Class 10 Maths NCERT Solutions Unit - 12

Areas Related to Circles

Exercise 12.1

Q1 :

The radii of two circles are 19 cm and 9 cm respectively. Find the radius of the circle which has circumference equal to the sum of the circumferences of the two circles.

Answer :

Radius (r_1) of 1st circle = 19 cm

Radius (r_2) or 2nd circle = 9 cm

Let the radius of 3rd circle be r.

Circumference of 1st circle = $2\pi r_1 = 2\pi(19) = 38\pi$

Circumference of 2nd circle = $2\pi r_2 = 2\pi(9) = 18\pi$

Circumference of 3rd circle = $2\pi r$

Given that,

Circumference of 3rd circle = Circumference of 1st circle + Circumference of 2nd circle

$2\pi r = 38\pi + 18\pi = 56\pi$

$$r = \frac{56\pi}{2\pi} = 28$$

Therefore, the radius of the circle which has circumference equal to the sum of the circumference of the given two circles is 28 cm.

Q2 :

The radii of two circles are 8 cm and 6 cm respectively. Find the radius of the circle having area equal to the sum of the areas of the two circles.

Answer :

Radius (r_1) of 1st circle = 8 cm

Radius (r_2) of 2nd circle = 6 cm

Let the radius of 3rd circle be r.

Area of 1st circle $= \pi r_1^2 = \pi(8)^2 = 64\pi$

Area of 2nd circle $= \pi r_2^2 = \pi(6)^2 = 36\pi$

Given that,

Area of 3rd circle = Area of 1st circle + Area of 2nd circle

$$\pi r^2 = \pi r_1^2 + \pi r_2^2$$

$$\pi r^2 = 64\pi + 36\pi$$

$$\pi r^2 = 100\pi$$

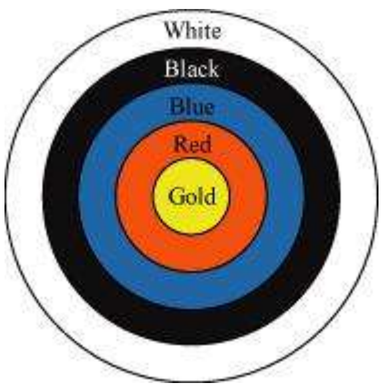
$$r^2 = 100$$

$$r = \pm 10$$

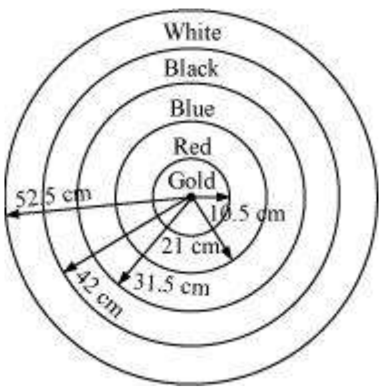
However, the radius cannot be negative. Therefore, the radius of the circle having area equal to the sum of the areas of the two circles is 10 cm.

Q3 :

Given figure depicts an archery target marked with its five scoring areas from the centre outwards as Gold, Red, Blue, Black and White. The diameter of the region representing Gold score is 21 cm and each of the other bands is 10.5 cm wide. Find the area of each of the five scoring regions. $\left[\text{Use } \pi = \frac{22}{7}\right]$



Answer :



Radius (r_1) of gold region (i.e., 1st circle) $= \frac{21}{2} = 10.5$ cm

Given that each circle is 10.5 cm wider than the previous circle.

Therefore, radius (r_2) of 2nd circle = 10.5 + 10.5

21 cm

Radius (r_3) of 3rd circle = 21 + 10.5

= 31.5 cm

Radius (r_4) of 4th circle = 31.5 + 10.5

= 42 cm

Radius (r_5) of 5th circle = 42 + 10.5

= 52.5 cm

Area of gold region = Area of 1st circle $= \pi r_1^2 = \pi(10.5)^2 = 346.5 \text{ cm}^2$

Area of red region = Area of 2nd circle - Area of 1st circle

$$= \pi r_2^2 - \pi r_1^2$$
$$= \pi(21)^2 - \pi(10.5)^2$$
$$= 441\pi - 110.25\pi = 330.75\pi$$
$$= 1039.5 \text{ cm}^2$$

Area of blue region = Area of 3rd circle - Area of 2nd circle

$$= \pi r_3^2 - \pi r_1^2$$
$$= \pi(31.5)^2 - \pi(21)^2$$
$$= 992.25\pi - 441\pi = 551.25\pi$$
$$= 1732.5 \text{ cm}^2$$

Area of black region = Area of 4th circle - Area of 3rd circle

$$= \pi r_4^2 - \pi r_3^2$$
$$= \pi(42)^2 - \pi(31.5)^2$$
$$= 1764\pi - 992.25\pi$$
$$= 771.75\pi = 2425.5 \text{ cm}^2$$

Area of white region = Area of 5th circle - Area of 4th circle

$$= \pi r_5^2 - \pi r_4^2$$
$$= \pi(52.5)^2 - \pi(42)^2$$
$$= 2756.25\pi - 1764\pi$$
$$= 992.25\pi = 3118.5 \text{ cm}^2$$

Therefore, areas of gold, red, blue, black, and white regions are 346.5 cm^2, 1039.5 cm^2, 1732.5 cm^2, 2425.5 cm^2, and 3118.5 cm^2 respectively.

Q4 :

The wheels of a car are of diameter 80 cm each. How many complete revolutions does each wheel make in 10 minutes when the car is traveling at a speed of 66 km per hour? $\left[\text{Use } \pi = \frac{22}{7}\right]$

Answer :

Diameter of the wheel of the car = 80 cm

Radius (*r*) of the wheel of the car = 40 cm

Circumference of wheel = 2π*r*

= 2π (40) = 80π cm

Speed of car = 66 km/hour

$$= \frac{66 \times 100000}{60} \text{ cm/min}$$

$$= 110000 \text{ cm/min}$$

Distance travelled by the car in 10 minutes

= 110000 × 10 = 1100000 cm

Let the number of revolutions of the wheel of the car be *n*.

n × Distance travelled in 1 revolution (i.e., circumference)

= Distance travelled in 10 minutes

$$n \times 80\pi = 1100000$$

$$n = \frac{1100000 \times 7}{80 \times 22}$$

$$= \frac{35000}{8} = 4375$$

Therefore, each wheel of the car will make 4375 revolutions.

Q5 :

Tick the correct answer in the following and justify your choice: If the perimeter and the area of a circle are numerically equal, then the radius of the circle is

(A) 2 units (B) π units (C) 4 units (D)7 units

Answer :

Let the radius of the circle be *r*

Circumference of circle = 2π*r*

Area of circle = πr^2

Given that, the circumference of the circle and the area of the circle are equal.

This implies $2\pi r = \pi r^2$

2 = *r*

Therefore, the radius of the circle is 2 units.

Hence, the correct answer is A.

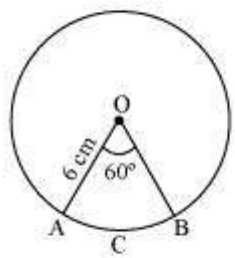
Exercise 12.2

Exercise 12.2 :

Q1 :

Find the area of a sector of a circle with radius 6 cm if angle of the sector is 60°. $\left[\text{Use } \pi = \frac{22}{7}\right]$

Answer :



Let OACB be a sector of the circle making 60° angle at centre O of the circle.

