Class 11 Maths NCERT Solutions Chapter - 3

Trigonometric Functions Class 11

Chapter 3 Trigonometric Functions Exercise 3.1, 3.2, 3.3, 3.4, miscellaneous Solutions

Exercise 3.1 : Solutions of Questions on Page Number : **54**

Q1 :

Find the radian measures corresponding to the following degree measures:

(i) 25° (ii) - 47° 30' (iii) 240° (iv) 520°

Answer :

(i) 25°

We know that 180° = π radian

$$\therefore 25° = \frac{\pi}{180} \times 25 \text{ radian} = \frac{5\pi}{36} \text{ radian}$$

(ii) - 47° 30'

- 47° 30' = $-47\frac{1}{2}$ degree [1° = 60']

$= \frac{-95}{2}$ degree

Since 180° = π radian

$$\frac{-95}{2} \text{ deg ree} = \frac{\pi}{180} \times \left(\frac{-95}{2}\right) \text{ radian} = \left(\frac{-19}{36 \times 2}\right)\pi \text{ radian} = \frac{-19}{72}\pi \text{ radian}$$

$$\therefore -47° \ 30' = \frac{-19}{72}\pi \text{ radian}$$

(iii) 240°

We know that 180° = π radian

$$\therefore 240° = \frac{\pi}{180} \times 240 \text{ radian} = \frac{4}{3}\pi \text{ radian}$$

(iv) 520°

We know that 180° = π radian

$$\therefore 520° = \frac{\pi}{180} \times 520 \text{ radian} = \frac{26\pi}{9} \text{ radian}$$

Q2 :

Find the degree measures corresponding to the following radian measures

$\left(\text{Use } \pi = \frac{22}{7}\right)$.

(i) $\frac{11}{16}$ **(ii) - 4 (iii)** $\frac{5\pi}{3}$ **(iv)** $\frac{7\pi}{6}$

Answer :

(i) $\frac{11}{16}$

We know that π radian = 180°

$$\therefore \frac{11}{16} \text{ radain} = \frac{180}{\pi} \times \frac{11}{16} \text{ deg ree} = \frac{45 \times 11}{\pi \times 4} \text{deg ree}$$

$$= \frac{45 \times 11 \times 7}{22 \times 4} \text{ deg ree} = \frac{315}{8} \text{ deg ree}$$

$$= 39\frac{3}{8} \text{ deg ree}$$

$$= 39^\circ + \frac{3 \times 60}{8} \text{ min utes} \qquad [1^\circ = 60']$$

$$= 39^\circ + 22' + \frac{1}{2} \text{ min utes}$$

$$= 39^\circ 22' 30'' \qquad [1' = 60'']$$

(ii) - 4

We know that π radian = 180°

$$-4 \text{ radian} = \frac{180}{\pi} \times (-4) \text{ deg ree} = \frac{180 \times 7(-4)}{22} \text{ deg ree}$$

$$= \frac{-2520}{11} \text{ deg ree} = -229\frac{1}{11} \text{ deg ree}$$

$$= -229^\circ + \frac{1 \times 60}{11} \text{ min utes} \qquad [1^\circ = 60']$$

$$= -229^\circ + 5' + \frac{5}{11} \text{ min utes}$$

$$= -229^\circ 5' 27'' \qquad [1' = 60'']$$

(iii) $\frac{5\pi}{3}$

We know that π radian = 180°

$$\therefore \frac{5\pi}{3} \text{radian} = \frac{180}{\pi} \times \frac{5\pi}{3} \text{ degree} = 300°$$

(iv) $\frac{7\pi}{6}$

We know that π radian = 180°

$$\therefore \frac{7\pi}{6} \text{radian} = \frac{180}{\pi} \times \frac{7\pi}{6} = 210°$$

Q3 :

A wheel makes 360 revolutions in one minute. Through how many radians does it turn in one second?

Answer :

Number of revolutions made by the wheel in 1 minute = 360

∴Number of revolutions made by the wheel in 1 second = $\frac{360}{60} = 6$

In one complete revolution, the wheel turns an angle of 2π radian.

Hence, in 6 complete revolutions, it will turn an angle of 6 × 2π radian, i.e.,

12 π radian

Thus, in one second, the wheel turns an angle of 12π radian.

Q4 :

Find the degree measure of the angle subtended at the centre of a circle of radius 100 cm by an arc of length 22 cm $\left(\text{Use } \pi = \frac{22}{7}\right)$.

Answer :

We know that in a circle of radius *r* unit, if an arc of length *l* unit subtends an angle *θ* radian at the centre, then

$$\theta = \frac{l}{r}$$

Therefore, forr = 100 cm, l = 22 cm, we have

$$\theta = \frac{22}{100} \text{ radian} = \frac{180}{\pi} \times \frac{22}{100} \text{ degree} = \frac{180 \times 7 \times 22}{22 \times 100} \text{ degree}$$

$$= \frac{126}{10} \text{ degree} = 12\frac{3}{5} \text{ degree} = 12°36' \qquad [1° = 60']$$

Thus, the required angle is 12°36'.

Q5 :

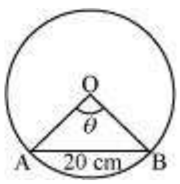
In a circle of diameter 40 cm, the length of a chord is 20 cm. Find the length of minor arc of the chord.

Answer :

Diameter of the circle = 40 cm

∴Radius (r) of the circle = $\frac{40}{2}\text{ cm} = 20\text{ cm}$

Let AB be a chord (length = 20 cm) of the circle.



In ΔOAB, OA = OB = Radius of circle = 20 cm

Also, AB = 20 cm

Thus, ΔOAB is an equilateral triangle.

∴θ = 60° = $\frac{\pi}{3}$ radian

We know that in a circle of radius r unit, if an arc of length l unit subtends an angle θ radian at the centre, then $\theta = \frac{l}{r}$.

$$\frac{\pi}{3} = \frac{\widehat{AB}}{20} \Rightarrow \widehat{AB} = \frac{20\pi}{3}\text{ cm}$$

Thus, the length of the minor arc of the chord is $\frac{20\pi}{3}$ cm .

Q6 :

If in two circles, arcs of the same length subtend angles 60° and 75° at the centre, find the ratio of their radii.

Answer :

Let the radii of the two circles be r_1 and r_2. Let an arc of length l subtend an angle of 60° at the centre of the circle of radiusr_1, while let an arc of length l subtend an angle of 75° at the centre of the circle of radius r_2.

Now, 60° = $\frac{\pi}{3}$ radian and 75° = $\frac{5\pi}{12}$ radian

We know that in a circle of radius *r* unit, if an arc of length *l* unit subtends an angle *θ* radian at the centre, then

$$\theta = \frac{l}{r} \text{ or } l = r\theta$$

.

$$\therefore l = \frac{r_1\pi}{3} \text{ and } l = \frac{r_2 5\pi}{12}$$

$$\Rightarrow \frac{r_1\pi}{3} = \frac{r_2 5\pi}{12}$$

$$\Rightarrow r_1 = \frac{r_2 5}{4}$$

$$\Rightarrow \frac{r_1}{r_2} = \frac{5}{4}$$

Thus, the ratio of the radii is 5:4.

