

## Class 10 Maths NCERT Solutions Unit -06

### Triangles

#### Exercise 6.1

Q1 :

Fill in the blanks using correct word given in the brackets:-

- (i) All circles are \_\_\_\_\_. (congruent, similar)
- (ii) All squares are \_\_\_\_\_. (similar, congruent)
- (iii) All \_\_\_\_\_ triangles are similar. (isosceles, equilateral)
- (iv) Two polygons of the same number of sides are similar, if (a) their corresponding angles are \_\_\_\_\_ and (b) their corresponding sides are \_\_\_\_\_. (equal, proportional)

Answer :

- (i) Similar
- (ii) Similar
- (iii) Equilateral
- (iv) (a) Equal
- (b) Proportional

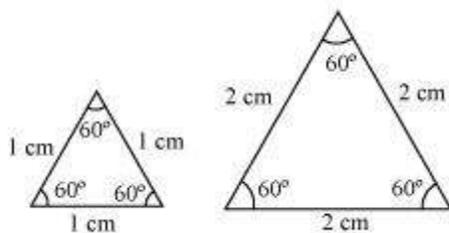
Q2 :

Give two different examples of pair of

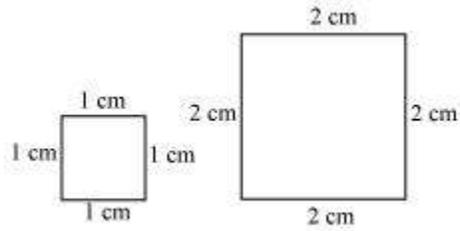
- (i) Similar figures
- (ii) Non-similar figures

Answer :

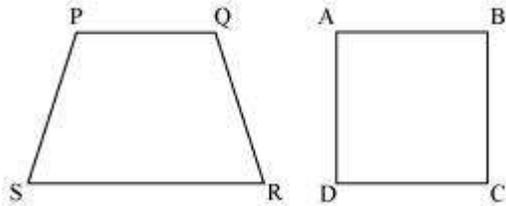
- (i) Two equilateral triangles with sides 1 cm and 2 cm



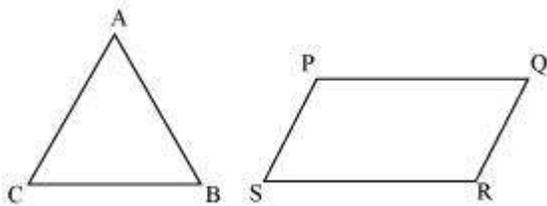
Two squares with sides 1 cm and 2 cm



(ii) Trapezium and square

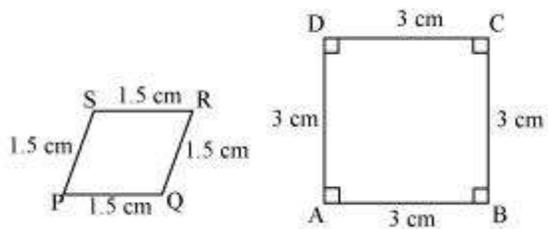


Triangle and parallelogram



Q3 :

State whether the following quadrilaterals are similar or not:



Answer :

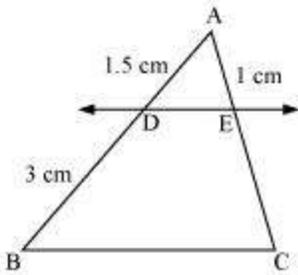
Quadrilateral PQRS and ABCD are not similar as their corresponding sides are proportional, i.e. 1:2, but their corresponding angles are not equal.

## Exercise 6.2

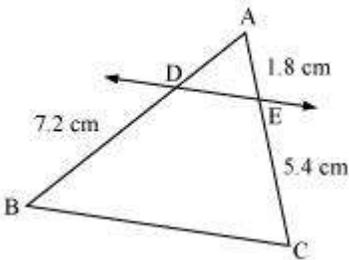
Q1 :

In figure.6.17. (i) and (ii),  $DE \parallel BC$ . Find EC in (i) and AD in (ii).

(i)

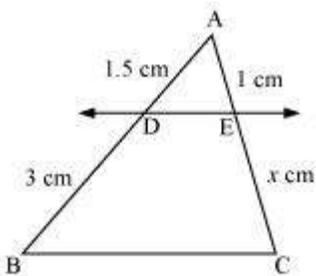


(ii)



Answer :

(i)



Let  $EC = x$  cm

It is given that  $DE \parallel BC$ .

By using basic proportionality theorem, we obtain

$$\frac{AD}{DB} = \frac{AE}{EC}$$

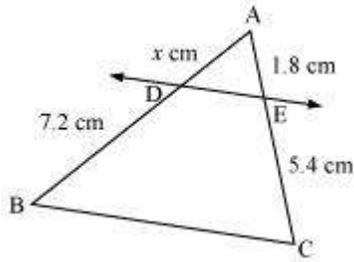
$$\frac{1.5}{3} = \frac{1}{x}$$

$$x = \frac{3 \times 1}{1.5}$$

$$x = 2$$

$$\therefore EC = 2 \text{ cm}$$

(ii)



Let  $AD = x$  cm

It is given that  $DE \parallel BC$ .

By using basic proportionality theorem, we obtain

$$\frac{AD}{DB} = \frac{AE}{EC}$$
$$\frac{x}{7.2} = \frac{1.8}{5.4}$$
$$x = \frac{1.8 \times 7.2}{5.4}$$
$$x = 2.4$$

$\therefore AD = 2.4$  cm

**Q2 :**

E and F are points on the sides PQ and PR respectively of a  $\Delta PQR$ . For each of the following cases, state whether  $EF \parallel QR$ .

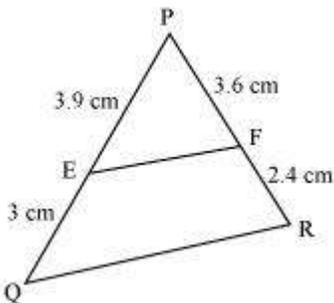
(i)  $PE = 3.9$  cm,  $EQ = 3$  cm,  $PF = 3.6$  cm and  $FR = 2.4$  cm

(ii)  $PE = 4$  cm,  $QE = 4.5$  cm,  $PF = 8$  cm and  $RF = 9$  cm

(iii)  $PQ = 1.28$  cm,  $PR = 2.56$  cm,  $PE = 0.18$  cm and  $PF = 0.63$  cm

**Answer :**

(i)



Given that,  $PE = 3.9$  cm,  $EQ = 3$  cm,  $PF = 3.6$  cm,  $FR = 2.4$  cm

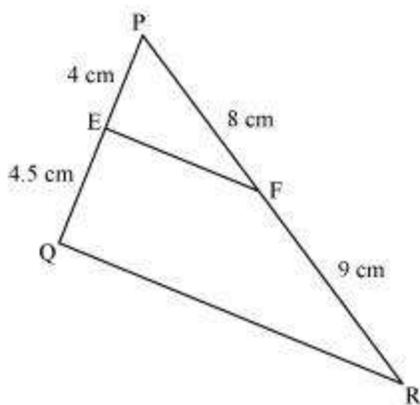
$$\frac{PE}{EQ} = \frac{3.9}{3} = 1.3$$

$$\frac{PF}{FR} = \frac{3.6}{2.4} = 1.5$$

Hence,  $\frac{PE}{EQ} \neq \frac{PF}{FR}$

Therefore, EF is not parallel to QR.

(ii)



PE = 4 cm, QE = 4.5 cm, PF = 8 cm, RF = 9 cm

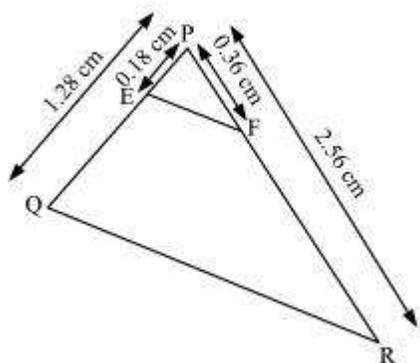
$$\frac{PE}{EQ} = \frac{4}{4.5} = \frac{8}{9}$$

$$\frac{PF}{FR} = \frac{8}{9}$$

Hence,  $\frac{PE}{EQ} = \frac{PF}{FR}$

Therefore, EF is parallel to QR.

(iii)



PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm, PF = 0.36 cm

$$\frac{PE}{PQ} = \frac{0.18}{1.28} = \frac{18}{128} = \frac{9}{64}$$

$$\frac{PF}{PR} = \frac{0.36}{2.56} = \frac{9}{64}$$

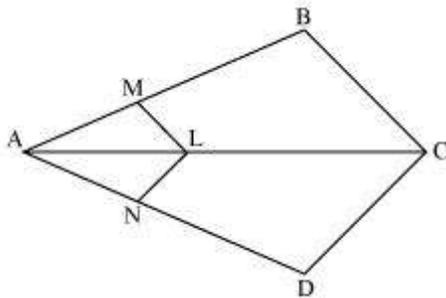
Hence,  $\frac{PE}{PQ} = \frac{PF}{PR}$

Therefore, EF is parallel to QR.

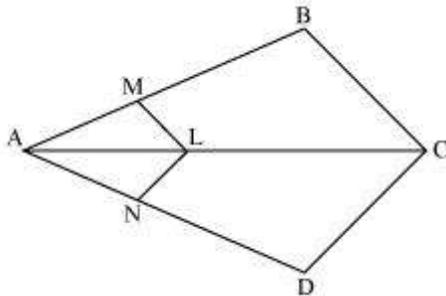
Q3 :

In the following figure, if  $LM \parallel CB$  and  $LN \parallel CD$ , prove that

$$\frac{AM}{AB} = \frac{AN}{AD}$$



Answer :



In the given figure,  $LM \parallel CB$

By using basic proportionality theorem, we obtain

$$\frac{AM}{AB} = \frac{AL}{AC} \quad (i)$$

Similarly,  $LN \parallel CD$

$$\therefore \frac{AN}{AD} = \frac{AL}{AC} \quad (ii)$$

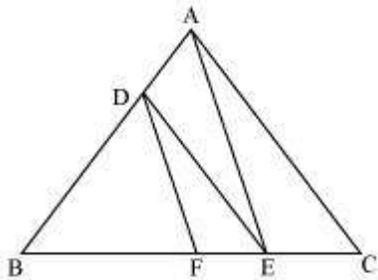
From (i) and (ii), we obtain

$$\frac{AM}{AB} = \frac{AN}{AD}$$

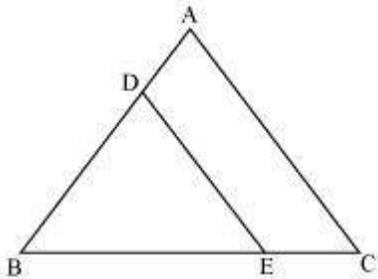
Q4 :

In the following figure,  $DE \parallel AC$  and  $DF \parallel AE$ . Prove that

$$\frac{BF}{FE} = \frac{BE}{EC}$$

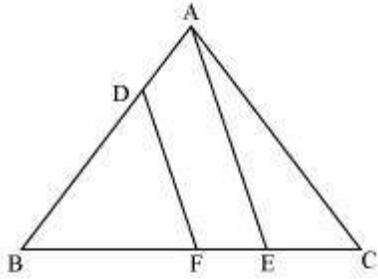


Answer :



In  $\triangle ABC$ ,  $DE \parallel AC$

$$\therefore \frac{BD}{DA} = \frac{BE}{EC} \quad (\text{Basic Proportionality Theorem}) \quad (i)$$



In  $\triangle BAE$ ,  $DF \parallel AE$

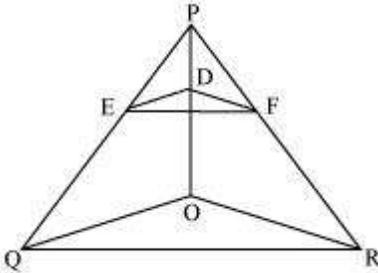
$$\therefore \frac{BD}{DA} = \frac{BF}{FE} \quad (\text{Basic Proportionality Theorem}) \quad (ii)$$

From (i) and (ii), we obtain

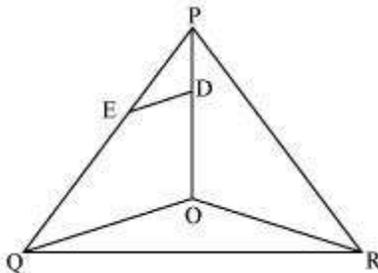
$$\frac{BE}{EC} = \frac{BF}{FE}$$

**Q5 :**

In the following figure,  $DE \parallel OQ$  and  $DF \parallel OR$ , show that  $EF \parallel QR$ .