Area of sector of angle θ = $\frac{\theta}{360°} \times \pi r^2$

Area of sector OACB = $\frac{60°}{360°} \times \frac{22}{7} \times (6)^2$

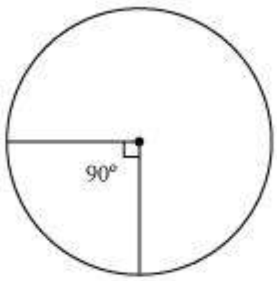
$= \frac{1}{6} \times \frac{22}{7} \times 6 \times 6 = \frac{132}{7}\ \text{cm}^2$

Therefore, the area of the sector of the circle making 60° at the centre of the circle is $\frac{132}{7}\ \text{cm}^2$

Q2 :

Find the area of a quadrant of a circle whose circumference is 22 cm. $\left[\text{Use } \pi = \frac{22}{7}\right]$

Answer :



Let the radius of the circle be *r*.

Circumference = 22 cm

$2\pi r = 22$

$$r = \frac{22}{2\pi} = \frac{11}{\pi}$$

Quadrant of circle will subtend 90° angle at the centre of the circle.

Area of such quadrant of the circle $= \frac{90°}{360°} \times \pi \times r^2$

$$= \frac{1}{4\pi} \times \pi \times \left(\frac{11}{\ }\right)^2$$

$$= \frac{121}{4\pi} = \frac{121 \times 7}{4 \times 22}$$

$$= \frac{77}{8} \text{ cm}^2$$

Q3 :

The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes. $\left[\text{Use } \pi = \frac{22}{7}\right]$

Answer :

We know that in 1 hour (i.e., 60 minutes), the minute hand rotates 360°.

In 5 minutes, minute hand will rotate = $\frac{360°}{60} \times 5 = 30°$

Therefore, the area swept by the minute hand in 5 minutes will be the area of a sector of 30° in a circle of 14 cm radius.

Area of sector of angle θ = $\frac{\theta}{360°} \times \pi r^2$

Area of sector of 30° $= \frac{30°}{360°} \times \frac{22}{7} \times 14 \times 14$

$= \frac{22}{12} \times 2 \times 14$

$= \frac{11 \times 14}{3}$

$= \frac{154}{3}\ \text{cm}^2$

Therefore, the area swept by the minute hand in 5 minutes is $\frac{154}{3}\ \text{cm}^2$.

Q4 :

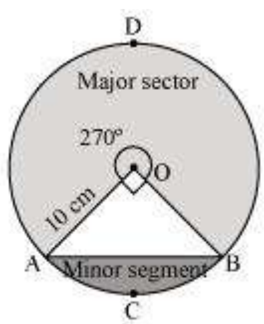
A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding:

(i) Minor segment

(ii) Major sector

[Use π = 3.14]

Answer :



Let AB be the chord of the circle subtending 90° angle at centre O of the circle.

Area of major sector OADB = $\left(\frac{360° - 90°}{360°}\right) \times \pi r^2 = \left(\frac{270°}{360°}\right) \pi r^2$

$$= \frac{3}{4} \times 3.14 \times 10 \times 10$$
$$= 235.5 \text{ cm}^2$$

Area of minor sector OACB = $\frac{90°}{360°} \times \pi r^2$

$$= \frac{1}{4} \times 3.14 \times 10 \times 10$$
$$= 78.5 \text{ cm}^2$$

Area of ΔOAB = $\frac{1}{2} \times \text{OA} \times \text{OB} = \frac{1}{2} \times 10 \times 10$

= 50 cm^2

Area of minor segment ACB = Area of minor sector OACB -

Area of ΔOAB = 78.5 - 50 = 28.5 cm^2

Q5 :

In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find:

(i) The length of the arc

(ii) Area of the sector formed by the arc

(iii) Area of the segment forced by the corresponding chord

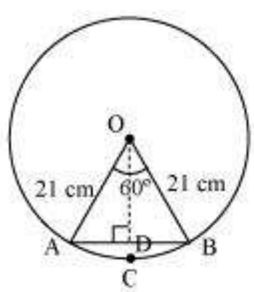
$$\left[\text{Use } \pi = \frac{22}{7}\right]$$

Answer :

Radius (r) of circle = 21 cm

Angle subtended by the given arc = 60°

Length of an arc of a sector of angle θ = $\frac{\theta}{360°} \times 2\pi r$



Length of arc ACB = $\frac{60°}{360°}\times 2\times\frac{22}{7}\times 21$

$=\frac{1}{6}\times 2\times 22\times 3$

= 22 cm

Area of sector OACB = $\frac{60°}{360°}\times\pi r^2$

$=\frac{1}{6}\times\frac{22}{7}\times 21\times 21$

$=231\ \text{cm}^2$

In ΔOAB,

∠OAB = ∠OBA (As OA = OB)

∠OAB + ∠AOB + ∠OBA = 180°

2∠OAB + 60° = 180°

∠OAB = 60°

Therefore, ΔOAB is an equilateral triangle.

Area of ΔOAB = $\frac{\sqrt{3}}{4}\times(\text{Side})^2$

$=\frac{\sqrt{3}}{4}\times(21)^2=\frac{441\sqrt{3}}{4}\ \text{cm}^2$

Area of segment ACB = Area of sector OACB - Area of ΔOAB

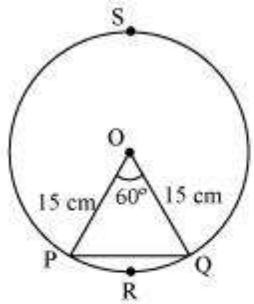
$=\left(231-\frac{441\sqrt{3}}{4}\right)\ \text{cm}^2$

Q6 :

A chord of a circle of radius 15 cm subtends an angle of 60° at the centre. Find the areas of the corresponding minor and major segments of the circle.

[Use π = 3.14 and $\sqrt{3}=1.73$]

Answer :



Radius (r) of circle = 15 cm

Area of sector OPRQ = $\frac{60°}{360°} \times \pi r^2$

$= \frac{1}{6} \times 3.14 \times (15)^2$

$= 117.75 \text{ cm}^2$

In ΔOPQ,

∠OPQ = ∠OQP (As OP = OQ)

∠OPQ + ∠OQP + ∠POQ = 180°

2 ∠OPQ = 120°

∠OPQ = 60°

ΔOPQ is an equilateral triangle.

Area of ΔOPQ = $\frac{\sqrt{3}}{4} \times (\text{side})^2$

$= \frac{\sqrt{3}}{4} \times (15)^2 = \frac{225\sqrt{3}}{4} \text{ cm}^2$

$= 56.25\sqrt{3}$

$= 97.3125 \text{ cm}^2$

Area of segment PRQ = Area of sector OPRQ - Area of ΔOPQ

= 117.75 - 97.3125

= 20.4375 cm^2

Area of major segment PSQ = Area of circle - Area of segment PRQ

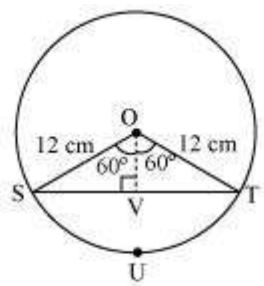
$$
\begin{aligned}
&= \pi(15)^2 - 20.4375 \\
&= 3.14 \times 225 - 20.4375 \\
&= 706.5 - 20.4375 \\
&= 686.0625 \text{ cm}^2
\end{aligned}
$$

Q7 :

A chord of a circle of radius 12 cm subtends an angle of 120° at the centre. Find the area of the corresponding segment of the circle.

[Use π = 3.14 and $\sqrt{3} = 1.73$ **]**

Answer :



Let us draw a perpendicular OV on chord ST. It will bisect the chord ST.