Q7 :

Find the angle in radian though which a pendulum swings if its length is 75 cm and the tip describes an arc of length

(i) 10 cm (ii) 15 cm (iii) 21 cm

Answer :

We know that in a circle of radius *r* unit, if an arc of length *l* unit subtends an angle *θ* radian at the centre, then $\theta = \frac{l}{r}$

.

It is given that *r* = 75 cm

(i) Here, *l* = 10 cm

$$\theta = \frac{10}{75} \text{ radian} = \frac{2}{15} \text{ radian}$$

(ii) Here, *l* = 15 cm

$$\theta = \frac{15}{75} \text{ radian} = \frac{1}{5} \text{ radian}$$

(iii) Here, *l* = 21 cm

$$\theta = \frac{21}{75} \text{ radian} = \frac{7}{25} \text{ radian}$$

Exercise 3.2 : Solutions of Questions on Page Number : **63**
Q1 :

Find the values of other five trigonometric functions if $\cos x = -\frac{1}{2}$, *x* lies in third quadrant.

Answer :

$$\cos x = -\frac{1}{2}$$

$$\therefore \sec x = \frac{1}{\cos x} = \frac{1}{\left(-\frac{1}{2}\right)} = -2$$

$$\sin^2 x + \cos^2 x = 1$$

$$\Rightarrow \sin^2 x = 1 - \cos^2 x$$

$$\Rightarrow \sin^2 x = 1 - \left(-\frac{1}{2}\right)^2$$

$$\Rightarrow \sin^2 x = 1 - \frac{1}{4} = \frac{3}{4}$$

$$\Rightarrow \sin x = \pm\frac{\sqrt{3}}{2}$$

Since *x* lies in the 3rd quadrant, the value of sin *x* will be negative.

$$\therefore \sin x = -\frac{\sqrt{3}}{2}$$

$$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{\left(-\frac{\sqrt{3}}{2}\right)} = -\frac{2}{\sqrt{3}}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{\left(-\frac{\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}\right)} = \sqrt{3}$$

$$\cot x = \frac{1}{\tan x} = \frac{1}{\sqrt{3}}$$

Q2 :

Find the values of other five trigonometric functions if $\sin x = \frac{3}{5}$, *x* lies in second quadrant.

Answer :

$$\sin x = \frac{3}{5}$$

$$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{\left(\frac{3}{5}\right)} = \frac{5}{3}$$

$$\sin^2 x + \cos^2 x = 1$$

$$\Rightarrow \cos^2 x = 1 - \sin^2 x$$

$$\Rightarrow \cos^2 x = 1 - \left(\frac{3}{5}\right)^2$$

$$\Rightarrow \cos^2 x = 1 - \frac{9}{25}$$

$$\Rightarrow \cos^2 x = \frac{16}{25}$$

$$\Rightarrow \cos x = \pm\frac{4}{5}$$

Since *x* lies in the 2nd quadrant, the value of cos *x* will be negative

$$\therefore \cos x = -\frac{4}{5}$$

$$\sec x = \frac{1}{\cos x} = \frac{1}{\left(-\frac{4}{5}\right)} = -\frac{5}{4}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{\left(\frac{3}{5}\right)}{\left(-\frac{4}{5}\right)} = -\frac{3}{4}$$

$$\cot x = \frac{1}{\tan x} = -\frac{4}{3}$$

Q3 :

Find the values of other five trigonometric functions if $\cot x = \frac{3}{4}$, *x* lies in third quadrant.

Answer :

$$\cot x = \frac{3}{4}$$

$$\tan x = \frac{1}{\cot x} = \frac{1}{\left(\frac{3}{4}\right)} = \frac{4}{3}$$

$$1 + \tan^2 x = \sec^2 x$$

$$\Rightarrow 1 + \left(\frac{4}{3}\right)^2 = \sec^2 x$$

$$\Rightarrow 1 + \frac{16}{9} = \sec^2 x$$

$$\Rightarrow \frac{25}{9} = \sec^2 x$$

$$\Rightarrow \sec x = \pm\frac{5}{3}$$

Since x lies in the 3rd quadrant, the value of sec x will be negative.

$$\therefore \sec x = -\frac{5}{3}$$

$$\cos x = \frac{1}{\sec x} = \frac{1}{\left(-\frac{5}{3}\right)} = -\frac{3}{5}$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\Rightarrow \frac{4}{3} = \frac{\sin x}{\left(\frac{-3}{5}\right)}$$

$$\Rightarrow \sin x = \left(\frac{4}{3}\right) \times \left(\frac{-3}{5}\right) = -\frac{4}{5}$$

$$\operatorname{cosec} x = \frac{1}{\sin x} = -\frac{5}{4}$$

Q4 :

Find the values of other five trigonometric functions if $\sec x = \frac{13}{5}$, *x* lies in fourth quadrant.

Answer :

$$\sec x = \frac{13}{5}$$

$$\cos x = \frac{1}{\sec x} = \frac{1}{\left(\frac{13}{5}\right)} = \frac{5}{13}$$

$$\sin^2 x + \cos^2 x = 1$$

$$\Rightarrow \sin^2 x = 1 - \cos^2 x$$

$$\Rightarrow \sin^2 x = 1 - \left(\frac{5}{13}\right)^2$$

$$\Rightarrow \sin^2 x = 1 - \frac{25}{169} = \frac{144}{169}$$

$$\Rightarrow \sin x = \pm\frac{12}{13}$$

Since *x* lies in the 4th quadrant, the value of sin *x* will be negative.

$$\therefore \sin x = -\frac{12}{13}$$

$$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{\left(-\frac{12}{13}\right)} = -\frac{13}{12}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{\left(\frac{-12}{13}\right)}{\left(\frac{5}{13}\right)} = -\frac{12}{5}$$

$$\cot x = \frac{1}{\tan x} = \frac{1}{\left(-\frac{12}{5}\right)} = -\frac{5}{12}$$

Q5 :

Find the values of other five trigonometric functions if $\tan x = -\frac{5}{12}$, *x* lies in second quadrant.

Answer :

$$\tan x = -\frac{5}{12}$$

$$\cot x = \frac{1}{\tan x} = \frac{1}{\left(-\frac{5}{12}\right)} = -\frac{12}{5}$$

$$1 + \tan^2 x = \sec^2 x$$

$$\Rightarrow 1 + \left(-\frac{5}{12}\right)^2 = \sec^2 x$$

$$\Rightarrow 1 + \frac{25}{144} = \sec^2 x$$

$$\Rightarrow \frac{169}{144} = \sec^2 x$$

$$\Rightarrow \sec x = \pm\frac{13}{12}$$

Since *x* lies in the 2nd quadrant, the value of sec *x* will be negative.

