, A_{13} = 3$

$A_{21} = 8, A_{22} = -10, A_{23} = -6$

$A_{31} = 1, A_{32} = 3, A_{33} = -5$

$$\therefore A^{-1} = \frac{1}{|A|}(adjA) = \frac{1}{34} \begin{bmatrix} 13 & 8 & 1 \\ 5 & -10 & 3 \\ 3 & -6 & -5 \end{bmatrix}$$

$$\therefore X = A^{-1}B = \frac{1}{34} \begin{bmatrix} 13 & 8 & 1 \\ 5 & -10 & 3 \\ 3 & -6 & -5 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 2 \\ 9 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{34} \begin{bmatrix} 13+12+9 \\ 5-15+27 \\ 3-9-45 \end{bmatrix}$$

$$= \frac{1}{34} \begin{bmatrix} 34 \\ 17 \\ -51 \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{1}{2} \\ -\frac{3}{2} \end{bmatrix}$$

Hence, $x = 1, y = \frac{1}{2}$, and $z = -\frac{3}{2}$.

Q 12:

Solve system of linear equations, using matrix method.

$x - y + z = 4$

$2x + y - 3z = 0$

$x + y + z = 2$

Answer:

The given system of equations can be written in the form of $AX = B$, where

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}.$$

Now,

$$|A| = 1(1+3) + 1(2+3) + 1(2-1) = 4 + 5 + 1 = 10 \neq 0$$

Thus, A is non-singular. Therefore, its inverse exists.

Now, $A_{11} = 4, A_{12} = -5, A_{13} = 1$

$A_{21} = 2, A_{22} = 0, A_{23} = -2$

$A_{31} = 2, A_{32} = 5, A_{33} = 3$

$$\therefore A^{-1} = \frac{1}{|A|}(adjA) = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

$$\therefore X = A^{-1}B = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 16+0+4 \\ -20+0+10 \\ 4+0+6 \end{bmatrix}$$

$$= \frac{1}{10}\begin{bmatrix} 20 \\ -10 \\ 10 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

Hence, $x = 2, y = -1,$ and $z = 1.$

Q 13:

Solve system of linear equations, using matrix method.

$2x + 3y + 3z = 5$

$x - 2y + z = -4$

$3x - y - 2z = 3$

Answer:

The given system of equations can be written in the form $AX = B$, where

$$A = \begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}.$$

Now,

$$|A| = 2(4+1) - 3(-2-3) + 3(-1+6) = 2(5) - 3(-5) + 3(5) = 10 + 15 + 15 = 40 \neq 0$$

Thus, A is non-singular. Therefore, its inverse exists.

Now, $A_{11} = 5, A_{12} = 5, A_{13} = 5$

$A_{21} = 3, A_{22} = -13, A_{23} = 11$

$A_{31} = 9, A_{32} = 1, A_{33} = -7$

$$\therefore A^{-1} = \frac{1}{|A|}(adjA) = \frac{1}{40}\begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}$$

$$\therefore X = A^{-1}B = \frac{1}{40}\begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}\begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{40}\begin{bmatrix} 25-12+27 \\ 25+52+3 \\ 25-44-21 \end{bmatrix}$$

$$= \frac{1}{40}\begin{bmatrix} 40 \\ 80 \\ -40 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

Hence, $x = 1, y = 2,$ and $z = -1$.

Q 14:

Solve system of linear equations, using matrix method.

$x - y + 2z = 7$

$3x + 4y - 5z = -5$

$2x - y + 3z = 12$

Answer:

The given system of equations can be written in the form of $AX = B$, where

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}.$$

Now,

$$|A| = 1(12-5) + 1(9+10) + 2(-3-8) = 7 + 19 - 22 = 4 \neq 0$$

Thus, A is non-singular. Therefore, its inverse exists.

Now, $A_{11} = 7, A_{12} = -19, A_{13} = -11$

$A_{21} = 1, A_{22} = -1, A_{23} = -1$

$A_{31} = -3, A_{32} = 11, A_{33} = 7$

$$\therefore A^{-1} = \frac{1}{|A|}(adjA) = \frac{1}{4}\begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}$$

$$\therefore X = A^{-1}B = \frac{1}{4}\begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}\begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 49-5-36 \\ -133+5+132 \\ -77+5+84 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 8 \\ 4 \\ 12 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

Hence, $x = 2, y = 1,$ and $z = 3$.