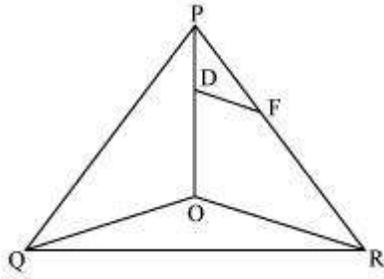


**Answer :**



In  $\triangle POQ$ ,  $DE \parallel OQ$

$$\therefore \frac{PE}{EQ} = \frac{PD}{DO} \quad (\text{Basic proportionality theorem}) \quad (i)$$



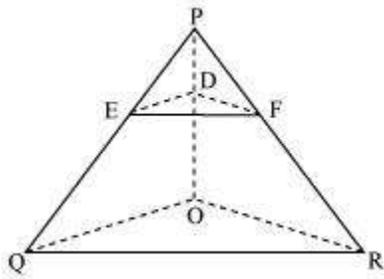
In  $\triangle POR$ ,  $DF \parallel OR$

$$\therefore \frac{PF}{FR} = \frac{PD}{DO} \quad (\text{Basic proportionality theorem}) \quad (ii)$$

From (i) and (ii), we obtain

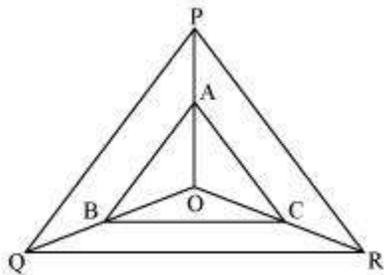
$$\frac{PE}{EQ} = \frac{PF}{FR}$$

$\therefore EF \parallel QR$  (Converse of basic proportionality theorem)

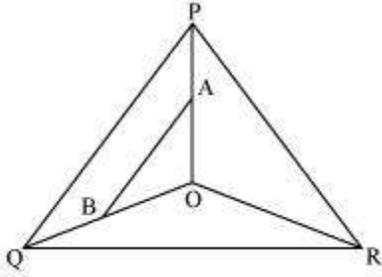


Q6 :

In the following figure, A, B and C are points on OP, OQ and OR respectively such that  $AB \parallel PQ$  and  $AC \parallel PR$ . Show that  $BC \parallel QR$ .

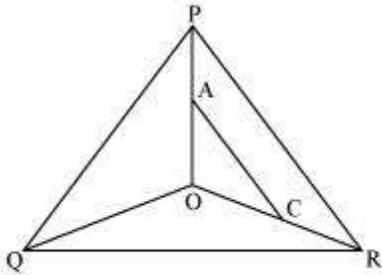


Answer :



In  $\Delta POQ$ ,  $AB \parallel PQ$

$$\therefore \frac{OA}{AP} = \frac{OB}{BQ} \quad (\text{Basic proportionality theorem}) \quad (i)$$



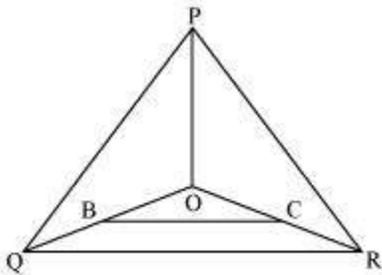
In  $\Delta POR$ ,  $AC \parallel PR$

$$\therefore \frac{OA}{AP} = \frac{OC}{CR} \quad (\text{By basic proportionality theorem}) \quad (ii)$$

From (i) and (ii), we obtain

$$\frac{OB}{BQ} = \frac{OC}{CR}$$

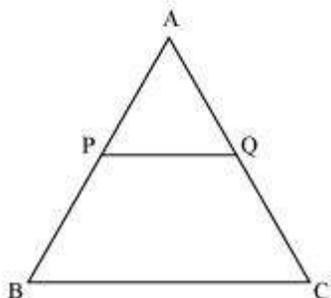
$$\therefore BC \parallel QR \quad (\text{By the converse of basic proportionality theorem})$$



Q7 :

Using Basic proportionality theorem, prove that a line drawn through the mid-points of one side of a triangle parallel to another side bisects the third side. (Recall that you have proved it in Class IX).

Answer :



Consider the given figure in which PQ is a line segment drawn through the mid-point P of line AB, such that  $PQ \parallel BC$

By using basic proportionality theorem, we obtain

$$\frac{AQ}{QC} = \frac{AP}{PB}$$

$$\frac{AQ}{QC} = \frac{1}{1} \quad (\text{P is the mid-point of AB. } \therefore AP = PB)$$

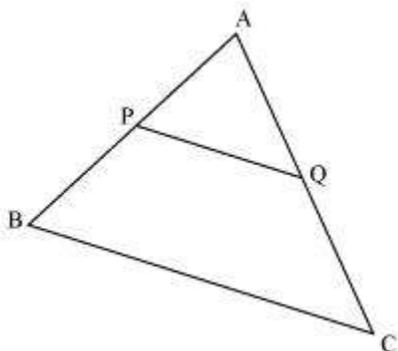
$$\Rightarrow AQ = QC$$

Or, Q is the mid-point of AC.

Q8 :

Using Converse of basic proportionality theorem, prove that the line joining the mid-points of any two sides of a triangle is parallel to the third side. (Recall that you have done it in Class IX).

Answer :



Consider the given figure in which PQ is a line segment joining the mid-points P and Q of line AB and AC respectively.

i.e.,  $AP = PB$  and  $AQ = QC$

It can be observed that

$$\frac{AP}{PB} = \frac{1}{1}$$

and  $\frac{AQ}{QC} = \frac{1}{1}$

$$\therefore \frac{AP}{PB} = \frac{AQ}{QC}$$

Hence, by using basic proportionality theorem, we obtain

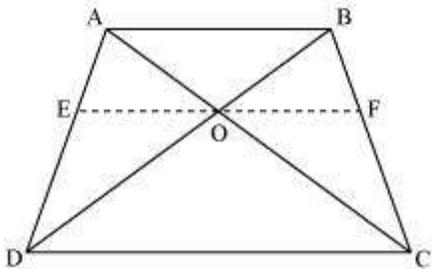
$$PQ \parallel BC$$

**Q9 :**

ABCD is a trapezium in which  $AB \parallel DC$  and its diagonals intersect each other at the point O. Show

that  $\frac{AO}{BO} = \frac{CO}{DO}$ .

**Answer :**



Draw a line EF through point O, such that  $EF \parallel CD$

In  $\triangle ADC$ ,  $EO \parallel CD$

By using basic proportionality theorem, we obtain

$$\frac{AE}{ED} = \frac{AO}{OC} \quad (1)$$

In  $\triangle ABD$ ,  $OE \parallel AB$

So, by using basic proportionality theorem, we obtain

$$\frac{ED}{AE} = \frac{OD}{BO}$$

$$\Rightarrow \frac{AE}{ED} = \frac{BO}{OD} \quad (2)$$

From equations (1) and (2), we obtain

$$\frac{AO}{OC} = \frac{BO}{OD}$$

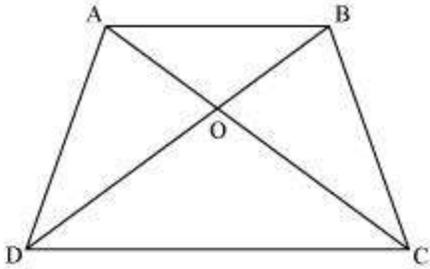
$$\Rightarrow \frac{AO}{BO} = \frac{OC}{OD}$$

Q10 :

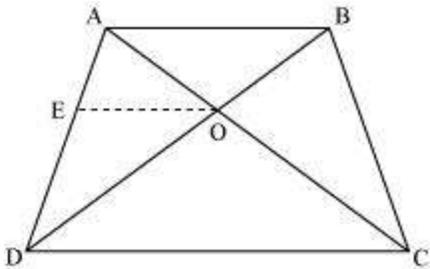
The diagonals of a quadrilateral ABCD intersect each other at the point O such that  $\frac{AO}{BO} = \frac{CO}{DO}$ . Show that ABCD is a trapezium.

Answer :

Let us consider the following figure for the given question.



Draw a line OE || AB



In  $\triangle ABD$ ,  $OE \parallel AB$

By using basic proportionality theorem, we obtain

$$\frac{AE}{ED} = \frac{BO}{OD} \quad (1)$$

However, it is given that

$$\frac{AO}{OC} = \frac{OB}{OD} \quad (2)$$

From equations (1) and (2), we obtain

$$\frac{AE}{ED} = \frac{AO}{OC}$$

$\Rightarrow EO \parallel DC$  [By the converse of basic proportionality theorem]

$\Rightarrow AB \parallel OE \parallel DC$

$\Rightarrow AB \parallel CD$

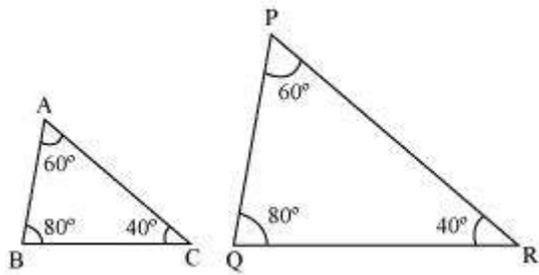
$\therefore$  ABCD is a trapezium.

### Exercise 6.3

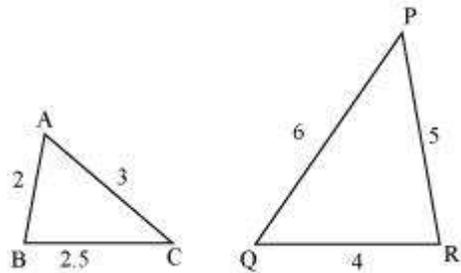
Q1 :

State which pairs of triangles in the following figure are similar? Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form:

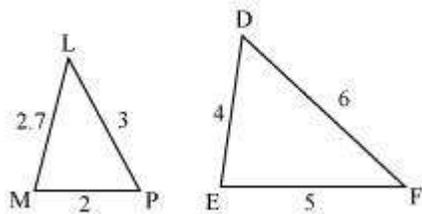
(i)



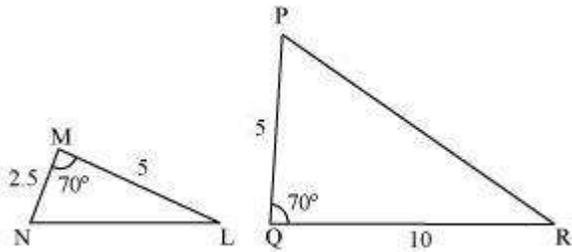
(ii)



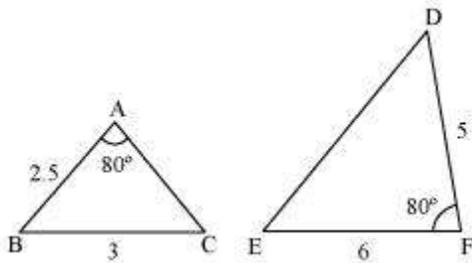
(iii)



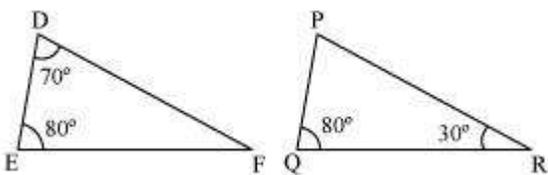
(iv)



(v)



(vi)



**Answer :**

(i)  $\angle A = \angle P = 60^\circ$

$\angle B = \angle Q = 80^\circ$

$\angle C = \angle R = 40^\circ$

Therefore,  $\triangle ABC \sim \triangle PQR$  [By AAA similarity criterion]

$$\frac{AB}{QR} = \frac{BC}{RP} = \frac{CA}{PQ}$$

(ii)

$\therefore \triangle ABC \sim \triangle QRP$  [By SSS similarity criterion]

(iii) The given triangles are not similar as the corresponding sides are not proportional.