SV = VT

In ΔOVS,

$$\frac{OV}{OS} = \cos 60°$$

$$\frac{OV}{12} = \frac{1}{2}$$

$$OV = 6 \text{ cm}$$

$$\frac{SV}{SO} = \sin 60° = \frac{\sqrt{3}}{2}$$

$$\frac{SV}{12} = \frac{\sqrt{3}}{2}$$

$$SV = 6\sqrt{3} \text{ cm}$$

$$ST = 2SV = 2 \times 6\sqrt{3} = 12\sqrt{3} \text{ cm}$$

Area of ΔOST = $\frac{1}{2} \times ST \times OV$

$$= \frac{1}{2} \times 12\sqrt{3} \times 6$$

$$= 36\sqrt{3} = 36 \times 1.73 = 62.28 \text{ cm}^2$$

Area of sector OSUT = $\frac{120°}{360°} \times \pi (12)^2$

$$= \frac{1}{3} \times 3.14 \times 144 = 150.72 \text{ cm}^2$$

Area of segment SUT = Area of sector OSUT - Area of ΔOST

= 150.72 - 62.28

= 88.44 cm^2

Q8 :

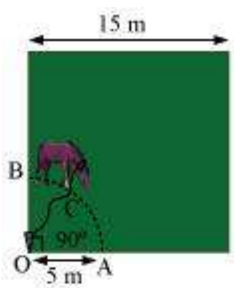
A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope (see the given figure). Find

(i) The area of that part of the field in which the horse can graze.

(ii) The increase in the grazing area of the rope were 10 m long instead of 5 m.

[Use Ãâ‚¬ = 3.14]

Answer :



From the figure, it can be observed that the horse can graze a sector of 90° in a circle of 5 m radius.

Area that can be grazed by horse = Area of sector OACB

$$= \frac{90°}{360°} \pi r^2$$

$$= \frac{1}{4} \times 3.14 \times (5)^2$$

$$= 19.625 \text{ m}^2$$

Area that can be grazed by the horse when length of rope is 10 m long

$$= \frac{90°}{360°} \times \pi \times (10)^2$$

$$= \frac{1}{4} \times 3.14 \times 100$$

$$= 78.5 \text{ m}^2$$

Increase in grazing area = (78.5 - 19.625) m^2

= 58.875 m^2

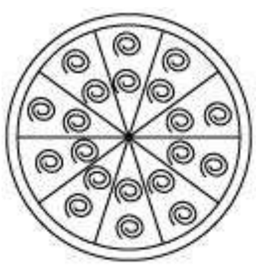
Q9 :

A brooch is made with silver wire in the form of a circle with diameter 35 mm. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors as shown in figure. Find.

(i) The total length of the silver wire required.

(ii) The area of each sector of the brooch

$$\left[\text{Use } \pi = \frac{22}{7}\right]$$



Answer :

Total length of wire required will be the length of 5 diameters and the circumference of the brooch.

Radius of circle = $\frac{35}{2}$ mm

Circumference of brooch = $2\pi r$

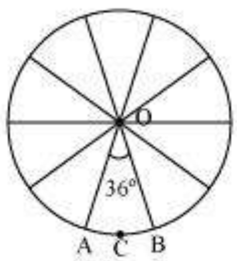
$$= 2 \times \frac{22}{7} \times \left(\frac{35}{2}\right)$$

= 110 mm

Length of wire required = 110 + 5 × 35

= 110 + 175 = 285 mm

It can be observed from the figure that each of 10 sectors of the circle is subtending 36° at the centre of the circle.



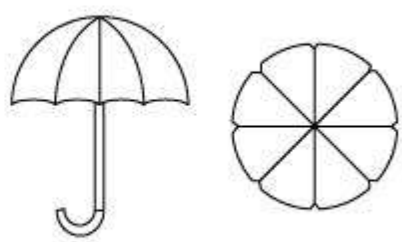
Therefore, area of each sector = $\dfrac{36°}{360°} \times \pi r^2$

$$= \frac{1}{10} \times \frac{22}{7} \times \left(\frac{35}{2}\right) \times \left(\frac{35}{2}\right)$$

$$= \frac{385}{4} \text{ mm}^2$$

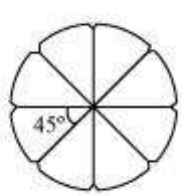
Q10 :

An umbrella has 8 ribs which are equally spaced (see figure). Assuming umbrella to be a flat circle of radius 45 cm, find the area between the two consecutive ribs of the umbrella. $\left[\text{Use } \pi = \dfrac{22}{7}\right]$



Answer :

There are 8 ribs in an umbrella. The area between two consecutive ribs is subtending $\dfrac{360°}{8} = 45°$ at the centre of the assumed flat circle.



Area between two consecutive ribs of circle = $\dfrac{45°}{360°} \times \pi r^2$

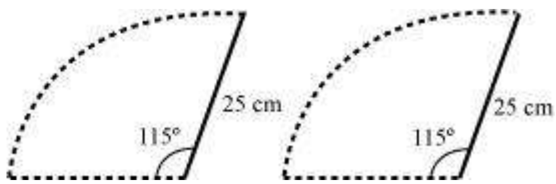
$$= \frac{1}{8} \times \frac{22}{7} \times (45)^2$$

$$= \frac{11}{28} \times 2025 = \frac{22275}{28} \text{ cm}^2$$

Q11 :

A car has two wipers which do not overlap. Each wiper has blade of length 25 cm sweeping through an angle of 115°. Find the total area cleaned at each sweep of the blades. $\left[\text{Use } \pi = \frac{22}{7}\right]$

Answer :



It can be observed from the figure that each blade of wiper will sweep an area of a sector of 115° in a circle of 25 cm radius.

Area of such sector = $\frac{115^\circ}{360^\circ} \times \pi \times (25)^2$

$$= \frac{23}{72} \times \frac{22}{7} \times 25 \times 25$$
$$= \frac{158125}{252} \text{ cm}^2$$

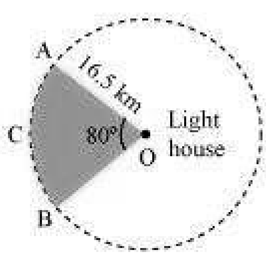
Area swept by 2 blades = $2 \times \frac{158125}{252}$

$$= \frac{158125}{126} \text{ cm}^2$$

Q12 :

To warn ships for underwater rocks, a lighthouse spreads a red coloured light over a sector of angle 80° to a distance of 16.5 km. Find the area of the sea over which the ships warned. [Use π = 3.14]

Answer :



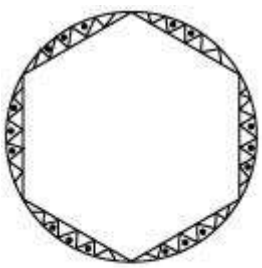
It can be observed from the figure that the lighthouse spreads light across a sector of 80° in a circle of 16.5 km radius.

Area of sector OACB = $\frac{80^\circ}{360^\circ} \times \pi r^2$

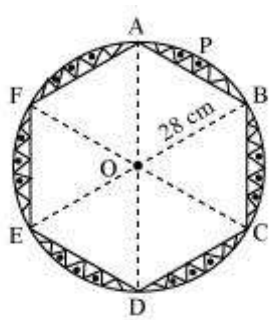
$$= \frac{2}{9} \times 3.14 \times 16.5 \times 16.5$$
$$= 189.97 \text{ km}^2$$

Q13 :

A round table cover has six equal designs as shown in figure. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of Rs.0.35 per cm^2. [Use $\sqrt{3} = 1.7$]



Answer :



It can be observed that these designs are segments of the circle.

Consider segment APB. Chord AB is a side of the hexagon. Each chord will substitut e $\frac{360°}{6} = 60°$ at the centre of the circle.

In ΔOAB,

∠OAB = ∠OBA (As OA = OB)

∠AOB = 60°

∠OAB + ∠OBA + ∠AOB = 180°

2∠OAB = 180° 60° = 120°

∠OAB = 60°

Therefore, ΔOAB is an equilateral triangle.