∴sec *x* = $-\frac{13}{12}$

$$\cos x = \frac{1}{\sec x} = \frac{1}{\left(-\frac{13}{12}\right)} = -\frac{12}{13}$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\Rightarrow -\frac{5}{12} = \frac{\sin x}{\left(-\frac{12}{13}\right)}$$

$$\Rightarrow \sin x = \left(-\frac{5}{12}\right) \times \left(-\frac{12}{13}\right) = \frac{5}{13}$$

$$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{\left(\frac{5}{13}\right)} = \frac{13}{5}$$

Q6 :

Find the value of the trigonometric function sin 765°

Answer :

It is known that the values of sin *x* repeat after an interval of 2π or 360°.

$$\therefore \sin 765° = \sin\left(2 \times 360° + 45°\right) = \sin 45° = \frac{1}{\sqrt{2}}$$

Q7 :

Find the value of the trigonometric function cosec (-1410°)

Answer :

It is known that the values of cosec *x* repeat after an interval of 2π or 360°.

$$\begin{aligned}\therefore \operatorname{cosec}\left(-1410^\circ\right) &= \operatorname{cosec}\left(-1410^\circ + 4\times 360^\circ\right)\\ &= \operatorname{cosec}\left(-1410^\circ + 1440^\circ\right)\\ &= \operatorname{cosec}30^\circ = 2\end{aligned}$$

Q8 :

Find the value of the trigonometric function $\tan\dfrac{19\pi}{3}$

Answer :

It isknown that the values of tan *x* repeat after an interval of π or 180°.

$$\therefore \tan\frac{19\pi}{3} = \tan 6\frac{1}{3}\pi = \tan\left(6\pi + \frac{\pi}{3}\right) = \tan\frac{\pi}{3} = \tan 60^\circ = \sqrt{3}$$

Q9 :

Find the value of the trigonometric function $\sin\left(-\dfrac{11\pi}{3}\right)$

Answer :

It is known that the values of sin *x* repeat after an interval of 2π or 360°.

$$\therefore \sin\left(-\frac{11\pi}{3}\right) = \sin\left(-\frac{11\pi}{3} + 2\times 2\pi\right) = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

Q10 :

Find the value of the trigonometric function $\cot\left(-\dfrac{15\pi}{4}\right)$

Answer :

It is known that the values of cot *x* repeat after an interval of π or 180°.

$$\therefore \cot\left(-\frac{15\pi}{4}\right) = \cot\left(-\frac{15\pi}{4} + 4\pi\right) = \cot\frac{\pi}{4} = 1$$

Exercise 3.3 : Solutions of Questions on Page Number : **73**

Q1 :

$$\sin^2\frac{\pi}{6} + \cos^2\frac{\pi}{3} - \tan^2\frac{\pi}{4} = -\frac{1}{2}$$

Answer :

L.H.S. = $\sin^2\frac{\pi}{6} + \cos^2\frac{\pi}{3} - \tan^2\frac{\pi}{4}$

$$= \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 - (1)^2$$

$$= \frac{1}{4} + \frac{1}{4} - 1 = -\frac{1}{2}$$

$$= \text{R.H.S.}$$

Q2 :

Prove that $2\sin^2\frac{\pi}{6} + \operatorname{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3} = \frac{3}{2}$

Answer :

L.H.S. = $2\sin^2\frac{\pi}{6} + \operatorname{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3}$

$$= 2\left(\frac{1}{2}\right)^2 + \operatorname{cosec}^2\left(\pi + \frac{\pi}{6}\right)\left(\frac{1}{2}\right)^2$$

$$= 2 \times \frac{1}{4} + \left(-\operatorname{cosec}\frac{\pi}{6}\right)^2\left(\frac{1}{4}\right)$$

$$= \frac{1}{2} + (-2)^2\left(\frac{1}{4}\right)$$

$$= \frac{1}{2} + \frac{4}{4} = \frac{1}{2} + 1 = \frac{3}{2}$$

$$= \text{R.H.S.}$$

Q3 :

Prove that $\cot^2\frac{\pi}{6} + \operatorname{cosec}\frac{5\pi}{6} + 3\tan^2\frac{\pi}{6} = 6$

Answer :

L.H.S. = $\cot^2\frac{\pi}{6} + \operatorname{cosec}\frac{5\pi}{6} + 3\tan^2\frac{\pi}{6}$

$$= \left(\sqrt{3}\right)^2 + \operatorname{cosec}\left(\pi - \frac{\pi}{6}\right) + 3\left(\frac{1}{\sqrt{3}}\right)^2$$

$$= 3 + \operatorname{cosec}\frac{\pi}{6} + 3 \times \frac{1}{3}$$

$$= 3 + 2 + 1 = 6$$

$$= \text{R.H.S}$$

Q4 :

Prove that $2\sin^2\frac{3\pi}{4} + 2\cos^2\frac{\pi}{4} + 2\sec^2\frac{\pi}{3} = 10$

Answer :

L.H.S = $2\sin^2\frac{3\pi}{4} + 2\cos^2\frac{\pi}{4} + 2\sec^2\frac{\pi}{3}$

$$= 2\left\{\sin\left(\pi - \frac{\pi}{4}\right)\right\}^2 + 2\left(\frac{1}{\sqrt{2}}\right)^2 + 2(2)^2$$

$$= 2\left\{\sin\frac{\pi}{4}\right\}^2 + 2\times\frac{1}{2} + 8$$

$$= 2\left(\frac{1}{\sqrt{2}}\right)^2 + 1 + 8$$

$$= 1 + 1 + 8$$

$$= 10$$

$$= \text{R.H.S}$$

Q5 :

Find the value of:

(i) sin 75°

(ii) tan 15°

Answer :

(i) sin 75° = sin (45° + 30°)

= sin 45° cos 30° + cos 45° sin 30°

[sin (*x* + *y*) = sin *x* cos *y* + cos *x* sin *y*]

$$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$$

$$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

(ii) tan 15° = tan (45° - 30°)

$$= \frac{\tan 45° - \tan 30°}{1 + \tan 45° \tan 30°} \qquad \left[\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}\right]$$

$$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1\left(\frac{1}{\sqrt{3}}\right)} = \frac{\frac{\sqrt{3}-1}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}}$$

$$= \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{\left(\sqrt{3}-1\right)^2}{\left(\sqrt{3}+1\right)\left(\sqrt{3}-1\right)} = \frac{3+1-2\sqrt{3}}{\left(\sqrt{3}\right)^2 - (1)^2}$$

$$= \frac{4-2\sqrt{3}}{3-1} = 2 - \sqrt{3}$$

Q6 :

Prove that: $\cos\left(\frac{\pi}{4} - x\right)\cos\left(\frac{\pi}{4} - y\right) - \sin\left(\frac{\pi}{4} - x\right)\sin\left(\frac{\pi}{4} - y\right) = \sin(x+y)$