(iv) In  $\triangle MNL$  and  $\triangle QPR$ , we observe that

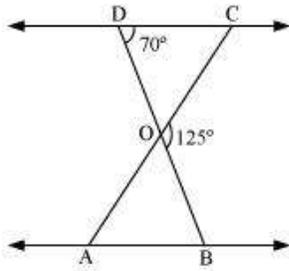
(iv) Yes. SAS,  $\triangle MNL \sim \triangle QPR$

(v) The given triangles are not similar as the corresponding sides are not proportional.

(vi) Yes. AA,  $\triangle DEF \sim \triangle PQR$

**Q2 :**

In the following figure,  $\triangle ODC \sim \triangle OBA$ ,  $\angle BOC = 125^\circ$  and  $\angle CDO = 70^\circ$ . Find  $\angle DOC$ ,  $\angle DCO$  and  $\angle OAB$



**Answer :**

DOB is a straight line.

$$\therefore \angle DOC + \angle COB = 180^\circ$$

$$\Rightarrow \angle DOC = 180^\circ - 125^\circ$$

$$= 55^\circ$$

In  $\triangle ODC$ ,

$$\angle DCO + \angle CDO + \angle DOC = 180^\circ$$

(Sum of the measures of the angles of a triangle is  $180^\circ$ .)

$$\Rightarrow \angle DCO + 70^\circ + 55^\circ = 180^\circ$$

$$\Rightarrow \angle DCO = 55^\circ$$

It is given that  $\triangle ODC \sim \triangle OBA$ .

$$\therefore \angle OAB = \angle OCD \text{ [Corresponding angles are equal in similar triangles.]}$$

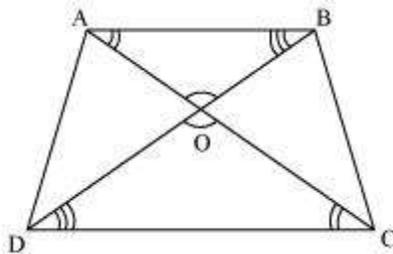
$$\Rightarrow \angle OAB = 55^\circ$$

**Q3 :**

Diagonals AC and BD of a trapezium ABCD with  $AB \parallel DC$  intersect each other at the point O. Using a

similarity criterion for two triangles, show that  $\frac{AO}{OC} = \frac{OB}{OD}$

**Answer :**



In  $\triangle ODC$  and  $\triangle OBA$ ,

$\angle CDO = \angle ABO$  [Alternate interior angles as  $AB \parallel CD$ ]

$\angle DCO = \angle BAO$  [Alternate interior angles as  $AB \parallel CD$ ]

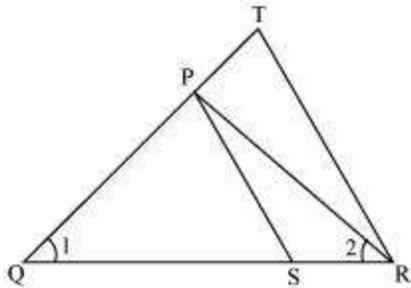
$\angle DOC = \angle BOA$  [Vertically opposite angles]

$\therefore \triangle DOC \sim \triangle BOA$  [AAA similarity criterion]

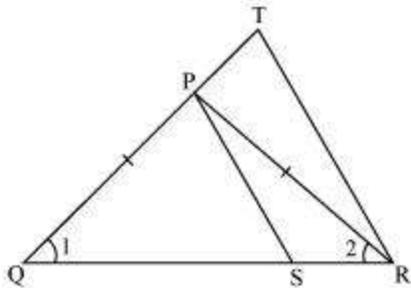
$$\begin{aligned} \therefore \frac{DO}{BO} &= \frac{OC}{OA} && \text{[Corresponding sides are proportional]} \\ \Rightarrow \frac{OA}{OC} &= \frac{OB}{OD} \end{aligned}$$

**Q4 :**

In the following figure,  $\frac{QR}{QS} = \frac{QT}{PR}$  and  $\angle 1 = \angle 2$ . Show that  $\triangle PQS \sim \triangle TQR$



**Answer :**



In  $\triangle PQR$ ,  $\angle PQR = \angle PRQ$

$\therefore PQ = PR$  (i)

Given,

$$\frac{QR}{QS} = \frac{QT}{PR}$$

Using (i), we obtain

$$\frac{QR}{QS} = \frac{QT}{QP} \quad (ii)$$

In  $\Delta PQS$  and  $\Delta TQR$ ,

$$\frac{QR}{QS} = \frac{QT}{QP} \quad [\text{Using (ii)}]$$

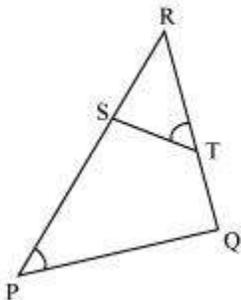
$$\angle Q = \angle Q$$

$\therefore \Delta PQS \sim \Delta TQR$  [SAS similarity criterion]

**Q5 :**

S and T are point on sides PR and QR of  $\Delta PQR$  such that  $\angle P = \angle RTS$ . Show that  $\Delta RPQ \sim \Delta RTS$ .

**Answer :**



In  $\Delta RPQ$  and  $\Delta RTS$ ,

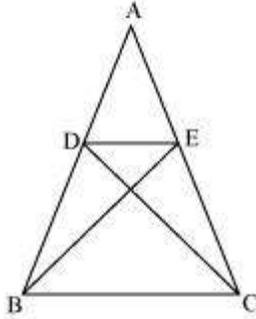
$$\angle RTS = \angle QPS \text{ (Given)}$$

$$\angle R = \angle R \text{ (Common angle)}$$

$\therefore \Delta RPQ \sim \Delta RTS$  (By AA similarity criterion)

Q6 :

In the following figure, if  $\triangle ABE \cong \triangle ACD$ , show that  $\triangle ADE \sim \triangle ABC$ .



Answer :

It is given that  $\triangle ABE \cong \triangle ACD$ .

$\therefore AB = AC$  [By CPCT] (1)

And,  $AD = AE$  [By CPCT] (2)

In  $\triangle ADE$  and  $\triangle ABC$ ,

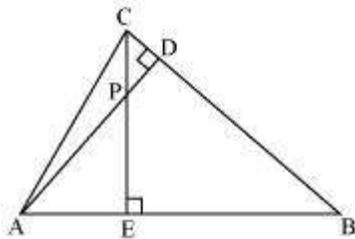
$$\frac{AD}{AB} = \frac{AE}{AC} \quad \text{[Dividing equation (2) by (1)]}$$

$\angle A = \angle A$  [Common angle]

$\therefore \triangle ADE \sim \triangle ABC$  [By SAS similarity criterion]

Q7 :

In the following figure, altitudes  $AD$  and  $CE$  of  $\triangle ABC$  intersect each other at the point  $P$ . Show that:



(i)  $\triangle AEP \sim \triangle CDP$

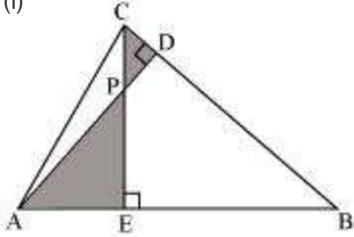
(ii)  $\triangle ABD \sim \triangle CBE$

(iii)  $\triangle AEP \sim \triangle ADB$

(v)  $\triangle PDC \sim \triangle BEC$

**Answer :**

(i)



In  $\triangle AEP$  and  $\triangle CDP$ ,

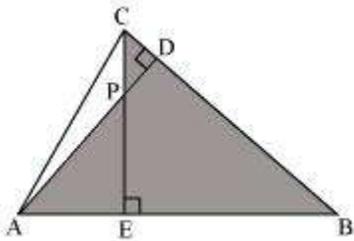
$$\angle AEP = \angle CDP \text{ (Each } 90^\circ\text{)}$$

$$\angle APE = \angle CPD \text{ (Vertically opposite angles)}$$

Hence, by using AA similarity criterion,

$$\triangle AEP \sim \triangle CDP$$

(ii)



In  $\triangle ABD$  and  $\triangle CBE$ ,

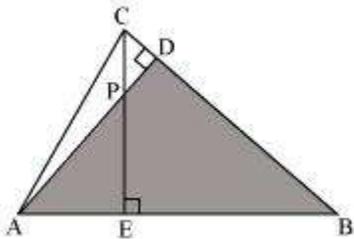
$$\angle ADB = \angle CEB \text{ (Each } 90^\circ\text{)}$$

$$\angle ABD = \angle CBE \text{ (Common)}$$

Hence, by using AA similarity criterion,

$$\triangle ABD \sim \triangle CBE$$

(iii)



In  $\triangle AEP$  and  $\triangle ADB$ ,

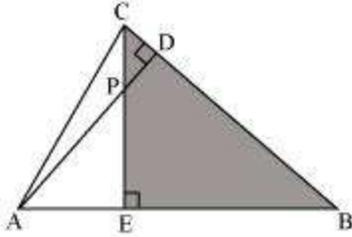
$$\angle AEP = \angle ADB \text{ (Each } 90^\circ\text{)}$$

$$\angle PAE = \angle DAB \text{ (Common)}$$

Hence, by using AA similarity criterion,

$$\triangle AEP \sim \triangle ADB$$

(iv)



In  $\triangle PDC$  and  $\triangle BEC$ ,

$\angle PDC = \angle BEC$  (Each  $90^\circ$ )

$\angle PCD = \angle BCE$  (Common angle)

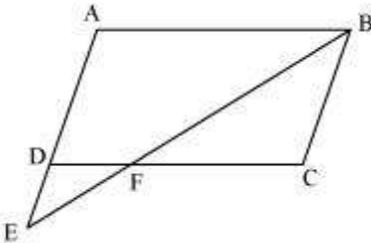
Hence, by using AA similarity criterion,

$\triangle PDC \sim \triangle BEC$

**Q8 :**

**E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that  $\triangle ABE \sim \triangle CFB$**

**Answer :**



In  $\triangle ABE$  and  $\triangle CFB$ ,

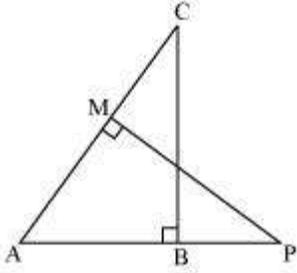
$\angle A = \angle C$  (Opposite angles of a parallelogram)

$\angle AEB = \angle CBF$  (Alternate interior angles as  $AE \parallel BC$ )

$\therefore \triangle ABE \sim \triangle CFB$  (By AA similarity criterion)

Q9 :

In the following figure, ABC and AMP are two right triangles, right angled at B and M respectively, prove that:



(i)  $\Delta ABC \sim \Delta AMP$

$$\frac{CA}{PA} = \frac{BC}{MP}$$

(ii)

Answer :

In  $\Delta ABC$  and  $\Delta AMP$ ,

$\angle ABC = \angle AMP$  (Each  $90^\circ$ )

$\angle A = \angle A$  (Common)

$\therefore \Delta ABC \sim \Delta AMP$  (By AA similarity criterion)

$$\Rightarrow \frac{CA}{PA} = \frac{BC}{MP} \quad (\text{Corresponding sides of similar triangles are proportional})$$

Q10 :

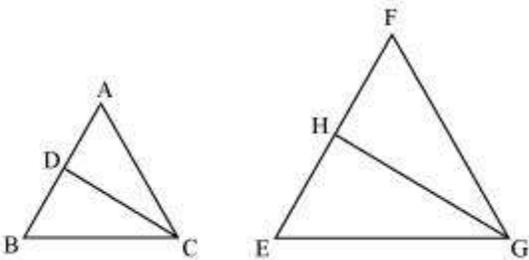
CD and GH are respectively the bisectors of  $\angle ACB$  and  $\angle EGF$  such that D and H lie on sides AB and FE of  $\Delta ABC$  and  $\Delta FEG$  respectively. If  $\Delta ABC \sim \Delta FEG$ , Show that:

$$\frac{CD}{GH} = \frac{AC}{FG}$$

(ii)  $\Delta DCB \sim \Delta HGE$

(iii)  $\Delta DCA \sim \Delta HGF$

Answer :



It is given that  $\Delta ABC \sim \Delta FEG$ .