Area of ΔOAB = $\frac{\sqrt{3}}{4} \times (\text{side})^2$

$= \frac{\sqrt{3}}{4} \times (28)^2 = 196\sqrt{3} = 196 \times 1.7$ = 333.2 cm^2

Area of sector OAPB = $\frac{60°}{360°} \times \pi r^2$

$$= \frac{1}{6} \times \frac{22}{7} \times 28 \times 28$$
$$= \frac{1232}{3} \text{ cm}^2$$

Area of segment APB = Area of sector OAPB   Area of ΔOAB

$$= \left(\frac{1232}{3} - 333.2 \right) \text{cm}^2$$

$$\text{Therefore, area of designs} = 6 \times \left(\frac{1232}{3} - 333.2 \right) \text{cm}^2$$
$$= (2464 - 1999.2) \text{cm}^2$$
$$= 464.8 \text{ cm}^2$$

Cost of making 1 cm^2 designs = Rs 0.35

Cost of making 464.76 cm^2 designs = 464.8×0.35 = Rs 162.68

Therefore, the cost of making such designs is Rs 162.68.

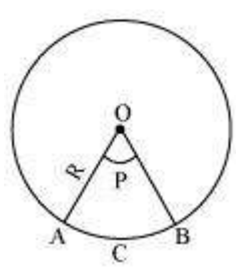
Q14 :

Tick the correct answer in the following:

Area of a sector of angle *p* (in degrees) of a circle with radius R is

(A) $\frac{p}{180} \times 2\pi R$ **, (B)** $\frac{p}{180} \times \pi R^2$ **, (C)** $\frac{p}{360} \times 2\pi R$ **, (D)** $\frac{p}{720} \times 2\pi R^2$

Answer :



We know that area of sector of angle θ = $\frac{\theta}{360°} \times \pi R^2$

Area of sector of angle P = $\frac{p}{360°}\left(\pi R^2\right)$

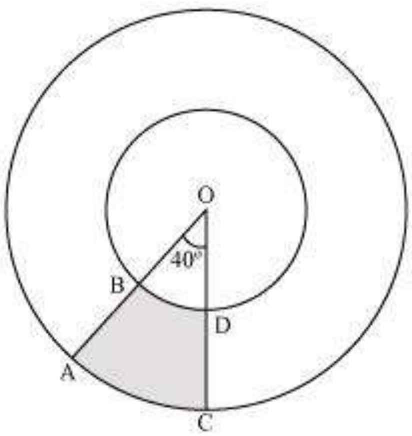
$$= \left(\frac{p}{720°} \right) \left(2\pi R^2 \right)$$

Hence, (D) is the correct answer.

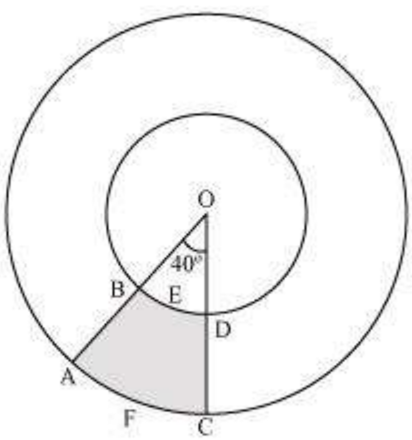
Exercise 12.3

Q1 :

Find the area of the shaded region in the given figure, if radii of the two concentric circles with centre O are 7 cm and 14 cm respectively and ∠AOC = 40°. $\left[\text{Use } \pi = \frac{22}{7}\right]$



Answer :



Radius of inner circle = 7 cm

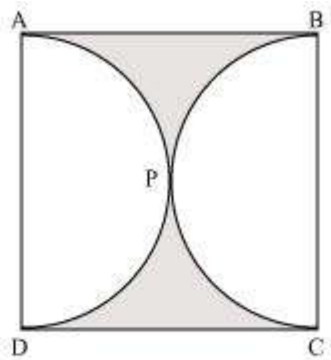
Radius of outer circle = 14 cm

Area of shaded region = Area of sector OAFC - Area of sector OBED

$= \frac{40°}{360°} \times \pi (14)^2 - 40°$

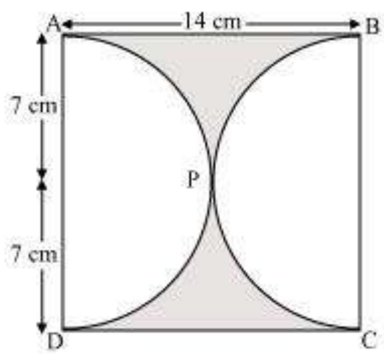
Q2 :

Find the area of the shaded region in the given figure, if ABCD is a square of side 14 cm and APD and BPC are semicircles. $\left[\text{Use } \pi = \frac{22}{7}\right]$



Answer :

It can be observed from the figure that the radius of each semi-circle is 7 cm.



Area of each semi-circle = $\frac{1}{2}\pi r^2$

$$= \frac{1}{2} \times \frac{22}{7} \times (7)^2$$
$$= 77 \text{ cm}^2$$

Area of square ABCD = $(\text{Side})^2 = (14)^2 = 196 \text{ cm}^2$

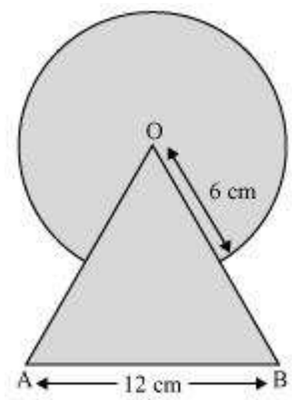
Area of the shaded region

= Area of square ABCD - Area of semi-circle APD - Area of semi-circle BPC

= 196 - 77 - 77 = 196 - 154 = 42 cm^2

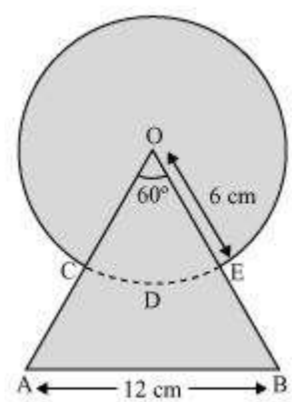
Q3 :

Find the area of the shaded region in the given figure, where a circular arc of radius 6 cm has been drawn with vertex O of an equilateral triangle OAB of side 12 cm as centre. $\left[\text{Use } \pi = \frac{22}{7}\right]$



Answer :

We know that each interior angle of an equilateral triangle is of measure 60°.