Q 15:

If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}. Using A^{-1} solve the system of equations

$2x - 3y + 5z = 11$

$3x + 2y - 4z = -5$

$x + y - 2z = -3$

Answer:

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$$

$$\therefore |A| = 2(-4+4) + 3(-6+4) + 5(3-2) = 0 - 6 + 5 = -1 \neq 0$$

Now, $A_{11} = 0$, $A_{12} = 2$, $A_{13} = 1$

$A_{21} = -1$, $A_{22} = -9$, $A_{23} = -5$

$A_{31} = 2$, $A_{32} = 23$, $A_{33} = 13$

$$\therefore A^{-1} = \frac{1}{|A|}(adjA) = -\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \qquad ...(1)$$

Now, the given system of equations can be written in the form of $AX = B$, where

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}.$$

The solution of the system of equations is given by $X = A^{-1}B$.

$X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix} \qquad \left[\text{Using (1)} \right]$$

$$= \begin{bmatrix} 0-5+6 \\ -22-45+69 \\ -11-25+39 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Hence, $x = 1$, $y = 2$, and $z = 3$.

Q 16:

The cost of 4 kg onion, 3 kg wheat and 2 kg rice is Rs 60. The cost of 2 kg onion, 4 kg wheat and 6 kg rice is Rs 90. The cost of 6 kg onion 2 kg wheat and 3 kg rice is Rs 70. Find cost of each item per kg by matrix method.

Answer:

Let the cost of onions, wheat, and rice per kg be Rs x, Rs y, and Rs z respectively.

Then, the given situation can be represented by a system of equations as:

$4x + 3y + 2z = 60$

$2x + 4y + 6z = 90$

$6x + 2y + 3z = 70$

This system of equations can be written in the form of $AX = B$, where

$$A = \begin{bmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}.$$

$|A| = 4(12-12) - 3(6-36) + 2(4-24) = 0 + 90 - 40 = 50 \neq 0$

Now, $\quad A_{11} = 0, A_{12} = 30, A_{13} = -20$

$\qquad\qquad A_{21} = -5, A_{22} = 0, A_{23} = 10$

$\qquad\qquad A_{31} = 10, A_{32} = -20, A_{33} = 10$

$$\therefore adjA = \begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} adjA = \frac{1}{50} \begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}$$

Now,

$X = A^{-1} B$

$$\Rightarrow X = \frac{1}{50} \begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix} \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{50} \begin{bmatrix} 0 - 450 + 700 \\ 1800 + 0 - 1400 \\ -1200 + 900 + 700 \end{bmatrix}$$

$$= \frac{1}{50} \begin{bmatrix} 250 \\ 400 \\ 400 \end{bmatrix}$$

$$= \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix}$$

$\therefore x = 5, y = 8,$ and $z = 8.$

Hence, the cost of onions is Rs 5 per kg, the cost of wheat is Rs 8 per kg, and the cost of rice is Rs 8 per kg.

Miscellaneous Solutions

Q 1:

Prove that the determinant $\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ is independent of θ.

Answer:

$$\Delta = \begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$$

$= x(x^2 - 1) - \sin\theta(-x\sin\theta - \cos\theta) + \cos\theta(-\sin\theta + x\cos\theta)$

$= x^3 - x + x\sin^2\theta + \sin\theta\cos\theta - \sin\theta\cos\theta + x\cos^2\theta$

$= x^3 - x + x(\sin^2\theta + \cos^2\theta)$

$= x^3 - x + x$

$= x^3$ (Independent of θ)

Hence, Δ is independent of θ.

Q 2:

Without expanding the determinant, prove that

$$\begin{vmatrix} a & a^2 & bc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix} = \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix}$$

Answer:

$$\text{L.H.S.} = \begin{vmatrix} a & a^2 & bc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix}$$

$$= \frac{1}{abc} \begin{vmatrix} a^2 & a^3 & abc \\ b^2 & b^3 & abc \\ c^2 & c^3 & abc \end{vmatrix} \qquad \left[R_1 \to aR_1, R_2 \to bR_2, \text{and } R_3 \to cR_3 \right]$$

$$= \frac{1}{abc} \cdot abc \begin{vmatrix} a^2 & a^3 & 1 \\ b^2 & b^3 & 1 \\ c^2 & c^3 & 1 \end{vmatrix} \qquad \left[\text{Taking out factor } abc \text{ from } C_3 \right]$$

$$= \begin{vmatrix} a^2 & a^3 & 1 \\ b^2 & b^3 & 1 \\ c^2 & c^3 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} \qquad \left[\text{Applying } C_1 \leftrightarrow C_3 \text{ and } C_2 \leftrightarrow C_3 \right]$$

$$= \text{R.H.S.}$$

Hence, the given result is proved.