$\therefore \angle A = \angle F$ ,  $\angle B = \angle E$ , and  $\angle ACB = \angle FGE$

$$\angle ACB = \angle FGE$$

$$\therefore \angle ACD = \angle FGH \text{ (Angle bisector)}$$

And,  $\angle DCB = \angle HGE$  (Angle bisector)

In  $\triangle ACD$  and  $\triangle FGH$ ,

$$\angle A = \angle F \text{ (Proved above)}$$

$$\angle ACD = \angle FGH \text{ (Proved above)}$$

$\therefore \triangle ACD \sim \triangle FGH$  (By AA similarity criterion)

$$\Rightarrow \frac{CD}{GH} = \frac{AC}{FG}$$

In  $\triangle DCB$  and  $\triangle HGE$ ,

$$\angle DCB = \angle HGE \text{ (Proved above)}$$

$$\angle B = \angle E \text{ (Proved above)}$$

$\therefore \triangle DCB \sim \triangle HGE$  (By AA similarity criterion)

In  $\triangle DCA$  and  $\triangle HGF$ ,

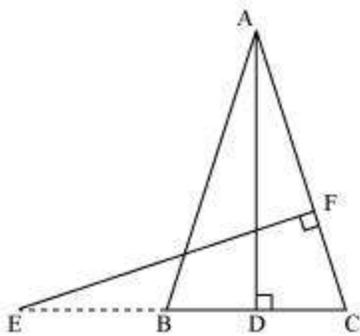
$$\angle ACD = \angle FGH \text{ (Proved above)}$$

$$\angle A = \angle F \text{ (Proved above)}$$

$\therefore \triangle DCA \sim \triangle HGF$  (By AA similarity criterion)

**Q11 :**

In the following figure, E is a point on side CB produced of an isosceles triangle ABC with  $AB = AC$ . If  $AD \perp BC$  and  $EF \perp AC$ , prove that  $\triangle ABD \sim \triangle ECF$



**Answer :**

It is given that ABC is an isosceles triangle.

$$\therefore AB = AC$$

$$\Rightarrow \angle ABD = \angle ECF$$

In  $\triangle ABD$  and  $\triangle ECF$ ,

$$\angle ADB = \angle EFC \text{ (Each } 90^\circ\text{)}$$

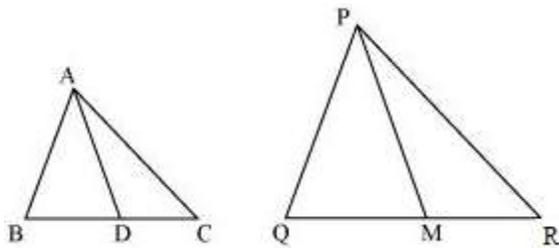
$$\angle BAD = \angle CEF \text{ (Proved above)}$$

$\therefore \triangle ABD \sim \triangle ECF$  (By using AA similarity criterion)

**Q12 :**

Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of  $\triangle PQR$  (see the given figure). Show that  $\triangle ABC \sim \triangle PQR$ .

**Answer :**



Median divides the opposite side.

$$\therefore BD = \frac{BC}{2} \text{ and } QM = \frac{QR}{2}$$

Given that,

$$\begin{aligned} \frac{AB}{PQ} &= \frac{BC}{QR} = \frac{AD}{PM} \\ \Rightarrow \frac{AB}{PQ} &= \frac{\frac{1}{2}BC}{\frac{1}{2}QR} = \frac{AD}{PM} \\ \Rightarrow \frac{AB}{PQ} &= \frac{BD}{QM} = \frac{AD}{PM} \end{aligned}$$

In  $\triangle ABD$  and  $\triangle PQM$ ,

$$\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM} \text{ (Proved above)}$$

$\therefore \triangle ABD \sim \triangle PQM$  (By SSS similarity criterion)

$\Rightarrow \angle ABD = \angle PQM$  (Corresponding angles of similar triangles)

In  $\triangle ABC$  and  $\triangle PQR$ ,

$\angle ABD = \angle PQM$  (Proved above)

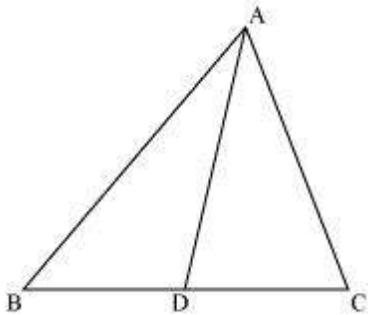
$$\frac{AB}{PQ} = \frac{BC}{QR}$$

$\therefore \triangle ABC \sim \triangle PQR$  (By SAS similarity criterion)

Q13 :

D is a point on the side BC of a triangle ABC such that  $\angle ADC = \angle BAC$ . Show that  $CA^2 = CB \cdot CD$ .

Answer :



In  $\triangle ADC$  and  $\triangle BAC$ ,

$\angle ADC = \angle BAC$  (Given)

$\angle ACD = \angle BCA$  (Common angle)

$\therefore \triangle ADC \sim \triangle BAC$  (By AA similarity criterion)

We know that corresponding sides of similar triangles are in proportion.

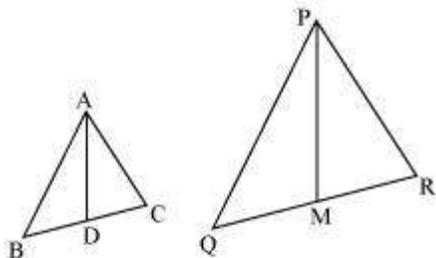
$$\therefore \frac{CA}{CB} = \frac{CD}{CA}$$

$$\Rightarrow CA^2 = CB \times CD$$

Q14 :

Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that  $\triangle ABC \sim \triangle PQR$

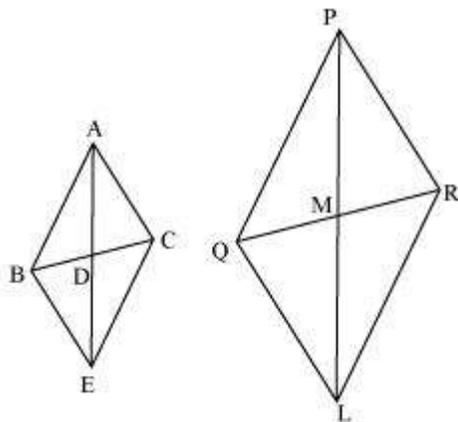
Answer :



Given that,

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$$

Let us extend AD and PM up to point E and L respectively, such that AD = DE and PM = ML. Then, join B to E, C to E, Q to L, and R to L.



We know that medians divide opposite sides.

Therefore, BD = DC and QM = MR

Also, AD = DE (By construction)

And, PM = ML (By construction)

In quadrilateral ABEC, diagonals AE and BC bisect each other at point D.

Therefore, quadrilateral ABEC is a parallelogram.

∴ AC = BE and AB = EC (Opposite sides of a parallelogram are equal)

Similarly, we can prove that quadrilateral PQLR is a parallelogram and PR = QL, PQ = LR

It was given that

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{BE}{QL} = \frac{2AD}{2PM}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{BE}{QL} = \frac{AE}{PL}$$

$\therefore \triangle ABE \sim \triangle PQL$  (By SSS similarity criterion)

We know that corresponding angles of similar triangles are equal.

$\therefore \angle BAE = \angle QPL \dots (1)$

Similarly, it can be proved that  $\triangle AEC \sim \triangle PLR$  and

$\angle CAE = \angle RPL \dots (2)$

Adding equation (1) and (2), we obtain

$\angle BAE + \angle CAE = \angle QPL + \angle RPL$

$\Rightarrow \angle CAB = \angle RPQ \dots (3)$

In  $\triangle ABC$  and  $\triangle PQR$ ,

$$\frac{AB}{PQ} = \frac{AC}{PR} \quad (\text{Given})$$

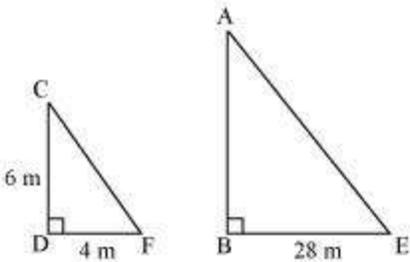
$\angle CAB = \angle RPQ$  [Using equation (3)]

$\therefore \triangle ABC \sim \triangle PQR$  (By SAS similarity criterion)

**Q15 :**

**A vertical pole of a length 6 m casts a shadow 4m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.**

**Answer :**



Let  $AB$  and  $CD$  be a tower and a pole respectively.

Let the shadow of  $BE$  and  $DF$  be the shadow of  $AB$  and  $CD$  respectively.

At the same time, the light rays from the sun will fall on the tower and the pole at the same angle.

Therefore,  $\angle DCF = \angle BAE$

And,  $\angle DFC = \angle BEA$

$\angle CDF = \angle ABE$  (Tower and pole are vertical to the ground)

$\therefore \triangle ABE \sim \triangle CDF$  (AAA similarity criterion)

$$\Rightarrow \frac{AB}{CD} = \frac{BE}{DF}$$

$$\Rightarrow \frac{AB}{6 \text{ m}} = \frac{28}{4}$$

$$\Rightarrow AB = 42 \text{ m}$$

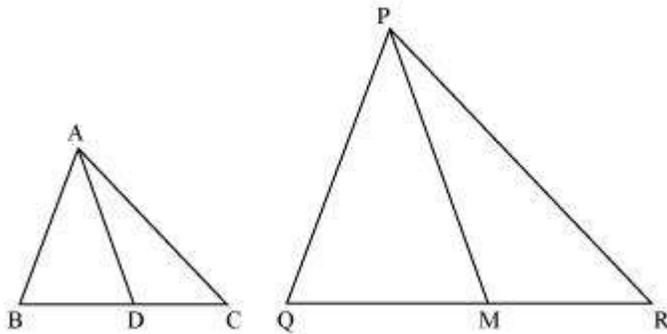
Therefore, the height of the tower will be 42 metres.