Area of sector OCDE $= \frac{60°}{360°}\pi r^2$

$$= \frac{1}{6} \times \frac{22}{7} \times 6 \times 6$$

$$= \frac{132}{7} \text{ cm}^2$$

Area of $\Delta OAB = \frac{\sqrt{3}}{4}(12)^2 = \frac{\sqrt{3} \times 12 \times 12}{4} = 36\sqrt{3} \text{ cm}^2$

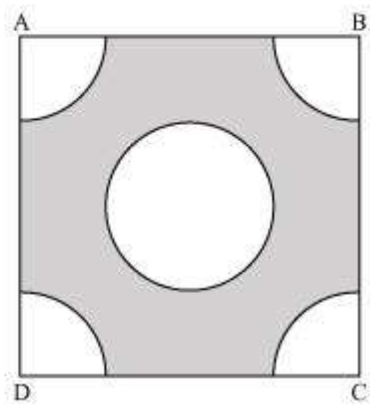
Area of circle = $\pi r^2 = \frac{22}{7} \times 6 \times 6 = \frac{792}{7} \text{ cm}^2$

Area of shaded region = Area of ΔOAB + Area of circle - Area of sector OCDE

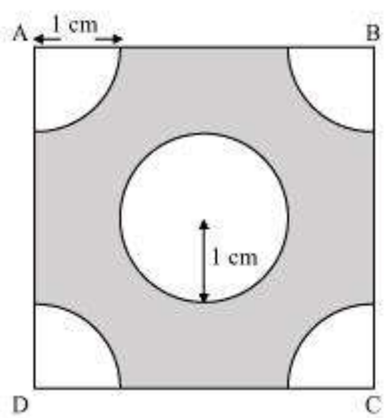
$$= 36\sqrt{3} + \frac{792}{7} - \frac{132}{7}$$
$$= \left(36\sqrt{3} + \frac{660}{7}\right) \text{ cm}^2$$

Q4 :

From each corner of a square of side 4 cm a quadrant of a circle of radius 1 cm is cut and also a circle of diameter 2 cm is cut as shown in the given figure. Find the area of the remaining portion of the square. $\left[\text{Use } \pi = \frac{22}{7}\right]$



Answer :



Each quadrant is a sector of 90° in a circle of 1 cm radius.

Area of each quadrant $= \frac{90°}{360°} \pi r^2$

$$= \frac{1}{4} \times \frac{22}{7} \times (1)^2 = \frac{22}{28} \text{cm}^2$$

Area of square = $(\text{Side})^2 = (4)^2 = 16\ \text{cm}^2$

Area of circle = $\pi r^2 = \pi (1)^2$

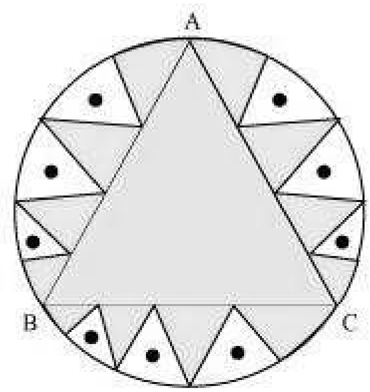
$$= \frac{22}{7}\ \text{cm}^2$$

Area of the shaded region = Area of square - Area of circle - 4 × Area of quadrant

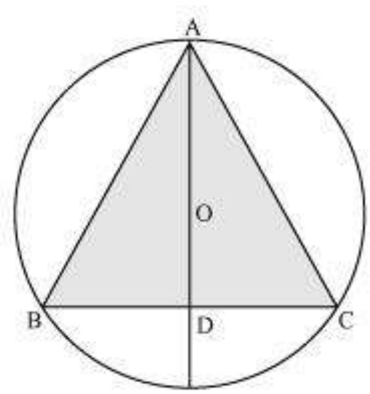
$$= 16 - \frac{22}{7} - 4 \times \frac{22}{28}$$
$$= 16 - \frac{22}{7} - \frac{22}{7} = 16 - \frac{44}{7}$$
$$= \frac{112 - 44}{7} = \frac{68}{7}\ \text{cm}^2$$

Q5 :

In a circular table cover of radius 32 cm, a design is formed leaving an equilateral triangle ABC in the middle as shown in the given figure. Find the area of the design (Shaded region). $\left[\text{Use } \pi = \frac{22}{7}\right]$



Answer :



Radius (r) of circle = 32 cm

AD is the median of ΔABC.

$$AO = \frac{2}{3}AD = 32$$

AD = 48 cm

In ΔABD,

$AB^2 = AD^2 + BD^2$

$$AB^2 = (48)^2 + \left(\frac{AB}{2}\right)^2$$

$$\frac{3AB^2}{4} = (48)^2$$

$$AB = \frac{48 \times 2}{\sqrt{3}} = \frac{96}{\sqrt{3}}$$

$$= 32\sqrt{3}\ \text{cm}$$

Area of equilateral triangle, $\Delta ABC = \frac{\sqrt{3}}{4}\left(32\sqrt{3}\right)^2$

$$= \frac{\sqrt{3}}{4} \times 32 \times 32 \times 3 = 96 \times 8 \times \sqrt{3}$$

$$= 768\sqrt{3}\ \text{cm}^2$$

Area of circle = πr^2

$$= \frac{22}{7} \times (32)^2$$

$$= \frac{22}{7} \times 1024$$

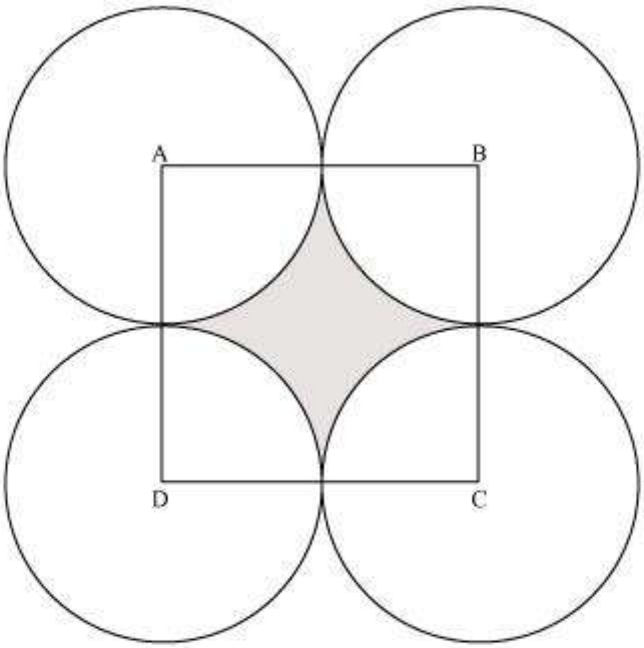
$$= \frac{22528}{7}\ \text{cm}^2$$

Area of design = Area of circle - Area of ΔABC

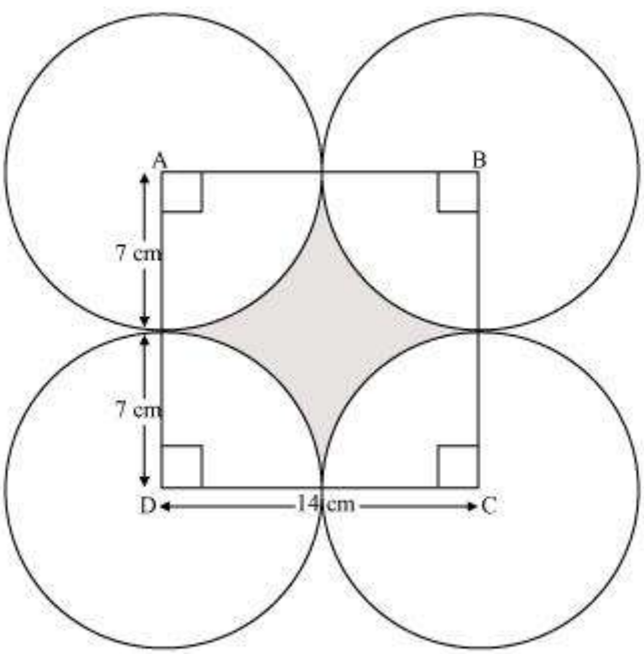
$$= \left(\frac{22528}{7} - 768\sqrt{3}\right)\ \text{cm}^2$$

Q6 :

In the given figure, ABCD is a square of side 14 cm. With centres A, B, C and D, four circles are drawn such that each circle touches externally two of the remaining three circles. Find the area of the shaded region. $\left[\text{Use } \pi = \frac{22}{7}\right]$



Answer :



Area of each of the 4 sectors is equal to each other and is a sector of 90° in a circle of 7 cm radius.