Answer :

$$\cos\left(\frac{\pi}{4} - x\right)\cos\left(\frac{\pi}{4} - y\right) - \sin\left(\frac{\pi}{4} - x\right)\sin\left(\frac{\pi}{4} - y\right)$$

$$= \frac{1}{2}\left[2\cos\left(\frac{\pi}{4} - x\right)\cos\left(\frac{\pi}{4} - y\right)\right] + \frac{1}{2}\left[-2\sin\left(\frac{\pi}{4} - x\right)\sin\left(\frac{\pi}{4} - y\right)\right]$$

$$= \frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4} - x\right) + \left(\frac{\pi}{4} - y\right)\right\} + \cos\left\{\left(\frac{\pi}{4} - x\right) - \left(\frac{\pi}{4} - y\right)\right\}\right]$$

$$+ \frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4} - x\right) + \left(\frac{\pi}{4} - y\right)\right\} - \cos\left\{\left(\frac{\pi}{4} - x\right) - \left(\frac{\pi}{4} - y\right)\right\}\right]$$

$$\begin{bmatrix} \because 2\cos A \cos B = \cos(A+B) + \cos(A-B) \\ -2\sin A \sin B = \cos(A+B) - \cos(A-B) \end{bmatrix}$$

$$= 2 \times \frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4} - x\right) + \left(\frac{\pi}{4} - y\right)\right\}\right]$$

$$= \cos\left[\frac{\pi}{2} - (x+y)\right]$$

$$= \sin(x+y)$$

$$= \text{R.H.S}$$

Q7 :

Prove that: $$\frac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)}=\left(\frac{1+\tan x}{1-\tan x}\right)^2$$

Answer :

It is known that $\tan(A+B)=\dfrac{\tan A+\tan B}{1-\tan A\tan B}$ and $\tan(A-B)=\dfrac{\tan A-\tan B}{1+\tan A\tan B}$

$$\therefore \text{L.H.S.} = \frac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)}=\frac{\left(\dfrac{\tan\frac{\pi}{4}+\tan x}{1-\tan\frac{\pi}{4}\tan x}\right)}{\left(\dfrac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}\right)}=\frac{\left(\dfrac{1+\tan x}{1-\tan x}\right)}{\left(\dfrac{1-\tan x}{1+\tan x}\right)}=\left(\frac{1+\tan x}{1-\tan x}\right)^2=\text{R.H.S.}$$

Q8 :

Prove that $$\frac{\cos(\pi+x)\cos(-x)}{\sin(\pi-x)\cos\left(\frac{\pi}{2}+x\right)}=\cot^2 x$$

Answer :

$$\text{L.H.S.}=\frac{\cos(\pi+x)\cos(-x)}{\sin(\pi-x)\cos\left(\frac{\pi}{2}+x\right)}$$
$$=\frac{[-\cos x][\cos x]}{(\sin x)(-\sin x)}$$
$$=\frac{-\cos^2 x}{-\sin^2 x}$$
$$=\cot^2 x$$
$$=\text{R.H.S.}$$

Q9 :

$$\cos\left(\frac{3\pi}{2}+x\right)\cos(2\pi+x)\left[\cot\left(\frac{3\pi}{2}-x\right)+\cot(2\pi+x)\right]=1$$

Answer :

L.H.S. = $\cos\left(\frac{3\pi}{2}+x\right)\cos(2\pi+x)\left[\cot\left(\frac{3\pi}{2}-x\right)+\cot(2\pi+x)\right]$

$$=\sin x\cos x[\tan x+\cot x]$$

$$=\sin x\cos x\left(\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}\right)$$

$$=(\sin x\cos x)\left[\frac{\sin^2 x+\cos^2 x}{\sin x\cos x}\right]$$

$$=1=\text{R.H.S.}$$

Q10 :

Prove that sin (*n* + 1)*x* sin (*n* + 2)*x* + cos (*n* + 1)*x* cos (*n* + 2)*x* = cos *x*

Answer :

L.H.S. = sin (n + 1)x sin(n + 2)x + cos (n + 1)x cos(n + 2)x

$$=\frac{1}{2}\left[2\sin(\mathrm{n}+1)\mathrm{x}\sin(\mathrm{n}+2)\mathrm{x}+2\cos(\mathrm{n}+1)\mathrm{x}\cos(\mathrm{n}+2)\mathrm{x}\right]$$

$$=\frac{1}{2}\left[\begin{array}{l}\cos\{(\mathrm{n}+1)\mathrm{x}-(\mathrm{n}+2)\mathrm{x}\}-\cos\{(\mathrm{n}+1)\mathrm{x}+(\mathrm{n}+2)\mathrm{x}\}\\ +\cos\{(\mathrm{n}+1)\mathrm{x}+(\mathrm{n}+2)\mathrm{x}\}+\cos\{(\mathrm{n}+1)\mathrm{x}-(\mathrm{n}+2)\mathrm{x}\}\end{array}\right]$$

$$\left[\begin{array}{l}\because -2\sin\mathrm{A}\sin\mathrm{B}=\cos(\mathrm{A}+\mathrm{B})-\cos(\mathrm{A}-\mathrm{B})\\ 2\cos\mathrm{A}\cos\mathrm{B}=\cos(\mathrm{A}+\mathrm{B})+\cos(\mathrm{A}-\mathrm{B})\end{array}\right]$$

$$=\frac{1}{2}\times 2\cos\{(\mathrm{n}+1)\mathrm{x}-(\mathrm{n}+2)\mathrm{x}\}$$

$$=\cos(-\mathrm{x})=\cos\mathrm{x}=\text{R.H.S.}$$

Q11 : Prove that

$$\cos\left(\frac{3\pi}{4}+x\right)-\cos\left(\frac{3\pi}{4}-x\right)=-\sqrt{2}\sin x$$

Answer :

It is known that $\cos A-\cos B=-2\sin\left(\frac{A+B}{2}\right).\sin\left(\frac{A-B}{2}\right)$.

$\therefore$L.H.S. = $\cos\left(\frac{3\pi}{4}+x\right)-\cos\left(\frac{3\pi}{4}-x\right)$

$$=-2\sin\left\{\frac{\left(\frac{3\pi}{4}+x\right)+\left(\frac{3\pi}{4}-x\right)}{2}\right\}.\sin\left\{\frac{\left(\frac{3\pi}{4}+x\right)-\left(\frac{3\pi}{4}-x\right)}{2}\right\}$$

$$=-2\sin\left(\frac{3\pi}{4}\right)\sin x$$

$$=-2\sin\left(\pi-\frac{\pi}{4}\right)\sin x$$

$$=-2\sin\frac{\pi}{4}\sin x$$

$$=-2\times\frac{1}{\sqrt{2}}\times\sin x$$

$$=-\sqrt{2}\sin x$$

$$=\text{R.H.S.}$$

Q12 :

Prove that $\sin^2 6x - \sin^2 4x = \sin 2x \sin 10x$

Answer :

It is known that

$$\sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right),\ \ \sin A-\sin B=2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$\therefore$L.H.S. = $\sin^2 6x - \sin^2 4x$

= $(\sin 6x + \sin 4x)(\sin 6x - \sin 4x)$

$$=\left[2\sin\left(\frac{6x+4x}{2}\right)\cos\left(\frac{6x-4x}{2}\right)\right]\left[2\cos\left(\frac{6x+4x}{2}\right).\sin\left(\frac{6x-4x}{2}\right)\right]$$

= (2 sin 5x cos x) (2 cos 5x sin x)

= (2 sin 5x cos 5x) (2 sin x cos x)

= sin 10x sin 2x

= R.H.S.