Q 3:

Evaluate $\begin{vmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta & -\sin\alpha \\ -\sin\beta & \cos\beta & 0 \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta & \cos\alpha \end{vmatrix}$

Answer:

$$\Delta = \begin{vmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta & -\sin\alpha \\ -\sin\beta & \cos\beta & 0 \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta & \cos\alpha \end{vmatrix}$$

Expanding along C_3, we have:

$$\Delta = -\sin\alpha\left(-\sin\alpha\sin^2\beta - \cos^2\beta\sin\alpha\right) + \cos\alpha\left(\cos\alpha\cos^2\beta + \cos\alpha\sin^2\beta\right)$$
$$= \sin^2\alpha\left(\sin^2\beta + \cos^2\beta\right) + \cos^2\alpha\left(\cos^2\beta + \sin^2\beta\right)$$
$$= \sin^2\alpha\left(1\right) + \cos^2\alpha\left(1\right)$$
$$= 1$$

Q 4:

If *a*, *b* and *c* are real numbers, and
Show that either *a* + *b* + *c* = 0 or *a* = *b* = *c*.

$$\Delta = \begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 0$$

Answer:

$$\Delta = \begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix}$$

Applying $R_1 \rightarrow R_1 + R_2 + R_3$, we have:

$$\Delta = \begin{vmatrix} 2(a+b+c) & 2(a+b+c) & 2(a+b+c) \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix}$$

$$= 2(a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix}$$

Applying $C_2 \rightarrow C_2 - C_1$ and $C_3 \rightarrow C_3 - C_1$, we have:

$$\Delta = 2(a+b+c)\begin{vmatrix} 1 & 0 & 0 \\ c+a & b-c & b-a \\ a+b & c-a & c-b \end{vmatrix}$$

Expanding along R_1, we have:

$\Delta = 2(a+b+c)(1)\left[(b-c)(c-b)-(b-a)(c-a)\right]$

$= 2(a+b+c)\left[-b^2-c^2+2bc-bc+ba+ac-a^2\right]$

$= 2(a+b+c)\left[ab+bc+ca-a^2-b^2-c^2\right]$

It is given that $\Delta = 0$.

$(a+b+c)\left[ab+bc+ca-a^2-b^2-c^2\right] = 0$

$\Rightarrow$ Either $a+b+c = 0$, or $ab+bc+ca-a^2-b^2-c^2 = 0$.

Now,

$ab+bc+ca-a^2-b^2-c^2 = 0$

$\Rightarrow -2ab-2bc-2ca+2a^2+2b^2+2c^2 = 0$

$\Rightarrow (a-b)^2+(b-c)^2+(c-a)^2 = 0$

$\Rightarrow (a-b)^2 = (b-c)^2 = (c-a)^2 = 0 \qquad \left[(a-b)^2,(b-c)^2,(c-a)^2 \text{ are non-negative}\right]$

$\Rightarrow (a-b) = (b-c) = (c-a) = 0$

$\Rightarrow a = b = c$

Hence, if $\Delta = 0$, then either $a + b + c = 0$ or $a = b = c$.

Q 5:

Solve the equation $\begin{vmatrix} x+a & x & x \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0, a \neq 0$

Answer:

$$\begin{vmatrix} x+a & x & x \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0$$

Applying $R_1 \to R_1 + R_2 + R_3$, we get:

$$\begin{vmatrix} 3x+a & 3x+a & 3x+a \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0$$

$$\Rightarrow (3x+a) \begin{vmatrix} 1 & 1 & 1 \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0$$

Applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$, we have:

$$(3x+a) \begin{vmatrix} 1 & 0 & 0 \\ x & a & 0 \\ x & 0 & a \end{vmatrix} = 0$$

Expanding along R_1, we have:

$$(3x+a)\left[1 \times a^2\right] = 0$$

$$\Rightarrow a^2(3x+a) = 0$$

But $a \neq 0$.