Area of each sector $= \frac{90^\circ}{360^\circ} \times \pi (7)^2$

$$= \frac{1}{4} \times \frac{22}{7} \times 7 \times 7$$

$$= \frac{77}{2} \text{ cm}^2$$

Area of square ABCD = (Side)^2 = (14)^2 = 196 cm^2

Area of shaded portion = Area of square ABCD - 4 × Area of each sector

$$= 196 - 4 \times \frac{77}{2} = 196 - 154$$

$$= 42 \text{ cm}^2$$

Therefore, the area of shaded portion is 42 cm^2.

Q7 :

The given figure depicts a racing track whose left and right ends are semicircular.

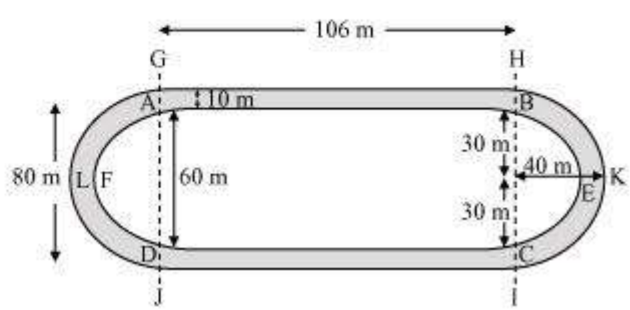
The distance between the two inner parallel line segments is 60 m and they are each 106 m long. If the track is 10 m wide, find:

(i) The distance around the track along its inner edge

(ii) The area of the track

$$\left[\text{Use } \pi = \frac{22}{7}\right]$$

Answer :



Distance around the track along its inner edge = AB + arc BEC + CD + arc DFA

$$= 106 + \frac{1}{2} \times 2\pi r + 106 + \frac{1}{2} \times 2\pi r$$
$$= 212 + \frac{1}{2} \times 2 \times \frac{22}{7} \times 30 + \frac{1}{2} \times 2 \times \frac{22}{7} \times 30$$
$$= 212 + 2 \times \frac{22}{7} \times 30$$
$$= 212 + \frac{1320}{7}$$
$$= \frac{1484 + 1320}{7} = \frac{2804}{7} \text{ m}$$

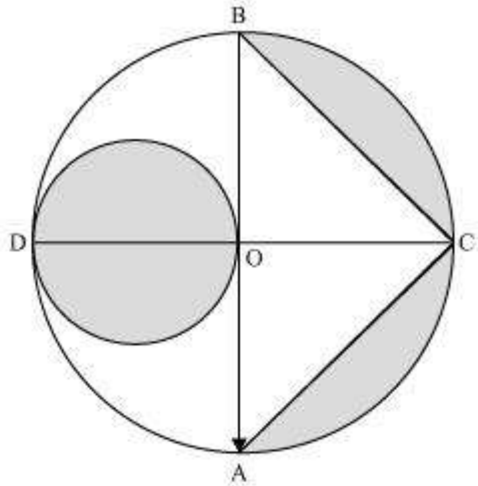
Area of the track = (Area of GHIJ - Area of ABCD) + (Area of semi-circle HKI - Area of semi-circle BEC) + (Area of semi-circle GLJ - Area of semi-circle AFD)

$$= 106 \times 80 - 106 \times 60 + \frac{1}{2} \times \frac{22}{7} \times (40)^2 - \frac{1}{2} \times \frac{22}{7} \times (30)^2 + \frac{1}{2} \times \frac{22}{7} \times (40)^2 - \frac{1}{2} \times \frac{22}{7} \times (30)^2$$
$$= 106(80 - 60) + \frac{22}{7} \times (40)^2 - \frac{22}{7} \times (30)^2$$
$$= 106(20) + \frac{22}{7}\left[(40)^2 - (30)^2\right]$$
$$= 2120 + \frac{22}{7}(40 - 30)(40 + 30)$$
$$= 2120 + \left(\frac{22}{7}\right)(10)(70)$$
$$= 2120 + 2200$$
$$= 4320 \text{ m}^2$$

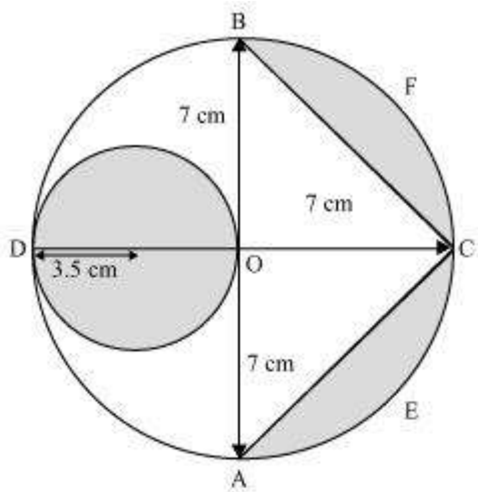
Therefore, the area of the track is 4320 m^2.

Q8 :

In the given figure, AB and CD are two diameters of a circle (with centre O) perpendicular to each other and OD is the diameter of the smaller circle. If OA = 7 cm, find the area of the shaded region. $\left[\text{Use } \pi = \frac{22}{7}\right]$



Answer :



Radius (r_1) of larger circle = 7 cm

Radius (r_2) of smaller circle $=\frac{7}{2}$ cm

Area of smaller circle $=\pi r_1^2$

$$=\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}$$
$$=\frac{77}{2}\text{ cm}^2$$

Area of semi-circle AECFB of larger circle $=\frac{1}{2}\pi r_2^2$

$$= \frac{1}{2} \times \frac{22}{7} \times (7)^2$$
$$= 77 \text{ cm}^2$$

Area of $\Delta ABC = \frac{1}{2} \times AB \times OC$

$$= \frac{1}{2} \times 14 \times 7 = 49 \text{ cm}^2$$

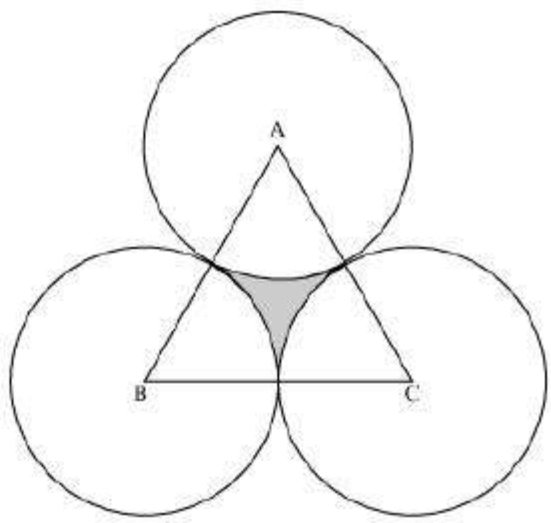
Area of the shaded region

= Area of smaller circle + Area of semi-circle AECFB - Area of ΔABC

$$= \frac{77}{2} + 77 - 49$$
$$= 28 + \frac{77}{2} = 28 + 38.5 = 66.5 \text{ cm}^2$$

Q9 :

The area of an equilateral triangle ABC is 17320.5 cm^2. With each vertex of the triangle as centre, a circle is drawn with radius equal to half the length of the side of the triangle (See the given figure). Find the area of shaded region. [Use π = 3.14 and $\sqrt{3} = 1.73205$]

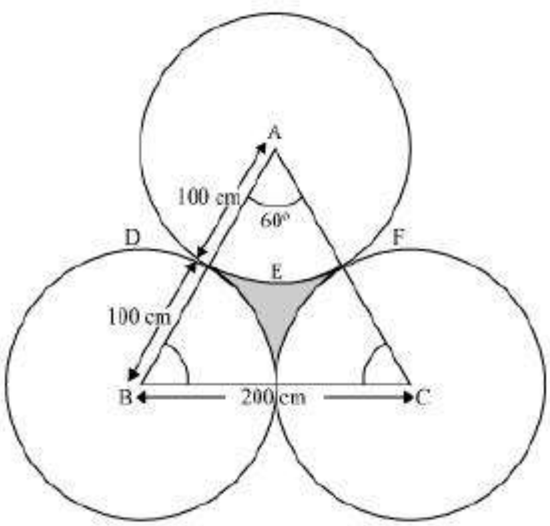


Answer :

Let the side of the equilateral triangle be *a*.