**Q16 :**

If AD and PM are medians of triangles ABC and PQR, respectively

$\Delta ABC \sim \Delta PQR$  prove that  $\frac{AB}{PQ} = \frac{AD}{PM}$   
where

**Answer :**



It is given that  $\Delta ABC \sim \Delta PQR$

We know that the corresponding sides of similar triangles are in proportion.

$$\therefore \frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR} \dots (1)$$

Also,  $\angle A = \angle P, \angle B = \angle Q, \angle C = \angle R \dots (2)$

Since AD and PM are medians, they will divide their opposite sides.

$$\therefore BD = \frac{BC}{2} \text{ and } QM = \frac{QR}{2} \dots (3)$$

From equations (1) and (3), we obtain

$$\frac{AB}{PQ} = \frac{BD}{QM} \dots (4)$$

In  $\Delta ABD$  and  $\Delta PQM$ ,

$\angle B = \angle Q$  [Using equation (2)]

$$\frac{AB}{PQ} = \frac{BD}{QM} \text{ [Using equation (4)]}$$

$\therefore \Delta ABD \sim \Delta PQM$  (By SAS similarity criterion)

$$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$$

## Exercise 6.4

Q1 :

Let  $\triangle ABC \sim \triangle DEF$  and their areas be, respectively,  $64 \text{ cm}^2$  and  $121 \text{ cm}^2$ . If  $EF = 15.4 \text{ cm}$ , find  $BC$ .

Answer :

It is given that  $\triangle ABC \sim \triangle DEF$ .

$$\therefore \frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DEF)} = \left(\frac{AB}{DE}\right)^2 = \left(\frac{BC}{EF}\right)^2 = \left(\frac{AC}{DF}\right)^2$$

Given that,

$$EF = 15.4 \text{ cm},$$

$$\text{ar}(\triangle ABC) = 64 \text{ cm}^2,$$

$$\text{ar}(\triangle DEF) = 121 \text{ cm}^2$$

$$\therefore \frac{\text{ar}(ABC)}{\text{ar}(DEF)} = \left(\frac{BC}{EF}\right)^2$$

$$\Rightarrow \left(\frac{64 \text{ cm}^2}{121 \text{ cm}^2}\right) = \frac{BC^2}{(15.4 \text{ cm})^2}$$

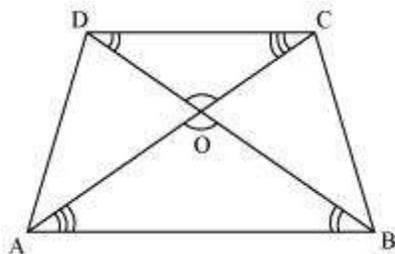
$$\Rightarrow \frac{BC}{15.4} = \left(\frac{8}{11}\right) \text{ cm}$$

$$\Rightarrow BC = \left(\frac{8 \times 15.4}{11}\right) \text{ cm} = (8 \times 1.4) \text{ cm} = 11.2 \text{ cm}$$

Q2 :

Diagonals of a trapezium  $ABCD$  with  $AB \parallel DC$  intersect each other at the point  $O$ . If  $AB = 2CD$ , find the ratio of the areas of triangles  $AOB$  and  $COD$ .

Answer :



Since  $AB \parallel CD$ ,

$\therefore \angle OAB = \angle OCD$  and  $\angle OBA = \angle ODC$  (Alternate interior angles)

In  $\triangle AOB$  and  $\triangle COD$ ,

$\angle AOB = \angle COD$  (Vertically opposite angles)

$\angle OAB = \angle OCD$  (Alternate interior angles)

$\angle OBA = \angle ODC$  (Alternate interior angles)

$\therefore \triangle AOB \sim \triangle COD$  (By AAA similarity criterion)

$$\therefore \frac{\text{ar}(\triangle AOB)}{\text{ar}(\triangle COD)} = \left(\frac{AB}{CD}\right)^2$$

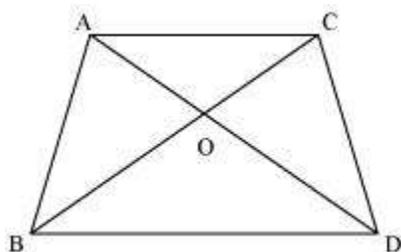
Since  $AB = 2 CD$ ,

$$\therefore \frac{\text{ar}(\triangle AOB)}{\text{ar}(\triangle COD)} = \left(\frac{2 CD}{CD}\right)^2 = \frac{4}{1} = 4 : 1$$

Q3 :

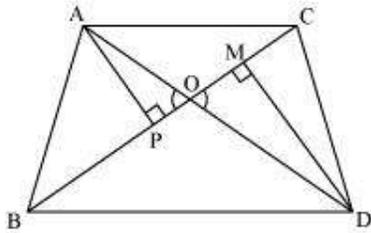
In the following figure,  $\triangle ABC$  and  $\triangle DBC$  are two triangles on the same base  $BC$ . If  $AD$  intersects  $BC$  at  $O$ , show

that 
$$\frac{\text{area}(\triangle ABC)}{\text{area}(\triangle DBC)} = \frac{AO}{DO}$$



Answer :

Let us draw two perpendiculars  $AP$  and  $DM$  on line  $BC$ .



We know that area of a triangle =  $\frac{1}{2} \times \text{Base} \times \text{Height}$

$$\therefore \frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DBC)} = \frac{\frac{1}{2} BC \times AP}{\frac{1}{2} BC \times DM} = \frac{AP}{DM}$$

In  $\triangle APO$  and  $\triangle DMO$ ,

$\angle APO = \angle DMO$  (Each =  $90^\circ$ )

$\angle AOP = \angle DOM$  (Vertically opposite angles)

$\therefore \triangle APO \sim \triangle DMO$  (By AA similarity criterion)

$$\begin{aligned} \therefore \frac{AP}{DM} &= \frac{AO}{DO} \\ \Rightarrow \frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DBC)} &= \frac{AO}{DO} \end{aligned}$$

**Q4 :**

If the areas of two similar triangles are equal, prove that they are congruent.

**Answer :**

Let us assume two similar triangles as  $\triangle ABC \sim \triangle PQR$ .

$$\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2 \quad (1)$$

Given that,  $\text{ar}(\triangle ABC) = \text{ar}(\triangle PQR)$

$$\Rightarrow \frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle PQR)} = 1$$

Putting this value in equation (1), we obtain

$$1 = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$$

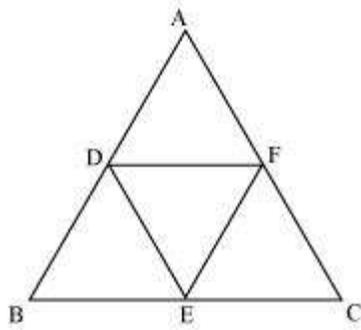
$\Rightarrow AB = PQ, BC = QR, \text{ and } AC = PR$

$\therefore \triangle ABC \cong \triangle PQR$  (By SSS congruence criterion)

**Q5 :**

D, E and F are respectively the mid-points of sides AB, BC and CA of  $\triangle ABC$ . Find the ratio of the area of  $\triangle DEF$  and  $\triangle ABC$ .

Answer :



D and E are the mid-points of  $\Delta ABC$ .

$$\therefore DE \parallel AC \text{ and } DE = \frac{1}{2} AC$$

In  $\Delta BED$  and  $\Delta BCA$ ,

$$\angle BED = \angle BCA \quad (\text{Corresponding angles})$$

$$\angle BDE = \angle BAC \quad (\text{Corresponding angles})$$

$$\angle EBD = \angle CBA \quad (\text{Common angles})$$

$$\therefore \Delta BED \sim \Delta BCA \quad (\text{AAA similarity criterion})$$

$$\frac{\text{ar}(\Delta BED)}{\text{ar}(\Delta BCA)} = \left(\frac{DE}{AC}\right)^2$$

$$\Rightarrow \frac{\text{ar}(\Delta BED)}{\text{ar}(\Delta BCA)} = \frac{1}{4}$$

$$\Rightarrow \text{ar}(\Delta BED) = \frac{1}{4} \text{ar}(\Delta BCA)$$

$$\text{Similarly, } \text{ar}(\Delta CFE) = \frac{1}{4} \text{ar}(\Delta CBA) \text{ and } \text{ar}(\Delta ADF) = \frac{1}{4} \text{ar}(\Delta ABC)$$

$$\text{Also, } \text{ar}(\Delta DEF) = \text{ar}(\Delta ABC) - [\text{ar}(\Delta BED) + \text{ar}(\Delta CFE) + \text{ar}(\Delta ADF)]$$

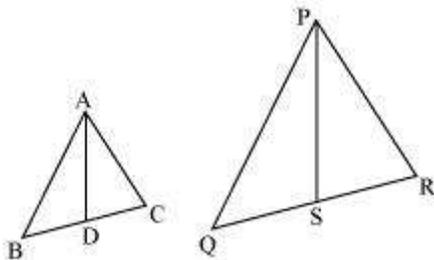
$$\Rightarrow \text{ar}(\Delta DEF) = \text{ar}(\Delta ABC) - \frac{3}{4} \text{ar}(\Delta ABC) = \frac{1}{4} \text{ar}(\Delta ABC)$$

$$\Rightarrow \frac{\text{ar}(\Delta DEF)}{\text{ar}(\Delta ABC)} = \frac{1}{4}$$

Q6 :

Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians.

Answer :



Let us assume two similar triangles as  $\Delta ABC \sim \Delta PQR$ . Let AD and PS be the medians of these triangles.

∴  $\triangle ABC \sim \triangle PQR$

$$\therefore \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} \dots (1)$$

$$\angle A = \angle P, \angle B = \angle Q, \angle C = \angle R \dots (2)$$

Since AD and PS are medians,

$$\therefore BD = DC = \frac{BC}{2}$$

$$\text{And, } QS = SR = \frac{QR}{2}$$

Equation (1) becomes

$$\frac{AB}{PQ} = \frac{BD}{QS} = \frac{AC}{PR} \dots (3)$$

In  $\triangle ABD$  and  $\triangle PQS$ ,

$$\angle B = \angle Q \text{ [Using equation (2)]}$$

$$\text{And, } \frac{AB}{PQ} = \frac{BD}{QS} \text{ [Using equation (3)]}$$

∴  $\triangle ABD \sim \triangle PQS$  (SAS similarity criterion)

Therefore, it can be said that

$$\frac{AB}{PQ} = \frac{BD}{QS} = \frac{AD}{PS} \dots (4)$$

$$\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$$

From equations (1) and (4), we may find that

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{AD}{PS}$$

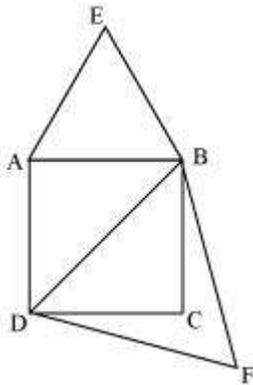
And hence,

$$\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle PQR)} = \left(\frac{AD}{PS}\right)^2$$

**Q7 :**

**Prove that the area of an equilateral triangle described on one side of a square is equal to half the area of the equilateral triangle described on one of its diagonals.**

Answer :



Let ABCD be a square of side  $a$ .

Therefore, its diagonal  $= \sqrt{2}a$

Two desired equilateral triangles are formed as  $\triangle ABE$  and  $\triangle DBF$ .