Therefore, we have:

$$3x+a = 0$$

$$\Rightarrow x = -\frac{a}{3}$$

Q 6:

Prove that $\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$

Answer:

$$\Delta = \begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix}$$

Taking out common factors a, b, and c from C_1, C_2, and C_3, we have:

$$\Delta = abc \begin{vmatrix} a & c & a+c \\ a+b & b & a \\ b & b+c & c \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, we have:

$$\Delta = abc \begin{vmatrix} a & c & a+c \\ b & b-c & -c \\ b-a & b & -a \end{vmatrix}$$

Applying $R_2 \to R_2 + R_1$, we have:

$$\Delta = abc \begin{vmatrix} a & c & a+c \\ a+b & b & a \\ b-a & b & -a \end{vmatrix}$$

Applying $R_3 \to R_3 + R_2$, we have:

$$\Delta = abc \begin{vmatrix} a & c & a+c \\ a+b & b & a \\ 2b & 2b & 0 \end{vmatrix}$$

$$= 2ab^2c \begin{vmatrix} a & c & a+c \\ a+b & b & a \\ 1 & 1 & 0 \end{vmatrix}$$

Applying $C_2 \to C_2 - C_1$, we have:

$$\Delta = 2ab^2c \begin{vmatrix} a & c-a & a+c \\ a+b & -a & a \\ 1 & 0 & 0 \end{vmatrix}$$

Expanding along R_3, we have:

$$\Delta = 2ab^2c \left[a(c-a) + a(a+c) \right]$$
$$= 2ab^2c \left[ac - a^2 + a^2 + ac \right]$$
$$= 2ab^2c(2ac)$$
$$= 4a^2b^2c^2$$

Hence, the given result is proved.

Q 8:

Let $A = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix}$ verify that

(i) $[adjA]^{-1} = adj(A^{-1})$

(ii) $(A^{-1})^{-1} = A$

Answer :

$A = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix}$

$\therefore |A| = 1(15-1) + 2(-10-1) + 1(-2-3) = 14 - 22 - 5 = -13$

Now, $A_{11} = 14, A_{12} = 11, A_{13} = -5$

$A_{21} = 11, A_{22} = 4, A_{23} = -3$

$A_{31} = -5, A_{32} = -3, A_{13} = -1$

$\therefore adjA = \begin{bmatrix} 14 & 11 & -5 \\ 11 & 4 & -3 \\ -5 & -3 & -1 \end{bmatrix}$

$\therefore A^{-1} = \frac{1}{|A|}(adjA)$

$= -\frac{1}{13}\begin{bmatrix} 14 & 11 & -5 \\ 11 & 4 & -3 \\ -5 & -3 & -1 \end{bmatrix} = \frac{1}{13}\begin{bmatrix} -14 & -11 & 5 \\ -11 & -4 & 3 \\ 5 & 3 & 1 \end{bmatrix}$

(i)

$|adjA| = 14(-4-9) - 11(-11-15) - 5(-33+20)$

$= 14(-13) - 11(-26) - 5(-13)$

$= -182 + 286 + 65 = 169$

We have,

$$adj(adjA) = \begin{bmatrix} -13 & 26 & -13 \\ 26 & -39 & -13 \\ -13 & -13 & -65 \end{bmatrix}$$

$$\therefore [adjA]^{-1} = \frac{1}{|adjA|}\big(adj(adjA)\big)$$

$$= \frac{1}{169}\begin{bmatrix} -13 & 26 & -13 \\ 26 & -39 & -13 \\ -13 & -13 & -65 \end{bmatrix}$$

$$= \frac{1}{13}\begin{bmatrix} -1 & 2 & -1 \\ 2 & -3 & -1 \\ -1 & -1 & -5 \end{bmatrix}$$

Now, $A^{-1} = \frac{1}{13}\begin{bmatrix} -14 & -11 & 5 \\ -11 & -4 & 3 \\ 5 & 3 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{14}{13} & -\frac{11}{13} & \frac{5}{13} \\ -\frac{11}{13} & -\frac{4}{13} & \frac{3}{13} \\ \frac{5}{13} & \frac{3}{13} & \frac{1}{13} \end{bmatrix}$

$$\therefore adj\big(A^{-1}\big) = \begin{bmatrix} -\frac{4}{169}-\frac{9}{169} & -\left(-\frac{11}{169}-\frac{15}{169}\right) & -\frac{33}{169}+\frac{20}{169} \\ -\left(-\frac{11}{169}-\frac{15}{169}\right) & -\frac{14}{169}-\frac{25}{169} & -\left(-\frac{42}{169}+\frac{55}{169}\right) \\ -\frac{33}{169}+\frac{20}{169} & -\left(-\frac{42}{169}+\frac{55}{169}\right) & \frac{56}{169}-\frac{121}{169} \end{bmatrix}$$

$$= \frac{1}{169}\begin{bmatrix} -13 & 26 & -13 \\ 26 & -39 & -13 \\ -13 & -13 & -65 \end{bmatrix} = \frac{1}{13}\begin{bmatrix} -1 & 2 & -1 \\ 2 & -3 & -1 \\ -1 & -1 & -5 \end{bmatrix}$$

Hence, $[adjA]^{-1} = adj\big(A^{-1}\big)$.