Area of equilateral triangle = 17320.5 cm^2

$$\frac{\sqrt{3}}{4}(a)^2 = 17320.5$$
$$\frac{1.73205}{4}a^2 = 17320.5$$
$$a^2 = 4 \times 10000$$
$$a = 200 \text{ cm}$$



Each sector is of measure 60°.

Area of sector ADEF $= \frac{60°}{360°} \times \pi \times r^2$

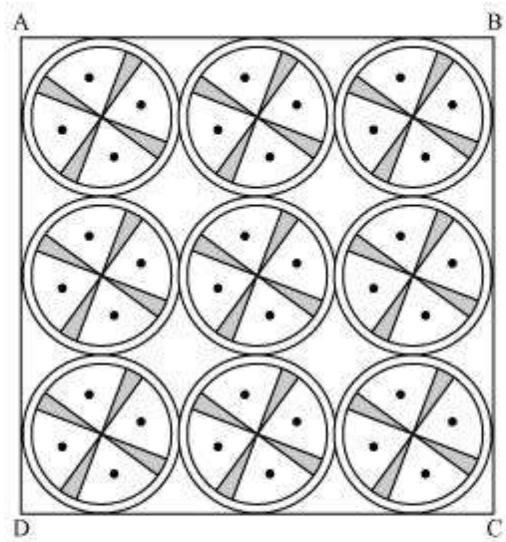
$$= \frac{1}{6} \times \pi \times (100)^2$$
$$= \frac{3.14 \times 10000}{6}$$
$$= \frac{15700}{3} \text{ cm}^2$$

Area of shaded region = Area of equilateral triangle - 3 × Area of each sector

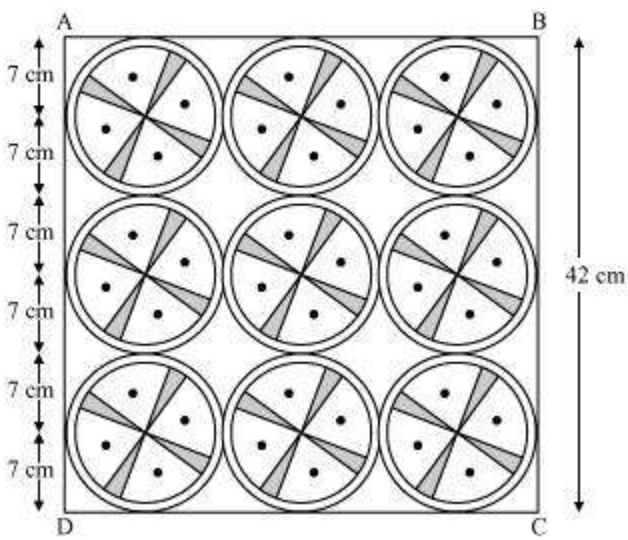
$$= 17320.5 - 3 \times \frac{15700}{3}$$
$$= 17320.5 - 15700 = 1620.5 \text{ cm}^2$$

Q10 :

On a square handkerchief, nine circular designs each of radius 7 cm are made (see the given figure). Find the area of the remaining portion of the handkerchief. $\left[\text{Use } \pi = \frac{22}{7}\right]$



Answer :



From the figure, it can be observed that the side of the square is 42 cm.

Area of square = $(\text{Side})^2 = (42)^2 = 1764$ cm^2

Area of each circle = $\pi r^2 = \frac{22}{7} \times (7)^2 = 154 \text{ cm}^2$

Area of 9 circles = $9 \times 154 = 1386$ cm^2

Area of the remaining portion of the handkerchief = $1764 - 1386 = 378$ cm^2

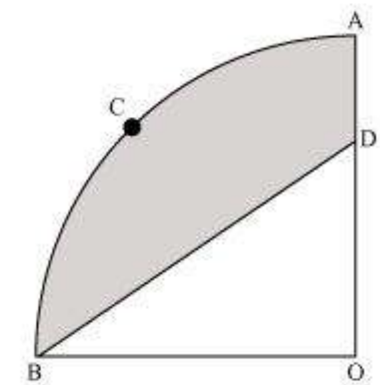
Q11 :

In the given figure, OACB is a quadrant of circle with centre O and radius 3.5 cm. If OD = 2 cm, find the area of the

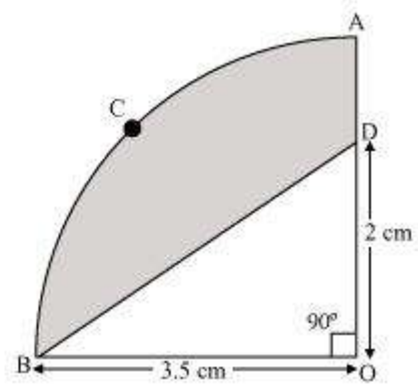
(i) Quadrant OACB

(ii) Shaded region

$$\left[\text{Use } \pi = \frac{22}{7}\right]$$



Answer :



(i) Since OACB is a quadrant, it will subtend 90° angle at O.

Area of quadrant OACB $= \frac{90^\circ}{360^\circ} \times \pi r^2$

$$= \frac{1}{4} \times \frac{22}{7} \times (3.5)^2 = \frac{1}{4} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2$$

$$= \frac{11 \times 7 \times 7}{2 \times 7 \times 2 \times 2} = \frac{77}{8} \text{ cm}^2$$

(ii) Area of ΔOBD $= \frac{1}{2} \times \text{OB} \times \text{OD}$

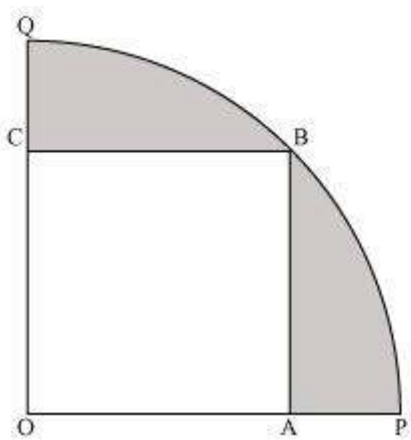
$$= \frac{1}{2} \times 3.5 \times 2$$
$$= \frac{1}{2} \times \frac{7}{2} \times 2$$
$$= \frac{7}{2} \text{ cm}^2$$

Area of the shaded region = Area of quadrant OACB - Area of ΔOBD

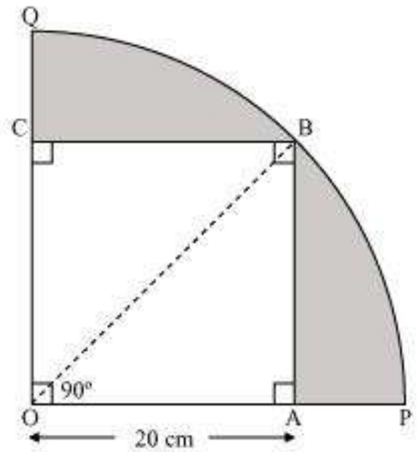
$$= \frac{77}{8} - \frac{7}{2}$$
$$= \frac{77 - 28}{8}$$
$$= \frac{49}{8} \text{ cm}^2$$

Q12 :

In the given figure, a square OABC is inscribed in a quadrant OPBQ. If OA = 20 cm, find the area of the shaded region. [Use π = 3.14]