Q13 :

Prove that $\cos^2 2x - \cos^2 6x = \sin 4x \sin 8x$

Answer :

It is known that

$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right),\ \ \cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

∴L.H.S. = $\cos^2 2x - \cos^2 6x$= (cos 2x + cos 6x) (cos 2x - 6x)

$$=\left[2\cos\left(\frac{2x+6x}{2}\right)\cos\left(\frac{2x-6x}{2}\right)\right]\left[-2\sin\left(\frac{2x+6x}{2}\right)\sin\frac{(2x-6x)}{2}\right]$$

$$=\left[2\cos 4x\cos(-2x)\right]\left[-2\sin 4x\sin(-2x)\right]$$

= [2 cos 4x cos 2x] [-2 sin 4x (- sin 2x)]

= (2 sin 4x cos 4x) (2 sin 2x cos 2x)

= sin 8x sin 4x

= R.H.S.

Q14 :

Prove that $\sin 2x + 2\sin 4x + \sin 6x = 4\cos^2 x \sin 4x$

Answer :

L.H.S. = sin 2x + 2 sin 4x + sin 6x

= [sin 2x + sin 6x] + 2 sin 4x

$$=\left[2\sin\left(\frac{2x+6x}{2}\right)\cos\left(\frac{2x-6x}{2}\right)\right]+2\sin 4x$$

$$\left[\because \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$$

= 2 sin 4x cos (- 2x) + 2 sin 4x

$= 2 \sin 4x \cos 2x + 2 \sin 4x$

$= 2 \sin 4x (\cos 2x + 1)$

$= 2 \sin 4x (2 \cos^2 x - 1 + 1)=$

$2 \sin 4x (2 \cos^2 x)$

$= 4\cos^2 x \sin 4x$

= R.H.S.

Q15 :

Prove that cot 4*x* (sin 5*x* + sin 3*x*) = cot *x* (sin 5*x* - sin 3*x*)

Answer :

L.H.S = $\cot 4x (\sin 5x + \sin 3x)$

$$= \frac{\cos 4x}{\sin 4x}\left[2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)\right]$$

$$\left[\because \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$$

$$= \left(\frac{\cos 4x}{\sin 4x}\right)\left[2\sin 4x \cos x\right]$$

$= 2 \cos 4x \cos x$

R.H.S. = $\cot x (\sin 5x - \sin 3x)$

$$= \frac{\cos x}{\sin x}\left[2\cos\left(\frac{5x+3x}{2}\right)\sin\left(\frac{5x-3x}{2}\right)\right]$$

$$\left[\because \sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\right]$$

$$= \frac{\cos x}{\sin x}\left[2\cos 4x \sin x\right]$$

$= 2 \cos 4x. \cos x$

L.H.S. = R.H.S.

Q16 :

Prove that $\frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x} = -\frac{\sin 2x}{\cos 10x}$

Answer :

It is known that

$$\cos A-\cos B=-2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right),\ \sin A-\sin B=2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

∴L.H.S = $\dfrac{\cos 9x-\cos 5x}{\sin 17x-\sin 3x}$

$$=\frac{-2\sin\left(\frac{9x+5x}{2}\right).\sin\left(\frac{9x-5x}{2}\right)}{2\cos\left(\frac{17x+3x}{2}\right).\sin\left(\frac{17x-3x}{2}\right)}$$

$$=\frac{-2\sin 7x.\sin 2x}{2\cos 10x.\sin 7x}$$

$$=-\frac{\sin 2x}{\cos 10x}$$

$$=\text{R.H.S.}$$

Q17 :

Prove that $\dfrac{\sin 5x+\sin 3x}{\cos 5x+\cos 3x}=\tan 4x$

Answer :

It is known that

$$\sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right),\ \cos A+\cos B=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

∴ L.H.S. = $\dfrac{\sin 5x+\sin 3x}{\cos 5x+\cos 3x}$

$$=\frac{2\sin\left(\frac{5x+3x}{2}\right).\cos\left(\frac{5x-3x}{2}\right)}{2\cos\left(\frac{5x+3x}{2}\right).\cos\left(\frac{5x-3x}{2}\right)}$$

$$=\frac{2\sin 4x.\cos x}{2\cos 4x.\cos x}$$

$$=\frac{\sin 4x}{\cos 4x}$$

$$=\tan 4x=\text{R.H.S.}$$

Q18 :

Prove that $\dfrac{\sin x - \sin y}{\cos x + \cos y} = \tan\dfrac{x-y}{2}$

Answer :

It is known that

$$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right), \quad \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

∴L.H.S. = $\dfrac{\sin x - \sin y}{\cos x + \cos y}$

$$= \frac{2\cos\left(\frac{x+y}{2}\right).\sin\left(\frac{x-y}{2}\right)}{2\cos\left(\frac{x+y}{2}\right).\cos\left(\frac{x-y}{2}\right)}$$

$$= \frac{\sin\left(\frac{x-y}{2}\right)}{\cos\left(\frac{x-y}{2}\right)}$$

$$= \tan\left(\frac{x-y}{2}\right) = \text{R.H.S.}$$

Q19 :

Prove that $\dfrac{\sin x + \sin 3x}{\cos x + \cos 3x} = \tan 2x$

Answer :

It is known that

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right), \quad \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

∴L.H.S. = $\dfrac{\sin x + \sin 3x}{\cos x + \cos 3x}$

$$= \frac{2\sin\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)}{2\cos\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)}$$

$$= \frac{\sin 2x}{\cos 2x}$$

$$= \tan 2x$$

$$= \text{R.H.S}$$

Q20 :

Prove that $\dfrac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} = 2\sin x$

Answer :

It is known that

$$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right), \ \cos^2 A - \sin^2 A = \cos 2A$$

∴L.H.S. = $\dfrac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x}$

$$= \frac{2\cos\left(\frac{x+3x}{2}\right)\sin\left(\frac{x-3x}{2}\right)}{-\cos 2x}$$

$$= \frac{2\cos 2x \sin(-x)}{-\cos 2x}$$

$$= -2 \times (-\sin x)$$

$$= 2\sin x = \text{R.H.S.}$$

Q21 :

Prove that $\dfrac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x} = \cot 3x$

Answer :

L.H.S. = $\dfrac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x}$

$$= \frac{(\cos 4x + \cos 2x) + \cos 3x}{(\sin 4x + \sin 2x) + \sin 3x}$$

$$= \frac{2\cos\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right) + \cos 3x}{2\sin\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right) + \sin 3x}$$

$$\left[\because \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right),\ \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$$

$$= \frac{2\cos 3x \cos x + \cos 3x}{2\sin 3x \cos x + \sin 3x}$$

$$= \frac{\cos 3x(2\cos x + 1)}{\sin 3x(2\cos x + 1)}$$

$$= \cot 3x = \text{R.H.S.}$$

Q22 :

Prove that cot *x* cot 2*x* - cot 2*x* cot 3*x* - cot 3*x* cot *x* = 1

Answer :

L.H.S. = cot x cot 2x - cot 2x cot 3x - cot 3x cot x

= cot x cot 2x - cot 3x (cot 2x + cot x)

= cot x cot 2x - cot (2x + x) (cot 2x + cot x)

$$= \cot x \cot 2x - \left[\frac{\cot 2x \cot x - 1}{\cot x + \cot 2x}\right](\cot 2x + \cot x)$$

$$\left[\because \cot(A+B) = \frac{\cot A \cot B - 1}{\cot A + \cot B}\right]$$

= cot x cot 2x - (cot 2x cot x - 1)

= 1 = R.H.S.