(ii) We have shown that:

$$A^{-1} = \frac{1}{13}\begin{bmatrix} -14 & -11 & 5 \\ -11 & -4 & 3 \\ 5 & 3 & 1 \end{bmatrix}$$

And, $adj A^{-1} = \frac{1}{13}\begin{bmatrix} -1 & 2 & -1 \\ 2 & -3 & -1 \\ -1 & -1 & -5 \end{bmatrix}$

Now,

$$|A^{-1}| = \left(\frac{1}{13}\right)^3 [-14 \times (-13) + 11 \times (-26) + 5 \times (-13)] = \left(\frac{1}{13}\right)^3 \times (-169) = -\frac{1}{13}$$

$$\therefore (A^{-1})^{-1} = \frac{adj A^{-1}}{|A^{-1}|} = \frac{1}{\left(-\frac{1}{13}\right)} \times \frac{1}{13}\begin{bmatrix} -1 & 2 & -1 \\ 2 & -3 & -1 \\ -1 & -1 & -5 \end{bmatrix} = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix} = A$$

$$\therefore (A^{-1})^{-1} = A$$

Q 9:

Evaluate $\begin{vmatrix} x & y & x+y \\ y & x+y & x \\ x+y & x & y \end{vmatrix}$

Answer:

$$\Delta = \begin{vmatrix} x & y & x+y \\ y & x+y & x \\ x+y & x & y \end{vmatrix}$$

Applying $R_1 \rightarrow R_1 + R_2 + R_3$, we have:

$$\Delta = \begin{vmatrix} 2(x+y) & 2(x+y) & 2(x+y) \\ y & x+y & x \\ x+y & x & y \end{vmatrix}$$

$$= 2(x+y)\begin{vmatrix} 1 & 1 & 1 \\ y & x+y & x \\ x+y & x & y \end{vmatrix}$$

Applying $C_2 \rightarrow C_2 - C_1$ and $C_3 \rightarrow C_3 - C_1$, we have:

$$\Delta = 2(x+y)\begin{vmatrix} 1 & 0 & 0 \\ y & x & x-y \\ x+y & -y & -x \end{vmatrix}$$

Expanding along R_1, we have:

$$\Delta = 2(x+y)\left[-x^2 + y(x-y)\right]$$
$$= -2(x+y)(x^2 + y^2 - yx)$$
$$= -2(x^3 + y^3)$$

Q 10:

Evaluate $\begin{vmatrix} 1 & x & y \\ 1 & x+y & y \\ 1 & x & x+y \end{vmatrix}$

Answer:

$$\Delta = \begin{vmatrix} 1 & x & y \\ 1 & x+y & y \\ 1 & x & x+y \end{vmatrix}$$

Applying $R_2 \rightarrow R_2 - R_1$ and $R_3 \rightarrow R_3 - R_1$, we have:

$$\Delta = \begin{vmatrix} 1 & x & y \\ 0 & y & 0 \\ 0 & 0 & x \end{vmatrix}$$

Expanding along C_1, we have:

$$\Delta = 1(xy - 0) = xy$$

Q 11:

Using properties of determinants, prove that:

$$\begin{vmatrix} \alpha & \alpha^2 & \beta+\gamma \\ \beta & \beta^2 & \gamma+\alpha \\ \gamma & \gamma^2 & \alpha+\beta \end{vmatrix} = (\beta-\gamma)(\gamma-\alpha)(\alpha-\beta)(\alpha+\beta+\gamma)$$

Answer:

$$\Delta = \begin{vmatrix} \alpha & \alpha^2 & \beta+\gamma \\ \beta & \beta^2 & \gamma+\alpha \\ \gamma & \gamma^2 & \alpha+\beta \end{vmatrix}$$