Answer :



In ΔOAB,

$OB^2 = OA^2 + AB^2$

$= (20)^2 + (20)^2$

$$OB = 20\sqrt{2}$$

Radius (r) of circle $= 20\sqrt{2}$ cm

Area of quadrant OPBQ $= \frac{90°}{360°} \times 3.14 \times \left(20\sqrt{2}\right)^2$

$$= \frac{1}{4} \times 3.14 \times 800$$
$$= 628 \text{ cm}^2$$

Area of OABC = $(\text{Side})^2 = (20)^2 = 400$ cm^2

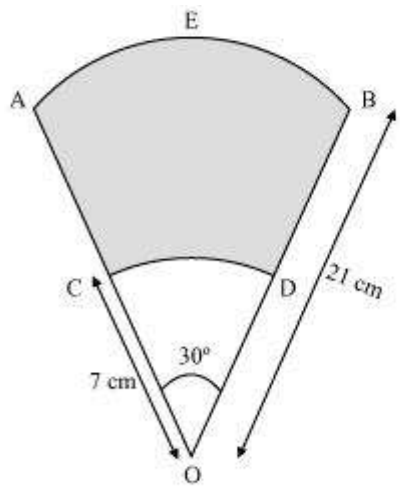
Area of shaded region = Area of quadrant OPBQ - Area of OABC

$= (628 - 400)$ cm^2

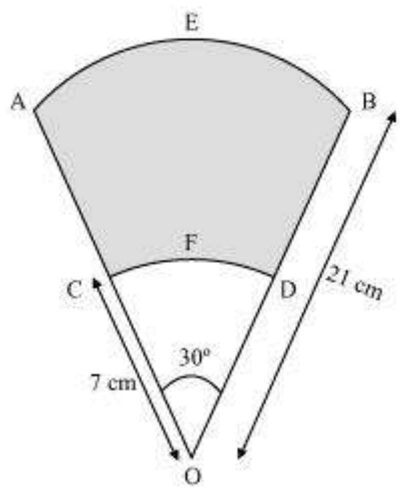
$= 228$ cm^2

Q13 :

AB and CD are respectively arcs of two concentric circles of radii 21 cm and 7 cm and centre O (see the given figure). If ∠AOB = 30°, find the area of the shaded region. $\left[\text{Use } \pi = \frac{22}{7}\right]$



Answer :



Area of the shaded region = Area of sector OAEB - Area of sector OCFD

$$=\frac{30°}{360°}\times\pi\times(21)^2-\frac{30°}{360°}\times\pi\times(7)^2$$

$$=\frac{1}{12}\times\pi\left[(21)^2-(7)^2\right]$$

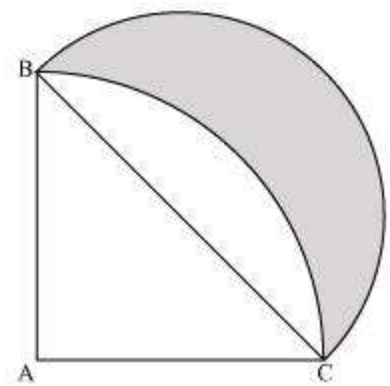
$$=\frac{1}{12}\times\frac{22}{7}\times\left[(21-7)(21+7)\right]$$

$$=\frac{22\times14\times28}{12\times7}$$

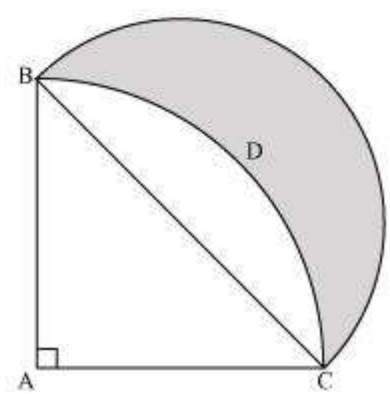
$$=\frac{308}{3}\text{ cm}^2$$

Q14 :

In the given figure, ABC is a quadrant of a circle of radius 14 cm and a semicircle is drawn with BC as diameter. Find the area of the shaded region. $\left[\text{Use } \pi = \frac{22}{7}\right]$



Answer :



As ABC is a quadrant of the circle, ∠BAC will be of measure 90 º.

In ΔABC,

$BC^2 = AC^2 + AB^2$

$= (14)^2 + (14)^2$

$BC = 14\sqrt{2}$

Radius (r_1) of semi-circle drawn on $BC = \frac{14\sqrt{2}}{2} = 7\sqrt{2}\text{ cm}$

Area of $\Delta ABC = \frac{1}{2} \times AB \times AC$

$$= \frac{1}{2} \times 14 \times 14$$
$$= 98\text{ cm}^2$$

Area of sector $ABDC = \frac{90°}{360°} \times \pi r^2$

$$= \frac{1}{4} \times \frac{22}{7} \times 14 \times 14$$
$$= 154 \text{ cm}^2$$

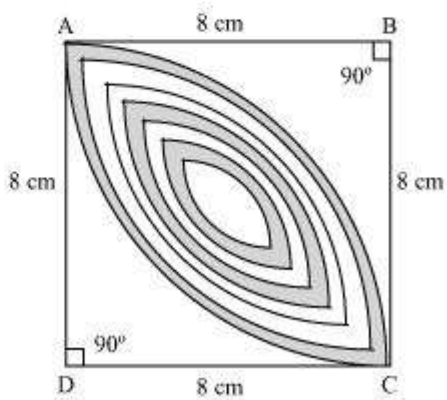
$$\text{Area of semi-circle drawn on BC} = \frac{1}{2} \times \pi \times r_1^2 = \frac{1}{2} \times \frac{22}{7} \times \left(7\sqrt{2}\right)^2$$
$$= \frac{1}{2} \times \frac{22}{7} \times 98 = 154 \text{ cm}^2$$

Area of shaded region = Area of semi-circle $-$ (Area of sector ABDC $-$ Area of ΔABC) = 154 - (154 - 98)

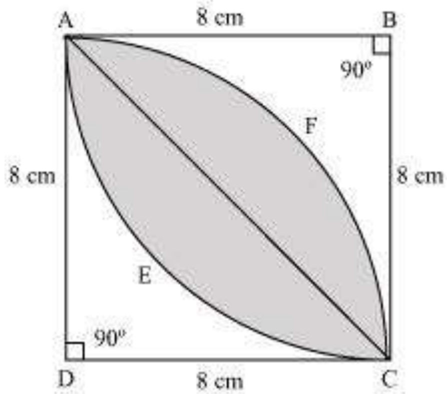
= 98 cm^2

Q15 :

Calculate the area of the designed region in the given figure common between the two quadrants of circles of radius 8 cm each. $\left[\text{Use } \pi = \frac{22}{7}\right]$



Answer :



The designed area is the common region between two sectors BAEC and DAFC.

Area of sector $\text{BAEC} = \frac{90°}{360°} \times \frac{22}{7} \times (8)^2$

$$= \frac{1}{4} \times \frac{22}{7} \times 64$$

$$= \frac{22 \times 16}{7}$$

$$= \frac{352}{7} \text{ cm}^2$$

Area of ΔBAC $= \frac{1}{2} \times \text{BA} \times \text{BC}$

$$= \frac{1}{2} \times 8 \times 8 = 32 \text{ cm}^2$$

Area of the designed portion = 2 × (Area of segment AEC)

= 2 × (Area of sector BAEC - Area of ΔBAC)

$$= 2 \times \left(\frac{352}{7} - 32\right) = 2\left(\frac{352 - 224}{7}\right)$$

$$= \frac{2 \times 128}{7}$$

$$= \frac{256}{7} \text{ cm}^2$$