Side of an equilateral triangle,  $\triangle ABE$ , described on one of its sides  $= a$

Side of an equilateral triangle,  $\triangle DBF$ , described on one of its diagonals  $= \sqrt{2}a$

We know that equilateral triangles have all its angles as  $60^\circ$  and all its sides of the same length. Therefore, all equilateral triangles are similar to each other. Hence, the ratio between the areas of these triangles will be equal to the square of the ratio between the sides of these triangles.

$$\frac{\text{Area of } \triangle ABE}{\text{Area of } \triangle DBF} = \left( \frac{a}{\sqrt{2}a} \right)^2 = \frac{1}{2}$$

Q8 :

ABC and BDE are two equilateral triangles such that D is the mid-point of BC. Ratio of the area of triangles ABC and BDE is

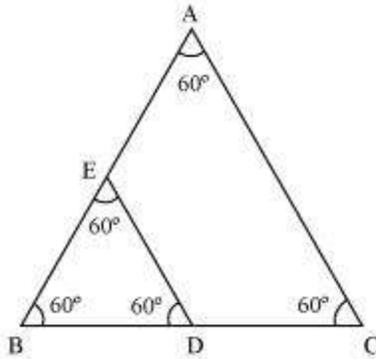
(A) 2 : 1

(B) 1 : 2

(C) 4 : 1

(D) 1 : 4

**Answer :**



We know that equilateral triangles have all its angles as  $60^\circ$  and all its sides of the same length. Therefore, all equilateral triangles are similar to each other. Hence, the ratio between the areas of these triangles will be equal to the square of the ratio between the sides of these triangles.

Let side of  $\triangle ABC = x$

Therefore, side of  $\triangle BDE = \frac{x}{2}$

$$\therefore \frac{\text{area}(\triangle ABC)}{\text{area}(\triangle BDE)} = \left(\frac{x}{\frac{x}{2}}\right)^2 = 4$$

Hence, the correct answer is (C).

**Q9 :**

**Sides of two similar triangles are in the ratio 4 : 9. Areas of these triangles are in the ratio**

(A) 2 : 3

(B) 4 : 9

(C) 81 : 16

(D) 16 : 81

**Answer :**

If two triangles are similar to each other, then the ratio of the areas of these triangles will be equal to the square of the ratio of the corresponding sides of these triangles.

It is given that the sides are in the ratio 4:9.

Therefore, ratio between areas of these triangles =  $\left(\frac{4}{9}\right)^2 = \frac{16}{81}$

Hence, the correct answer is (D).

## Exercise 6.5

Q1 :

Sides of triangles are given below. Determine which of them are right triangles? In case of a right triangle, write the length of its hypotenuse.

(i) 7 cm, 24 cm, 25 cm

(ii) 3 cm, 8 cm, 6 cm

(iii) 50 cm, 80 cm, 100 cm

(iv) 13 cm, 12 cm, 5 cm

**Answer :**

(i) It is given that the sides of the triangle are 7 cm, 24 cm, and 25 cm.

Squaring the lengths of these sides, we will obtain 49, 576, and 625.

$$49 + 576 = 625$$

$$\text{Or, } 7^2 + 24^2 = 25^2$$

The sides of the given triangle are satisfying Pythagoras theorem.

Therefore, it is a right triangle.

We know that the longest side of a right triangle is the hypotenuse.

Therefore, the length of the hypotenuse of this triangle is 25 cm.

(ii) It is given that the sides of the triangle are 3 cm, 8 cm, and 6 cm.

Squaring the lengths of these sides, we will obtain 9, 64, and 36.

$$\text{However, } 9 + 36 \neq 64$$

$$\text{Or, } 3^2 + 6^2 \neq 8^2$$

Clearly, the sum of the squares of the lengths of two sides is not equal to the square of the length of the third side.

Therefore, the given triangle is not satisfying Pythagoras theorem.

Hence, it is not a right triangle.

(iii) Given that sides are 50 cm, 80 cm, and 100 cm.

Squaring the lengths of these sides, we will obtain 2500, 6400, and 10000.

$$\text{However, } 2500 + 6400 \neq 10000$$

$$\text{Or, } 50^2 + 80^2 \neq 100^2$$

Clearly, the sum of the squares of the lengths of two sides is not equal to the square of the length of the third side.

Therefore, the given triangle is not satisfying Pythagoras theorem.

Hence, it is not a right triangle.

(iv) Given that sides are 13 cm, 12 cm, and 5 cm.

Squaring the lengths of these sides, we will obtain 169, 144, and 25.

Clearly,  $144 + 25 = 169$

$$\text{Or, } 12^2 + 5^2 = 13^2$$

The sides of the given triangle are satisfying Pythagoras theorem.

Therefore, it is a right triangle.

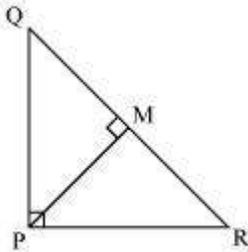
We know that the longest side of a right triangle is the hypotenuse.

Therefore, the length of the hypotenuse of this triangle is 13 cm.

**Q2 :**

**PQR is a triangle right angled at P and M is a point on QR such that  $PM \perp QR$ . Show that  $PM^2 = QM \times MR$ .**

**Answer :**



Let  $\angle MPR = x$

In  $\triangle MPR$ ,

$$\angle MRP = 180^\circ - 90^\circ - x$$

$$\angle MRP = 90^\circ - x$$

Similarly, in  $\triangle MPQ$ ,

$$\angle MPQ = 90^\circ - \angle MPR$$

$$= 90^\circ - x$$

$$\angle MQP = 180^\circ - 90^\circ - (90^\circ - x)$$

$$\angle MQP = x$$

In  $\triangle QMP$  and  $\triangle PMR$ ,

$$\angle MPQ = \angle MRP$$

$$\angle PMQ = \angle RMP$$

$$\angle MQP = \angle MPR$$

$\therefore \triangle QMP \sim \triangle PMR$  (By AAA similarity criterion)

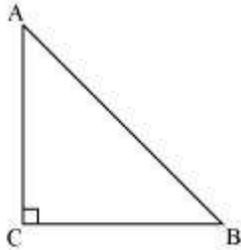
$$\Rightarrow \frac{QM}{PM} = \frac{MP}{MR}$$

$$\Rightarrow PM^2 = QM \times MR$$

Q3 :

ABC is an isosceles triangle right angled at C. prove that  $AB^2 = 2 AC^2$ .

Answer :



Given that  $\triangle ABC$  is an isosceles triangle.

$\therefore AC = CB$

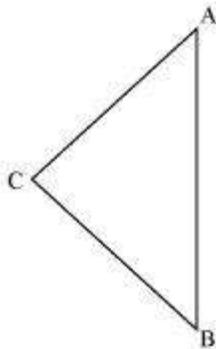
Applying Pythagoras theorem in  $\triangle ABC$  (i.e., right-angled at point C), we obtain

$$\begin{aligned} AC^2 + CB^2 &= AB^2 \\ \Rightarrow AC^2 + AC^2 &= AB^2 \quad (AC=CB) \\ \Rightarrow 2AC^2 &= AB^2 \end{aligned}$$

Q4 :

ABC is an isosceles triangle with  $AC = BC$ . If  $AB^2 = 2 AC^2$ , prove that ABC is a right triangle.

Answer :



Given that,

$$\begin{aligned} AB^2 &= 2AC^2 \\ \Rightarrow AB^2 &= AC^2 + AC^2 \\ \Rightarrow AB^2 &= AC^2 + BC^2 \quad (\text{As } AC = BC) \end{aligned}$$

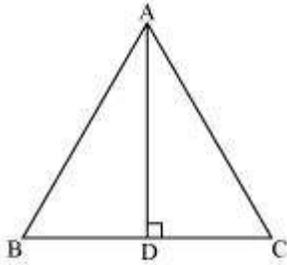
The triangle is satisfying the pythagoras theorem.

Therefore, the given triangle is a right - angled triangle.

Q5 :

ABC is an equilateral triangle of side  $2a$ . Find each of its altitudes.

Answer :



Let AD be the altitude in the given equilateral triangle,  $\Delta ABC$ .

We know that altitude bisects the opposite side.

$$\therefore BD = DC = a$$

In  $\Delta ADB$ ,

$$\angle ADB = 90^\circ$$

Applying pythagoras theorem, we obtain

$$AD^2 + DB^2 = AB^2$$

$$\Rightarrow AD^2 + a^2 = (2a)^2$$

$$\Rightarrow AD^2 + a^2 = 4a^2$$

$$\Rightarrow AD^2 = 3a^2$$

$$\Rightarrow AD = a\sqrt{3}$$

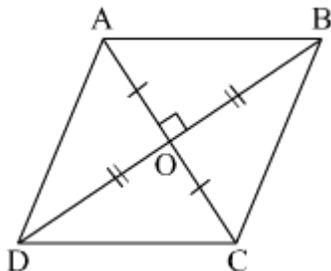
In an equilateral triangle, all the altitudes are equal in length.

Therefore, the length of each altitude will be  $\sqrt{3}a$ .

Q6 :

Prove that the sum of the squares of the sides of rhombus is equal to the sum of the squares of its diagonals.

Answer :



In  $\Delta AOB$ ,  $\Delta BOC$ ,  $\Delta COD$ ,  $\Delta AOD$ ,

Applying Pythagoras theorem, we obtain

$$AB^2 = AO^2 + OB^2 \quad \dots (1)$$

$$BC^2 = BO^2 + OC^2 \quad \dots (2)$$

$$CD^2 = CO^2 + OD^2 \quad \dots (3)$$

$$AD^2 = AO^2 + OD^2 \quad \dots (4)$$

Adding all these equations, we obtain

$$AB^2 + BC^2 + CD^2 + AD^2 = 2(AO^2 + OB^2 + OC^2 + OD^2)$$

$$= 2\left(\left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2 + \left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2\right)$$

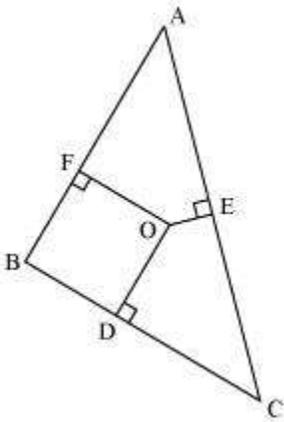
(Diagonals bisect each other)

$$= 2\left(\frac{(AC)^2}{2} + \frac{(BD)^2}{2}\right)$$

$$= (AC)^2 + (BD)^2$$

Q7 :

In the following figure, O is a point in the interior of a triangle ABC,  $OD \perp BC$ ,  $OE \perp AC$  and  $OF \perp AB$ . Show that

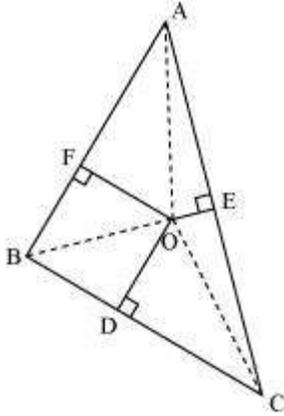


(i)  $OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$

(ii)  $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$

Answer :

Join OA, OB, and OC.



(i) Applying Pythagoras theorem in  $\triangle AOF$ , we obtain

$$OA^2 = OF^2 + AF^2$$

Similarly, in  $\triangle BOD$ ,

$$OB^2 = OD^2 + BD^2$$

Similarly, in  $\triangle COE$ ,

$$OC^2 = OE^2 + EC^2$$

Adding these equations,

$$OA^2 + OB^2 + OC^2 = OF^2 + AF^2 + OD^2 + BD^2 + OE^2 + EC^2$$

$$OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + EC^2$$

(ii) From the above result,

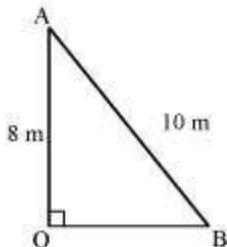
$$AF^2 + BD^2 + EC^2 = (OA^2 - OE^2) + (OC^2 - OD^2) + (OB^2 - OF^2)$$

$$\therefore AF^2 + BD^2 + EC^2 = AE^2 + CD^2 + BF^2$$

**Q8 :**

A ladder 10 m long reaches a window 8 m above the ground. Find the distance of the foot of the ladder from base of the wall.

**Answer :**



Let OA be the wall and AB be the ladder.