Applying $R_2 \rightarrow R_2 - R_1$ and $R_3 \rightarrow R_3 - R_1$, we have:

$$\Delta = \begin{vmatrix} \alpha & \alpha^2 & \beta+\gamma \\ \beta-\alpha & \beta^2-\alpha^2 & \alpha-\beta \\ \gamma-\alpha & \gamma^2-\alpha^2 & \alpha-\gamma \end{vmatrix}$$

$$= (\beta-\alpha)(\gamma-\alpha)\begin{vmatrix} \alpha & \alpha^2 & \beta+\gamma \\ 1 & \beta+\alpha & -1 \\ 1 & \gamma+\alpha & -1 \end{vmatrix}$$

Applying $R_3 \rightarrow R_3 - R_2$, we have:

$$\Delta = (\beta - \alpha)(\gamma - \alpha)\begin{vmatrix} \alpha & \alpha^2 & \beta + \gamma \\ 1 & \beta + \alpha & -1 \\ 0 & \gamma - \beta & 0 \end{vmatrix}$$

Expanding along R_3, we have:

$$\Delta = (\beta - \alpha)(\gamma - \alpha)\left[-(\gamma - \beta)(-\alpha - \beta - \gamma)\right]$$
$$= (\beta - \alpha)(\gamma - \alpha)(\gamma - \beta)(\alpha + \beta + \gamma)$$
$$= (\alpha - \beta)(\beta - \gamma)(\gamma - \alpha)(\alpha + \beta + \gamma)$$

Hence, the given result is proved.

Q 12:

Using properties of determinants, prove that:

$$\begin{vmatrix} x & x^2 & 1 + px^3 \\ y & y^2 & 1 + py^3 \\ z & z^2 & 1 + pz^3 \end{vmatrix} = (1 + pxyz)(x - y)(y - z)(z - x)$$

Answer:

$$\Delta = \begin{vmatrix} x & x^2 & 1 + px^3 \\ y & y^2 & 1 + py^3 \\ z & z^2 & 1 + pz^3 \end{vmatrix}$$

Applying $R_2 \rightarrow R_2 - R_1$ and $R_3 \rightarrow R_3 - R_1$, we have:

$$\Delta = \begin{vmatrix} x & x^2 & 1 + px^3 \\ y - x & y^2 - x^2 & p(y^3 - x^3) \\ z - x & z^2 - x^2 & p(z^3 - x^3) \end{vmatrix}$$

$$= (y - x)(z - x)\begin{vmatrix} x & x^2 & 1 + px^3 \\ 1 & y + x & p(y^2 + x^2 + xy) \\ 1 & z + x & p(z^2 + x^2 + xz) \end{vmatrix}$$

Applying $R_3 \rightarrow R_3 - R_2$, we have:

$$\Delta = (y-x)(z-x)\begin{vmatrix} x & x^2 & 1+px^3 \\ 1 & y+x & p(y^2+x^2+xy) \\ 0 & z-y & p(z-y)(x+y+z) \end{vmatrix}$$

$$= (y-x)(z-x)(z-y)\begin{vmatrix} x & x^2 & 1+px^3 \\ 1 & y+x & p(y^2+x^2+xy) \\ 0 & 1 & p(x+y+z) \end{vmatrix}$$

Expanding along R₃, we have:

$$\Delta = (x-y)(y-z)(z-x)\left[(-1)(p)(xy^2+x^3+x^2y)+1+px^3+p(x+y+z)(xy)\right]$$

$$= (x-y)(y-z)(z-x)\left[-pxy^2-px^3-px^2y+1+px^3+px^2y+pxy^2+pxyz\right]$$

$$= (x-y)(y-z)(z-x)(1+pxyz)$$

Hence, the given result is proved.

Q 13:

Using properties of determinants, prove that:

$$\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix} = 3(a+b+c)(ab+bc+ca)$$

Answer:

$$\Delta = \begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix}$$

Applying $C_1 \to C_1 + C_2 + C_3$, we have:

$$\Delta = \begin{vmatrix} a+b+c & -a+b & -a+c \\ a+b+c & 3b & -b+c \\ a+b+c & -c+b & 3c \end{vmatrix}$$

$$= (a+b+c) \begin{vmatrix} 1 & -a+b & -a+c \\ 1 & 3b & -b+c \\ 1 & -c+b & 3c \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, we have:

$$\Delta = (a+b+c) \begin{vmatrix} 1 & -a+b & -a+c \\ 0 & 2b+a & a-b \\ 0 & a-c & 2c+a \end{vmatrix}$$

Expanding along C_1, we have:

$$\Delta = (a+b+c)\left[(2b+a)(2c+a)-(a-b)(a-c)\right]$$
$$= (a+b+c)\left[4bc+2ab+2ac+a^2-a^2+ac+ba-bc\right]$$
$$= (a+b+c)(3ab+3bc+3ac)$$
$$= 3(a+b+c)(ab+bc+ca)$$

Hence, the given result is proved.