Q23 :

Prove that $\tan 4x = \frac{4\tan x(1-\tan^2 x)}{1 - 6\tan^2 x + \tan^4 x}$

Answer :

It is known that $\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$.

$\therefore$L.H.S. = tan 4x = tan 2(2x)

$$= \frac{2\tan 2x}{1-\tan^2(2x)}$$

$$= \frac{2\left(\dfrac{2\tan x}{1-\tan^2 x}\right)}{1-\left(\dfrac{2\tan x}{1-\tan^2 x}\right)^2}$$

$$= \frac{\left(\dfrac{4\tan x}{1-\tan^2 x}\right)}{\left[1-\dfrac{4\tan^2 x}{\left(1-\tan^2 x\right)^2}\right]}$$

$$= \frac{\left(\dfrac{4\tan x}{1-\tan^2 x}\right)}{\left[\dfrac{\left(1-\tan^2 x\right)^2-4\tan^2 x}{\left(1-\tan^2 x\right)^2}\right]}$$

$$= \frac{4\tan x\left(1-\tan^2 x\right)}{\left(1-\tan^2 x\right)^2-4\tan^2 x}$$

$$= \frac{4\tan x\left(1-\tan^2 x\right)}{1+\tan^4 x-2\tan^2 x-4\tan^2 x}$$

$$= \frac{4\tan x\left(1-\tan^2 x\right)}{1-6\tan^2 x+\tan^4 x} = \text{R.H.S.}$$

Q24 :

Prove that cos 4*x* = 1 - 8sin^2 *x* cos^2 *x*

Answer :

L.H.S. = cos 4*x*

= cos 2(2*x*)

= 1 - 2 sin^2 2*x* [cos 2*A* = 1 - 2 sin^2 *A*]

= 1 - 2(2 sin *x* cos *x*)^2 [sin2*A* = 2sin *A* cos*A*]

= 1 - 8 sin^2*x* cos^2*x* = R.H.S.

Q25 :

Prove that: cos 6x = 32 $\cos^6 x$ - 48 $\cos^4 x$ + 18 $\cos^2 x$ - 1

Answer :

L.H.S. = cos 6x

= cos 3(2x)

= 4 $\cos^3 2x$ - 3 cos 2x [cos 3A = 4 $\cos^3 A$ - 3 cos A]

= 4 [$(2 \cos^2 x - 1)^3$ - 3 (2 $\cos^2 x$ - 1) [cos 2x = 2 $\cos^2 x$ - 1]

= 4 [$(2 \cos^2 x)^3$ - $(1)^3$ - 3 $(2 \cos^2 x)^2$ + 3 (2 $\cos^2 x$)] - 6$\cos^2 x$ + 3

= 4 [8$\cos^6 x$ - 1 - 12 $\cos^4 x$ + 6 $\cos^2 x$] - 6 $\cos^2 x$ + 3

= 32 $\cos^6 x$ - 4 - 48 $\cos^4 x$ + 24 $\cos^2 x$ - 6 $\cos^2 x$ + 3

= 32 $\cos^6 x$ - 48 $\cos^4 x$ + 18 $\cos^2 x$ - 1

= R.H.S.

Exercise 3.4 : Solutions of Questions on Page Number : **78**

Q1 :

Find the principal and general solutions of the equation $\tan x = \sqrt{3}$

Answer :

$\tan x = \sqrt{3}$

It is known that $\tan\frac{\pi}{3} = \sqrt{3}$ and $\tan\left(\frac{4\pi}{3}\right) = \tan\left(\pi + \frac{\pi}{3}\right) = \tan\frac{\pi}{3} = \sqrt{3}$

Therefore, the principal solutions are $x = \frac{\pi}{3}$ and $\frac{4\pi}{3}$.

Now, $\tan x = \tan\frac{\pi}{3}$

$\Rightarrow x = n\pi + \frac{\pi}{3}$, where $n \in Z$

Therefore, the general solution is $x = n\pi + \frac{\pi}{3}$, where $n \in Z$

Q2 :

Find the principal and general solutions of the equation $\sec x = 2$

Answer :

$\sec x = 2$

It is known that $\sec\frac{\pi}{3} = 2$ and $\sec\frac{5\pi}{3} = \sec\left(2\pi - \frac{\pi}{3}\right) = \sec\frac{\pi}{3} = 2$

Therefore, the principal solutions are *x* = $\frac{\pi}{3}$ and $\frac{5\pi}{3}$.

Now, $\sec x = \sec\frac{\pi}{3}$

$\Rightarrow \cos x = \cos\frac{\pi}{3}$ $\qquad \left[\sec x = \frac{1}{\cos x}\right]$

$\Rightarrow x = 2n\pi \pm \frac{\pi}{3}$, where $n \in Z$

Therefore, the general solution is $x = 2n\pi \pm \frac{\pi}{3}$, where $n \in$ **Z**

Q3 :

Find the principal and general solutions of the equation $\cot x = -\sqrt{3}$

Answer :

$\cot x = -\sqrt{3}$

It is known that $\cot\frac{\pi}{6} = \sqrt{3}$

$\therefore \cot\left(\pi - \frac{\pi}{6}\right) = -\cot\frac{\pi}{6} = -\sqrt{3}$ and $\cot\left(2\pi - \frac{\pi}{6}\right) = -\cot\frac{\pi}{6} = -\sqrt{3}$

i.e., $\cot\frac{5\pi}{6} = -\sqrt{3}$ and $\cot\frac{11\pi}{6} = -\sqrt{3}$

Therefore, the principal solutions are *x* = $\frac{5\pi}{6}$ and $\frac{11\pi}{6}$.

$$\text{Now, } \cot x = \cot\frac{5\pi}{6}$$

$$\Rightarrow \tan x = \tan\frac{5\pi}{6} \qquad \left[\cot x = \frac{1}{\tan x}\right]$$

$$\Rightarrow x = n\pi + \frac{5\pi}{6}, \text{ where } n \in Z$$

Therefore, the general solution is $x = n\pi + \frac{5\pi}{6}, \text{ where } n \in Z$

Q4 :