Therefore, by Pythagoras theorem,

$$AB^2 = OA^2 + BO^2$$

$$(10 \text{ m})^2 = (8 \text{ m})^2 + OB^2$$

$$100 \text{ m}^2 = 64 \text{ m}^2 + OB^2$$

$$OB^2 = 36 \text{ m}^2$$

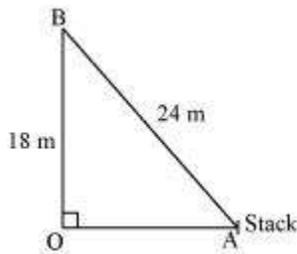
$$OB = 6 \text{ m}$$

Therefore, the distance of the foot of the ladder from the base of the wall is 6 m.

**Q9 :**

A guy wire attached to a vertical pole of height 18 m is 24 m long and has a stake attached to the other end. How far from the base of the pole should the stake be driven so that the wire will be taut?

**Answer :**



Let OB be the pole and AB be the wire.

By Pythagoras theorem,

$$AB^2 = OB^2 + OA^2$$

$$(24 \text{ m})^2 = (18 \text{ m})^2 + OA^2$$

$$OA^2 = (576 - 324) \text{ m}^2 = 252 \text{ m}^2$$

$$OA = \sqrt{252} \text{ m} = \sqrt{6 \times 6 \times 7} \text{ m} = 6\sqrt{7} \text{ m}$$

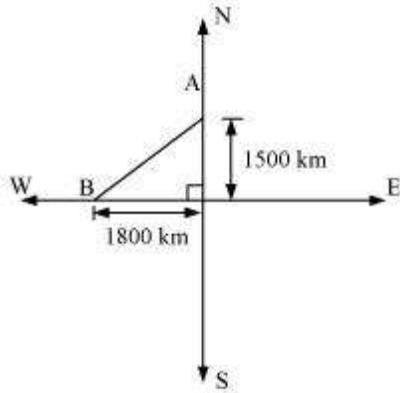
Therefore, the distance from the base is  $6\sqrt{7}$  m.

**Q10 :**

An aeroplane leaves an airport and flies due north at a speed of 1,000 km per hour. At the same time, another aeroplane leaves the same airport and flies due west at a speed of 1,200 km per hour. How far apart will be

the two planes after  $1\frac{1}{2}$  hours?

Answer :



Distance travelled by the plane flying towards north in  $1\frac{1}{2}$  hrs =  $1,000 \times 1\frac{1}{2} = 1,500$  km

Similarly, distance travelled by the plane flying towards west in  $1\frac{1}{2}$  hrs =  $1,200 \times 1\frac{1}{2} = 1,800$  km

Let these distances be represented by OA and OB respectively.

Applying Pythagoras theorem,

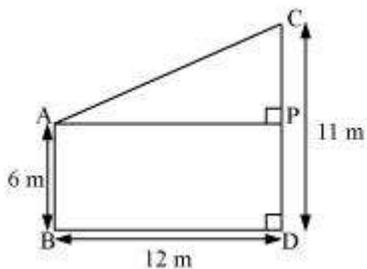
$$\begin{aligned} \text{Distance between these planes after } 1\frac{1}{2} \text{ hrs, } AB &= \sqrt{OA^2 + OB^2} \\ &= \left( \sqrt{(1,500)^2 + (1,800)^2} \right) \text{ km} = \left( \sqrt{2250000 + 3240000} \right) \text{ km} \\ &= \left( \sqrt{5490000} \right) \text{ km} = \left( \sqrt{9 \times 610000} \right) \text{ km} = 300\sqrt{61} \text{ km} \end{aligned}$$

Therefore, the distance between these planes will be  $300\sqrt{61}$  km after  $1\frac{1}{2}$  hrs

Q11 :

Two poles of heights 6 m and 11 m stand on a plane ground. If the distance between the feet of the poles is 12 m, find the distance between their tops.

Answer :



Let CD and AB be the poles of height 11 m and 6 m.

Therefore,  $CP = 11 - 6 = 5$  m

From the figure, it can be observed that  $AP = 12$  m

Applying Pythagoras theorem for  $\triangle APC$ , we obtain

$$AP^2 + PC^2 = AC^2$$

$$(12 \text{ m})^2 + (5 \text{ m})^2 = AC^2$$

$$AC^2 = (144 + 25) \text{ m}^2 = 169 \text{ m}^2$$

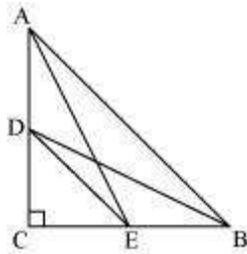
$$AC = 13 \text{ m}$$

Therefore, the distance between their tops is 13 m.

**Q12 :**

D and E are points on the sides CA and CB respectively of a triangle ABC right angled at C. Prove that  $AE^2 + BD^2 = AB^2 + DE^2$

**Answer :**



Applying Pythagoras theorem in  $\triangle ACE$ , we obtain

$$AC^2 + CE^2 = AE^2 \quad \dots (1)$$

Applying Pythagoras theorem in  $\triangle BCD$ , we obtain

$$BC^2 + CD^2 = BD^2 \quad \dots (2)$$

Using equation (1) and equation (2), we obtain

$$AC^2 + CE^2 + BC^2 + CD^2 = AE^2 + BD^2 \quad \dots (3)$$

Applying Pythagoras theorem in  $\triangle CDE$ , we obtain

$$DE^2 = CD^2 + CE^2$$

Applying Pythagoras theorem in  $\triangle ABC$ , we obtain

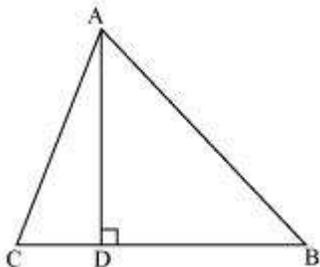
$$AB^2 = AC^2 + CB^2$$

Putting the values in equation (3), we obtain

$$DE^2 + AB^2 = AE^2 + BD^2$$

**Q13 :**

The perpendicular from A on side BC of a  $\triangle ABC$  intersect BC at D such that  $DB = 3 CD$ . Prove that  $2 AB^2 = 2 AC^2 + BC^2$



**Answer :**

Applying Pythagoras theorem for  $\triangle ACD$ , we obtain

$$AC^2 = AD^2 + DC^2$$

$$AD^2 = AC^2 - DC^2 \quad \dots (1)$$

Applying Pythagoras theorem in  $\triangle ABD$ , we obtain

$$AB^2 = AD^2 + DB^2$$

$$AD^2 = AB^2 - DB^2 \quad \dots (2)$$

From equation (1) and equation (2), we obtain

$$AC^2 - DC^2 = AB^2 - DB^2 \quad \dots (3)$$

It is given that  $3DC = DB$

$$\therefore DC = \frac{BC}{4} \text{ and } DB = \frac{3BC}{4}$$

Putting these values in equation (3), we obtain

$$AC^2 - \left(\frac{BC}{4}\right)^2 = AB^2 - \left(\frac{3BC}{4}\right)^2$$

$$AC^2 - \frac{BC^2}{16} = AB^2 - \frac{9BC^2}{16}$$

$$16AC^2 - BC^2 = 16AB^2 - 9BC^2$$

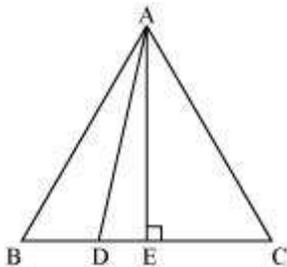
$$16AB^2 - 16AC^2 = 8BC^2$$

$$2AB^2 = 2AC^2 + BC^2$$

**Q14 :**

In an equilateral triangle  $ABC$ ,  $D$  is a point on side  $BC$  such that  $BD = \frac{1}{3} BC$ . Prove that  $9 AD^2 = 7 AB^2$ .

**Answer :**



Let the side of the equilateral triangle be  $a$ , and  $AE$  be the altitude of  $\triangle ABC$ .

$$\therefore BE = EC = \frac{BC}{2} = \frac{a}{2}$$

$$\text{And, } AE = \frac{a\sqrt{3}}{2}$$

$$\text{Given that, } BD = \frac{1}{3} BC$$

$$\therefore BD = \frac{a}{3}$$

$$DE = BE - BD = \frac{a}{2} - \frac{a}{3} = \frac{a}{6}$$

Applying Pythagoras theorem in  $\triangle ADE$ , we obtain

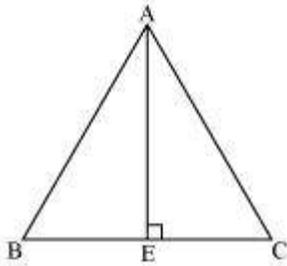
$$AD^2 = AE^2 + DE^2$$

$$\begin{aligned} AD^2 &= \left(\frac{a\sqrt{3}}{2}\right)^2 + \left(\frac{a}{6}\right)^2 \\ &= \left(\frac{3a^2}{4}\right) + \left(\frac{a^2}{36}\right) \\ &= \frac{28a^2}{36} \\ &= \frac{7}{9}AB^2 \end{aligned}$$

$$\Rightarrow 9 AD^2 = 7 AB^2$$

In an equilateral triangle, prove that three times the square of one side is equal to four times the square of one of its altitudes.

Answer :



Let the side of the equilateral triangle be  $a$ , and  $AE$  be the altitude of  $\triangle ABC$ .

$$\therefore BE = EC = \frac{BC}{2} = \frac{a}{2}$$

Applying Pythagoras theorem in  $\triangle ABE$ , we obtain

$$AB^2 = AE^2 + BE^2$$

$$a^2 = AE^2 + \left(\frac{a}{2}\right)^2$$

$$AE^2 = a^2 - \frac{a^2}{4}$$

$$AE^2 = \frac{3a^2}{4}$$

$$4AE^2 = 3a^2$$

$\Rightarrow 4 \times (\text{Square of altitude}) = 3 \times (\text{Square of one side})$

**Q16 :**

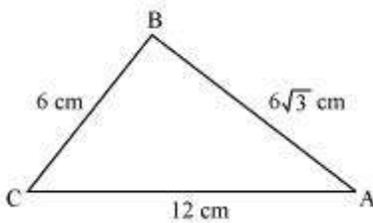
Tick the correct answer and justify: In  $\Delta ABC$ ,  $AB = 6\sqrt{3}$  cm,  $AC = 12$  cm and  $BC = 6$  cm.

The angle B is:

(A)  $120^\circ$  (B)  $60^\circ$

(C)  $90^\circ$  (D)  $45^\circ$

**Answer :**



Given that,  $AB = 6\sqrt{3}$  cm,  $AC = 12$  cm, and  $BC = 6$  cm

It can be observed that

$$AB^2 = 108$$

$$AC^2 = 144$$

$$\text{And, } BC^2 = 36$$

$$AB^2 + BC^2 = AC^2$$

The given triangle,  $\Delta ABC$ , is satisfying Pythagoras theorem.

Therefore, the triangle is a right triangle, right-angled at B.

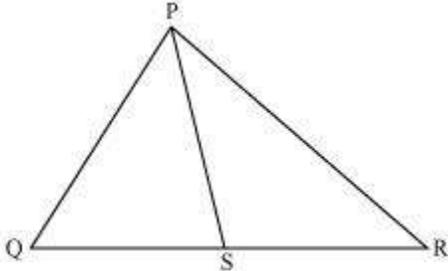
$$\therefore \angle B = 90^\circ$$

Hence, the correct answer is (C).