Q 14:

Using properties of determinants, prove that:

$$\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 4+3p+2q \\ 3 & 6+3p & 10+6p+3q \end{vmatrix} = 1$$

Answer:

$$\Delta = \begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 4+3p+2q \\ 3 & 6+3p & 10+6p+3q \end{vmatrix}$$

Applying $R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 - 3R_1$, we have:

$$\Delta = \begin{vmatrix} 1 & 1+p & 1+p+q \\ 0 & 1 & 2+p \\ 0 & 3 & 7+3p \end{vmatrix}$$

Applying $R_3 \to R_3 - 3R_2$, we have:

$$\Delta = \begin{vmatrix} 1 & 1+p & 1+p+q \\ 0 & 1 & 2+p \\ 0 & 0 & 1 \end{vmatrix}$$

Expanding along C_1, we have:

$$\Delta = 1 \begin{vmatrix} 1 & 2+p \\ 0 & 1 \end{vmatrix} = 1(1-0) = 1$$

Hence, the given result is proved.

Q 15:

Using properties of determinants, prove that:

$$\begin{vmatrix} \sin\alpha & \cos\alpha & \cos(\alpha+\delta) \\ \sin\beta & \cos\beta & \cos(\beta+\delta) \\ \sin\gamma & \cos\gamma & \cos(\gamma+\delta) \end{vmatrix} = 0$$

Answer:

$$\Delta = \begin{vmatrix} \sin\alpha & \cos\alpha & \cos(\alpha+\delta) \\ \sin\beta & \cos\beta & \cos(\beta+\delta) \\ \sin\gamma & \cos\gamma & \cos(\gamma+\delta) \end{vmatrix}$$

$$= \frac{1}{\sin\delta\cos\delta} \begin{vmatrix} \sin\alpha\sin\delta & \cos\alpha\cos\delta & \cos\alpha\cos\delta - \sin\alpha\sin\delta \\ \sin\beta\sin\delta & \cos\beta\cos\delta & \cos\beta\cos\delta - \sin\beta\sin\delta \\ \sin\gamma\sin\delta & \cos\gamma\cos\delta & \cos\gamma\cos\delta - \sin\gamma\sin\delta \end{vmatrix}$$

Applying $C_1 \to C_1 + C_3$, we have:

$$\Delta = \frac{1}{\sin\delta\cos\delta} \begin{vmatrix} \cos\alpha\cos\delta & \cos\alpha\cos\delta & \cos\alpha\cos\delta - \sin\alpha\sin\delta \\ \cos\beta\cos\delta & \cos\beta\cos\delta & \cos\beta\cos\delta - \sin\beta\sin\delta \\ \cos\gamma\cos\delta & \cos\gamma\cos\delta & \cos\gamma\cos\delta - \sin\gamma\sin\delta \end{vmatrix}$$

Here, two columns C_1 and C_2 are identical.

$\therefore \Delta = 0.$

Hence, the given result is proved.

Q 16:

Solve the system of the following equations

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$$

$$\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$$

$$\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$$

Answer:

Let $\dfrac{1}{x} = p, \dfrac{1}{y} = q, \dfrac{1}{z} = r$.

Then the given system of equations is as follows:

$2p + 3q + 10r = 4$

$4p - 6q + 5r = 1$

$6p + 9q - 20r = 2$

This system can be written in the form of $AX = B$, where

$$A = \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}, X = \begin{bmatrix} p \\ q \\ r \end{bmatrix} \text{ and } B = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}.$$

Now,

$$|A| = 2(120 - 45) - 3(-80 - 30) + 10(36 + 36)$$
$$= 150 + 330 + 720$$
$$= 1200$$

Thus, A is non-singular. Therefore, its inverse exists.