Find the general solution of cosec *x* = -2

Answer :

cosecx= - 2

It is known that

$$\operatorname{cosec}\frac{\pi}{6} = 2$$

$$\therefore \operatorname{cosec}\left(\pi + \frac{\pi}{6}\right) = -\operatorname{cosec}\frac{\pi}{6} = -2 \text{ and } \operatorname{cosec}\left(2\pi - \frac{\pi}{6}\right) = -\operatorname{cosec}\frac{\pi}{6} = -2$$

$$\text{i.e., } \operatorname{cosec}\frac{7\pi}{6} = -2 \text{ and } \operatorname{cosec}\frac{11\pi}{6} = -2$$

Therefore, the principal solutions are *x* = $\frac{7\pi}{6}$ and $\frac{11\pi}{6}$.

$$\text{Now, } \operatorname{cosec} x = \operatorname{cosec}\frac{7\pi}{6}$$

$$\Rightarrow \sin x = \sin\frac{7\pi}{6} \qquad \left[\operatorname{cosec} x = \frac{1}{\sin x}\right]$$

$$\Rightarrow x = n\pi + (-1)^n \frac{7\pi}{6}, \text{ where } n \in Z$$

Therefore, the general solution is $x = n\pi + (-1)^n \frac{7\pi}{6}, \text{ where } n \in Z$

Q5 :

Find the general solution of the equation $\cos 4x = \cos 2x$

Answer :

$$\cos 4x = \cos 2x$$

$$\Rightarrow \cos 4x - \cos 2x = 0$$

$$\Rightarrow -2\sin\left(\frac{4x+2x}{2}\right)\sin\left(\frac{4x-2x}{2}\right) = 0$$

$$\left[\because \cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\right]$$

$$\Rightarrow \sin 3x \sin x = 0$$

$$\Rightarrow \sin 3x = 0 \quad \text{or} \quad \sin x = 0$$

$$\therefore 3x = n\pi \quad \text{or} \quad x = n\pi,\ \text{where } n \in Z$$

$$\Rightarrow x = \frac{n\pi}{3} \quad \text{or} \quad x = n\pi,\ \text{where } n \in Z$$

Q6 :

Find the general solution of the equation $\cos 3x + \cos x - \cos 2x = 0$

Answer :

$$\cos 3x + \cos x - \cos 2x = 0$$

$$\Rightarrow 2\cos\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right) - \cos 2x = 0 \quad \left[\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$$

$$\Rightarrow 2\cos 2x \cos x - \cos 2x = 0$$

$$\Rightarrow \cos 2x(2\cos x - 1) = 0$$

$$\Rightarrow \cos 2x = 0 \quad \text{or} \quad 2\cos x - 1 = 0$$

$$\Rightarrow \cos 2x = 0 \quad \text{or} \quad \cos x = \frac{1}{2}$$

$$\therefore 2x = (2n+1)\frac{\pi}{2} \quad \text{or} \quad \cos x = \cos\frac{\pi}{3},\ \text{where } n \in Z$$

$$\Rightarrow x = (2n+1)\frac{\pi}{4} \quad \text{or} \quad x = 2n\pi \pm \frac{\pi}{3},\ \text{where } n \in Z$$

Q7 :

Find the general solution of the equation $\sin 2x + \cos x = 0$

Answer :

$$\sin 2x + \cos x = 0$$
$$\Rightarrow 2\sin x \cos x + \cos x = 0$$
$$\Rightarrow \cos x(2\sin x + 1) = 0$$
$$\Rightarrow \cos x = 0 \quad \text{or} \quad 2\sin x + 1 = 0$$
$$\text{Now, } \cos x = 0 \Rightarrow \cos x = (2n+1)\frac{\pi}{2}, \text{ where } n \in Z$$
$$2\sin x + 1 = 0$$
$$\Rightarrow \sin x = \frac{-1}{2} = -\sin\frac{\pi}{6} = \sin\left(\pi + \frac{\pi}{6}\right) = \sin\left(\pi + \frac{\pi}{6}\right) = \sin\frac{7\pi}{6}$$
$$\Rightarrow x = n\pi + (-1)^n \frac{7\pi}{6}, \text{ where } n \in Z$$

Therefore, the general solution is $(2n+1)\frac{\pi}{2} \text{ or } n\pi + (-1)^n \frac{7\pi}{6}, \ n \in Z$.

Q8 :

Find the general solution of the equation $\sec^2 2x = 1 - \tan 2x$

Answer :

$$\sec^2 2x = 1 - \tan 2x$$
$$\Rightarrow 1 + \tan^2 2x = 1 - \tan 2x$$
$$\Rightarrow \tan^2 2x + \tan 2x = 0$$
$$\Rightarrow \tan 2x(\tan 2x + 1) = 0$$
$$\Rightarrow \tan 2x = 0 \quad \text{or} \quad \tan 2x + 1 = 0$$

$$\text{Now, } \tan 2x = 0$$
$$\Rightarrow \tan 2x = \tan 0$$
$$\Rightarrow 2x = n\pi + 0, \text{ where } n \in Z$$
$$\Rightarrow x = \frac{n\pi}{2}, \text{ where } n \in Z$$

$$\tan 2x + 1 = 0$$

$$\Rightarrow \tan 2x = -1 = -\tan\frac{\pi}{4} = \tan\left(\pi - \frac{\pi}{4}\right) = \tan\frac{3\pi}{4}$$

$$\Rightarrow 2x = n\pi + \frac{3\pi}{4}, \text{ where } n \in Z$$

$$\Rightarrow x = \frac{n\pi}{2} + \frac{3\pi}{8}, \text{ where } n \in Z$$

Therefore, the general solution is $\frac{n\pi}{2}$ or $\frac{n\pi}{2} + \frac{3\pi}{8},\ n \in Z$.

Q9 :

Find the general solution of the equation $\sin x + \sin 3x + \sin 5x = 0$

Answer :

$$\sin x + \sin 3x + \sin 5x = 0$$

$$(\sin x + \sin 5x) + \sin 3x = 0$$

$$\Rightarrow \left[2\sin\left(\frac{x+5x}{2}\right)\cos\left(\frac{x-5x}{2}\right)\right] + \sin 3x = 0 \qquad \left[\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$$

$$\Rightarrow 2\sin 3x \cos(-2x) + \sin 3x = 0$$

$$\Rightarrow 2\sin 3x \cos 2x + \sin 3x = 0$$

$$\Rightarrow \sin 3x (2\cos 2x + 1) = 0$$

$$\Rightarrow \sin 3x = 0 \quad \text{or} \quad 2\cos 2x + 1 = 0$$

$$\text{Now, } \sin 3x = 0 \Rightarrow 3x = n\pi, \text{ where } n \in Z$$

$$\text{i.e., } x = \frac{n\pi}{3}, \text{ where } n \in Z$$

$$2\cos 2x + 1 = 0$$

$$\Rightarrow \cos 2x = \frac{-1}{2} = -\cos\frac{\pi}{3} = \cos\left(\pi - \frac{\pi}{3}\right)$$

$$\Rightarrow \cos 2x = \cos\frac{2\pi}{3}$$

$$\Rightarrow 2x = 2n\pi \pm \frac{2\pi}{3}, \text{ where } n \in Z$$

$$\Rightarrow x = n\pi \pm \frac{\pi}{3}, \text{ where } n \in Z$$

Therefore, the general solution is $\frac{n\pi}{3}$ or $n\pi \pm \frac{\pi}{3},\ n \in Z$