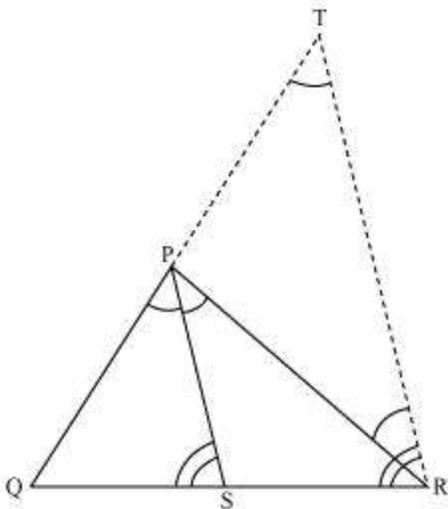
**Exercise 6.6 :**

**Q1 :**

In the given figure, PS is the bisector of  $\angle QPR$  of  $\Delta PQR$ . Prove that  $\frac{QS}{SR} = \frac{PQ}{PR}$ .



**Answer :**



Let us draw a line segment RT parallel to SP which intersects extended line segment QP at point T.

Given that, PS is the angle bisector of  $\angle QPR$ .

$$\angle QPS = \angle SPR \dots (1)$$

By construction,

$$\angle SPR = \angle PRT \text{ (As } PS \parallel TR \text{) } \dots (2)$$

$$\angle QPS = \angle QTR \text{ (As } PS \parallel TR \text{) } \dots (3)$$

Using these equations, we obtain

$$\angle PRT = \angle QTR$$

$$\therefore PT = PR$$

By construction,

$$PS \parallel TR$$

By using basic proportionality theorem for  $\Delta QTR$ ,

$$\Rightarrow QS/SR=QP/PT$$

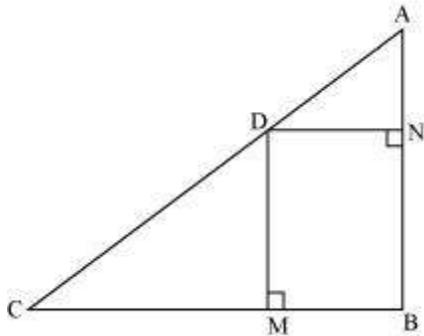
$$\Rightarrow \frac{QS}{SR} = \frac{PQ}{PR}$$

**Q2 :**

In the given figure, D is a point on hypotenuse AC of  $\Delta ABC$ ,  $DM \perp BC$  and  $DN \perp AB$ , Prove that:

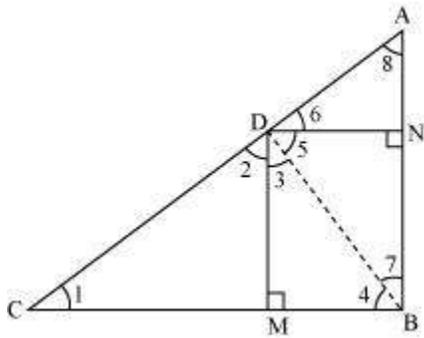
(i)  $DM^2 = DN \cdot MC$

(ii)  $DN^2 = DM \cdot AN$



**Answer :**

(i) Let us join DB.



We have,  $DN \parallel CB$ ,  $DM \parallel AB$ , and  $\angle B = 90^\circ$

$\therefore$  DMBN is a rectangle.

$\therefore DN = MB$  and  $DM = NB$

The condition to be proved is the case when D is the foot of the perpendicular drawn from B to AC.

$\therefore \angle CDB = 90^\circ$

$$\Rightarrow \angle 2 + \angle 3 = 90^\circ \dots (1)$$

In  $\Delta CDM$ ,

$$\angle 1 + \angle 2 + \angle DMC = 180^\circ$$

$$\Rightarrow \angle 1 + \angle 2 = 90^\circ \dots (2)$$

In  $\triangle DMB$ ,

$$\angle 3 + \angle DMB + \angle 4 = 180^\circ$$

$$\Rightarrow \angle 3 + \angle 4 = 90^\circ \dots (3)$$

From equation (1) and (2), we obtain

$$\angle 1 = \angle 3$$

From equation (1) and (3), we obtain

$$\angle 2 = \angle 4$$

In  $\triangle DCM$  and  $\triangle BDM$ ,

$$\angle 1 = \angle 3 \text{ (Proved above)}$$

$$\angle 2 = \angle 4 \text{ (Proved above)}$$

$\therefore \triangle DCM \sim \triangle BDM$  (AA similarity criterion)

$$\Rightarrow \frac{BM}{DM} = \frac{DM}{MC}$$
$$\Rightarrow \frac{DN}{DM} = \frac{DM}{MC} \quad (BM = DN)$$

$$\Rightarrow DM^2 = DN \times MC$$

(ii) In right triangle  $DBN$ ,

$$\angle 5 + \angle 7 = 90^\circ \dots (4)$$

In right triangle  $DAN$ ,

$$\angle 6 + \angle 8 = 90^\circ \dots (5)$$

$D$  is the foot of the perpendicular drawn from  $B$  to  $AC$ .

$$\therefore \angle ADB = 90^\circ$$

$$\Rightarrow \angle 5 + \angle 6 = 90^\circ \dots (6)$$

From equation (4) and (6), we obtain

$$\angle 6 = \angle 7$$

From equation (5) and (6), we obtain

$$\angle 8 = \angle 5$$

In  $\triangle DNA$  and  $\triangle BND$ ,

$$\angle 6 = \angle 7 \text{ (Proved above)}$$

$$\angle 8 = \angle 5 \text{ (Proved above)}$$

$\therefore \triangle DNA \sim \triangle BND$  (AA similarity criterion)

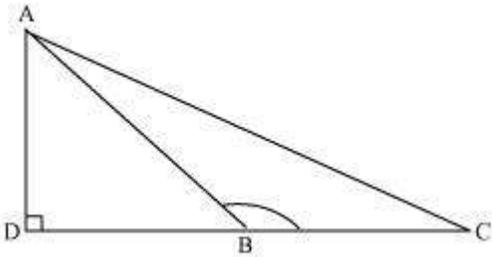
$$\Rightarrow \frac{AN}{DN} = \frac{DN}{NB}$$

$$\Rightarrow DN^2 = AN \times NB$$

$$\Rightarrow DN^2 = AN \times DM \text{ (As } NB = DM)$$

**Q3 :**

In the given figure, ABC is a triangle in which  $\angle ABC > 90^\circ$  and  $AD \perp CB$  produced. Prove that  $AC^2 = AB^2 + BC^2 + 2BC \cdot BD$ .



**Answer :**

Applying Pythagoras theorem in  $\triangle ADB$ , we obtain

$$AB^2 = AD^2 + DB^2 \dots (1)$$

Applying Pythagoras theorem in  $\triangle ADC$ , we obtain

$$AC^2 = AD^2 + DC^2$$

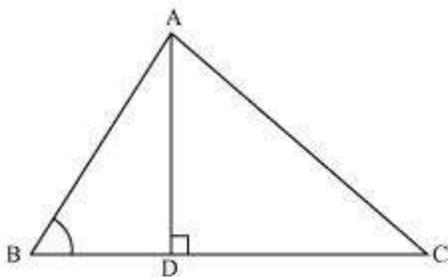
$$AC^2 = AD^2 + (DB + BC)^2$$

$$AC^2 = AD^2 + DB^2 + BC^2 + 2DB \times BC$$

$$AC^2 = AB^2 + BC^2 + 2DB \times BC \text{ [Using equation (1)]}$$

**Q4 :**

In the given figure, ABC is a triangle in which  $\angle ABC < 90^\circ$  and  $AD \perp BC$ . Prove that  $AC^2 = AB^2 + BC^2 - 2BC \cdot BD$ .



**Answer :**

Applying Pythagoras theorem in  $\triangle ADB$ , we obtain

$$AD^2 + DB^2 = AB^2$$

$$\Rightarrow AD^2 = AB^2 - DB^2 \dots (1)$$

Applying Pythagoras theorem in  $\triangle ADC$ , we obtain

$$AD^2 + DC^2 = AC^2$$

$$AB^2 - BD^2 + DC^2 = AC^2 \text{ [Using equation (1)]}$$

$$AB^2 - BD^2 + (BC - BD)^2 = AC^2$$

$$AC^2 = AB^2 - BD^2 + BC^2 + BD^2 - 2BC \times BD$$

$$= AB^2 + BC^2 - 2BC \times BD$$

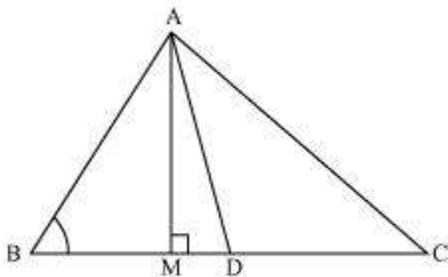
**Q5 :**

In the given figure, AD is a median of a triangle ABC and  $AM \perp BC$ . Prove that:

$$(i) \quad AC^2 = AD^2 + BC \cdot DM + \left(\frac{BC}{2}\right)^2$$

$$(ii) \quad AB^2 = AD^2 - BC \cdot DM + \left(\frac{BC}{2}\right)^2$$

$$(iii) \quad AC^2 + AB^2 = 2AD^2 + \frac{1}{2}BC^2$$



**Answer :**

(i) Applying Pythagoras theorem in  $\triangle AMD$ , we obtain

$$AM^2 + MD^2 = AD^2 \dots (1)$$

Applying Pythagoras theorem in  $\triangle AMC$ , we obtain

$$AM^2 + MC^2 = AC^2$$

$$AM^2 + (MD + DC)^2 = AC^2$$

$$(AM^2 + MD^2) + DC^2 + 2MD \cdot DC = AC^2$$

$$AD^2 + DC^2 + 2MD \cdot DC = AC^2 \text{ [Using equation (1)]}$$

Using the result,  $DC = \frac{BC}{2}$ , we obtain

$$AD^2 + \left(\frac{BC}{2}\right)^2 + 2MD \cdot \left(\frac{BC}{2}\right) = AC^2$$

$$AD^2 + \left(\frac{BC}{2}\right)^2 + MD \times BC = AC^2$$

(ii) Applying Pythagoras theorem in  $\triangle ABM$ , we obtain

$$AB^2 = AM^2 + MB^2$$

$$= (AD^2 - DM^2) + MB^2$$

$$= (AD^2 - DM^2) + (BD - MD)^2$$

$$= AD^2 - DM^2 + BD^2 + MD^2 - 2BD \times MD$$

$$= AD^2 + BD^2 - 2BD \times MD$$

$$= AD^2 + \left(\frac{BC}{2}\right)^2 - 2\left(\frac{BC}{2}\right) \times MD$$

$$= AD^2 + \left(\frac{BC}{2}\right)^2 - BC \times MD$$

(iii) Applying Pythagoras theorem in  $\triangle ABM$ , we obtain

$$AM^2 + MB^2 = AB^2 \dots (1)$$

Applying Pythagoras theorem in  $\triangle AMC$ , we obtain

$$AM^2 + MC^2 = AC^2 \dots (2)$$

Adding equations (1) and (2), we obtain

$$2AM^2 + MB^2 + MC^2 = AB^2 + AC^2$$

$$2AM^2 + (BD - DM)^2 + (MD + DC)^2 = AB^2 + AC^2$$

$$2AM^2 + BD^2 + DM^2 - 2BD \cdot DM + MD^2 + DC^2 + 2MD \cdot DC = AB^2 + AC^2$$

$$2AM^2 + 2MD^2 + BD^2 + DC^2 + 2MD(-BD + DC) = AB^2 + AC^2$$

$$2\left(AM^2 + MD^2\right) + \left(\frac{BC}{2}\right)^2 + \left(\frac{BC}{2}\right)^2 + 2MD\left(-\frac{BC}{2} + \frac{BC}{2}\right) = AB^2 + AC^2$$

$$2AD^2 + \frac{BC^2}{2} = AB^2 + AC^2$$