Now,

$A_{11} = 75, A_{12} = 110, A_{13} = 72$

$A_{21} = 150, A_{22} = -100, A_{23} = 0$

$A_{31} = 75, A_{32} = 30, A_{33} = -24$

$$\therefore A^{-1} = \frac{1}{|A|} adjA$$

$$= \frac{1}{1200} \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

Now,

$X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} p \\ q \\ r \end{bmatrix} = \frac{1}{1200} \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$= \frac{1}{1200} \begin{bmatrix} 300 + 150 + 150 \\ 440 - 100 + 60 \\ 288 + 0 - 48 \end{bmatrix}$$

$$= \frac{1}{1200} \begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5} \end{bmatrix}$$

$$\therefore p = \frac{1}{2}, q = \frac{1}{3}, \text{ and } r = \frac{1}{5}$$

Hence, $x = 2, y = 3,$ and $z = 5.$

Q 17:

Choose the correct Answer:.

If *a*, *b*, *c*, are in A.P., then the determinant

$$\begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix}$$

A. 0 **B.** 1 **C.** *x* **D.** 2*x*

Answer:

Answer:: A

$$\Delta = \begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix}$$

$$= \begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+(a+c) \\ x+4 & x+5 & x+2c \end{vmatrix} \qquad (2b = a+c \text{ as } a, b, \text{ and } c \text{ are in A.P.})$$

Applying $R_1 \rightarrow R_1 - R_2$ and $R_3 \rightarrow R_3 - R_2$, we have:

$$\Delta = \begin{vmatrix} -1 & -1 & a-c \\ x+3 & x+4 & x+(a+c) \\ 1 & 1 & c-a \end{vmatrix}$$

Applying $R_1 \rightarrow R_1 + R_3$, we have:

$$\Delta = \begin{vmatrix} 0 & 0 & 0 \\ x+3 & x+4 & x+a+c \\ 1 & 1 & c-a \end{vmatrix}$$

Here, all the elements of the first row (R_1) are zero.

Hence, we have $\Delta = 0$.

The correct Answer: is A.

Q 18:

Choose the correct
Answer:.

If x, y, z are nonzero real numbers, then the inverse of matrix $A = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$ is

A. $\begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$ **B.** $xyz \begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$

C. $\dfrac{1}{xyz} \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$ **D.** $\dfrac{1}{xyz} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Answer:

Answer::

A

$A = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$

$\therefore |A| = x(yz - 0) = xyz \neq 0$

Now, $A_{11} = yz, A_{12} = 0, A_{13} = 0$

$\qquad A_{21} = 0, A_{22} = xz, A_{23} = 0$

$\qquad A_{31} = 0, A_{32} = 0, A_{33} = xy$

$\therefore adjA = \begin{bmatrix} yz & 0 & 0 \\ 0 & xz & 0 \\ 0 & 0 & xy \end{bmatrix}$

$\therefore A^{-1} = \dfrac{1}{|A|} adjA$

$$= \frac{1}{xyz} \begin{bmatrix} yz & 0 & 0 \\ 0 & xz & 0 \\ 0 & 0 & xy \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{yz}{xyz} & 0 & 0 \\ 0 & \dfrac{xz}{xyz} & 0 \\ 0 & 0 & \dfrac{xy}{xyz} \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{1}{x} & 0 & 0 \\ 0 & \dfrac{1}{y} & 0 \\ 0 & 0 & \dfrac{1}{z} \end{bmatrix} = \begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$$

The correct Answer: is A.

Q 19:

Choose the correct Answer:.

Let $A = \begin{bmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{bmatrix}$, where $0 \leq \theta \leq 2\pi$, then

A. Det (A) = 0
B. Det (A) ∈ (2, ∞)
C. Det (A) ∈ (2, 4)
D. Det (A)∈ [2, 4]

Answer:

Answer::

D
$$A = \begin{bmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{bmatrix}$$

$\therefore |A| = 1(1+\sin^2\theta) - \sin\theta(-\sin\theta+\sin\theta) + 1(\sin^2\theta+1)$

$= 1 + \sin^2\theta + \sin^2\theta + 1$

$= 2 + 2\sin^2\theta$

$= 2(1+\sin^2\theta)$

Now, $0 \le \theta \le 2\pi$

$\Rightarrow 0 \le \sin\theta \le 1$

$\Rightarrow 0 \le \sin^2\theta \le 1$

$\Rightarrow 1 \le 1+\sin^2\theta \le 2$

$\Rightarrow 2 \le 2(1+\sin^2\theta) \le 4$

$\therefore \text{Det}(A) \in [2,4]$

The correct Answer: is D.