Exercise Miscellaneous : Solutions of Questions on Page Number : **81**

Q1 :

Prove that: $2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}=0$

Answer :

L.H.S.

$$=2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$$

$$=2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+2\cos\left(\frac{\frac{3\pi}{13}+\frac{5\pi}{13}}{2}\right)\cos\left(\frac{\frac{3\pi}{13}-\frac{5\pi}{13}}{2}\right) \quad \left[\cos x+\cos y=2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)\right]$$

$$=2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+2\cos\frac{4\pi}{13}\cos\left(\frac{-\pi}{13}\right)$$

$$=2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+2\cos\frac{4\pi}{13}\cos\frac{\pi}{13}$$

$$=2\cos\frac{\pi}{13}\left[\cos\frac{9\pi}{13}+\cos\frac{4\pi}{13}\right]$$

$$=2\cos\frac{\pi}{13}\left[2\cos\left(\frac{\frac{9\pi}{13}+\frac{4\pi}{13}}{2}\right)\cos\left(\frac{\frac{9\pi}{13}-\frac{4\pi}{13}}{2}\right)\right]$$

$$=2\cos\frac{\pi}{13}\left[2\cos\frac{\pi}{2}\cos\frac{5\pi}{26}\right]$$

$$=2\cos\frac{\pi}{13}\times 2\times 0\times\cos\frac{5\pi}{26}$$

= 0 = R.H.S

Q2 :

Prove that: (sin 3*x* + sin *x*) sin *x* + (cos 3*x* - cos *x*) cos *x* = 0

Answer :

L.H.S.

= (sin 3*x* + sin *x*) sin *x* + (cos 3*x* - cos *x*) cos *x*

$$= \sin 3x \sin x + \sin^2 x + \cos 3x \cos x - \cos^2 x$$

$$= \cos 3x \cos x + \sin 3x \sin x - \left(\cos^2 x - \sin^2 x\right)$$

$$= \cos\left(3x - x\right) - \cos 2x \qquad \left[\cos\left(A - B\right) = \cos A \cos B + \sin A \sin B\right]$$

$$= \cos 2x - \cos 2x$$

$$= 0$$

= RH.S.

Q3 :

Prove that: $\left(\cos x + \cos y\right)^2 + \left(\sin x - \sin y\right)^2 = 4\cos^2 \frac{x + y}{2}$

Answer :

L.H.S. = $\left(\cos x + \cos y\right)^2 + \left(\sin x - \sin y\right)^2$

$$= \cos^2 x + \cos^2 y + 2\cos x \cos y + \sin^2 x + \sin^2 y - 2\sin x \sin y$$

$$= \left(\cos^2 x + \sin^2 x\right) + \left(\cos^2 y + \sin^2 y\right) + 2\left(\cos x \cos y - \sin x \sin y\right)$$

$$= 1 + 1 + 2\cos\left(x + y\right) \qquad \left[\cos\left(A + B\right) = \left(\cos A \cos B - \sin A \sin B\right)\right]$$

$$= 2 + 2\cos\left(x + y\right)$$

$$= 2\left[1 + \cos\left(x + y\right)\right]$$

$$= 2\left[1 + 2\cos^2\left(\frac{x + y}{2}\right) - 1\right] \qquad \left[\cos 2A = 2\cos^2 A - 1\right]$$

$$= 4\cos^2\left(\frac{x + y}{2}\right) = \text{R.H.S.}$$

Q4 :

Prove that: $\left(\cos x - \cos y\right)^2 + \left(\sin x - \sin y\right)^2 = 4\sin^2 \frac{x - y}{2}$

Answer :

L.H.S. = $\left(\cos x - \cos y\right)^2 + \left(\sin x - \sin y\right)^2$

$$= \cos^2 x + \cos^2 y - 2\cos x \cos y + \sin^2 x + \sin^2 y - 2\sin x \sin y$$

$$= \left(\cos^2 x + \sin^2 x\right) + \left(\cos^2 y + \sin^2 y\right) - 2\left[\cos x \cos y + \sin x \sin y\right]$$

$$= 1 + 1 - 2\left[\cos(x - y)\right] \qquad \left[\cos(A - B) = \cos A \cos B + \sin A \sin B\right]$$

$$= 2\left[1 - \cos(x - y)\right]$$

$$= 2\left[1 - \left\{1 - 2\sin^2\left(\frac{x - y}{2}\right)\right\}\right] \qquad \left[\cos 2A = 1 - 2\sin^2 A\right]$$

$$= 4\sin^2\left(\frac{x - y}{2}\right) = \text{R.H.S.}$$

Q5 :

Prove that: $\sin x + \sin 3x + \sin 5x + \sin 7x = 4\cos x \cos 2x \sin 4x$

Answer :

It is known that $\sin A + \sin B = 2\sin\left(\frac{A + B}{2}\right) \cdot \cos\left(\frac{A - B}{2}\right)$.

∴L.H.S. = $\sin x + \sin 3x + \sin 5x + \sin 7x$

$$= \left(\sin x + \sin 5x\right) + \left(\sin 3x + \sin 7x\right)$$

$$= 2\sin\left(\frac{x + 5x}{2}\right) \cdot \cos\left(\frac{x - 5x}{2}\right) + 2\sin\left(\frac{3x + 7x}{2}\right)\cos\left(\frac{3x - 7x}{2}\right)$$

$$= 2\sin 3x \cos(-2x) + 2\sin 5x \ \cos(-2x)$$

$$= 2\sin 3x \cos 2x + 2\sin 5x \cos 2x$$

$$= 2\cos 2x\left[\sin 3x + \sin 5x\right]$$

$$= 2\cos 2x\left[2\sin\left(\frac{3x + 5x}{2}\right) \cdot \cos\left(\frac{3x - 5x}{2}\right)\right]$$

$$= 2\cos 2x\left[2\sin 4x \cdot \cos(-x)\right]$$

$$= 4\cos 2x \sin 4x \cos x = \text{R.H.S.}$$

Q6 :

Prove that: $\dfrac{\left(\sin 7x + \sin 5x\right) + \left(\sin 9x + \sin 3x\right)}{\left(\cos 7x + \cos 5x\right) + \left(\cos 9x + \cos 3x\right)} = \tan 6x$

Answer :

It is known that

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right),\ \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right).$$

L.H.S. = $\dfrac{(\sin 7x + \sin 5x) + (\sin 9x + \sin 3x)}{(\cos 7x + \cos 5x) + (\cos 9x + \cos 3x)}$

$$= \frac{\left[2\sin\left(\frac{7x+5x}{2}\right)\cdot\cos\left(\frac{7x-5x}{2}\right)\right] + \left[2\sin\left(\frac{9x+3x}{2}\right)\cdot\cos\left(\frac{9x-3x}{2}\right)\right]}{\left[2\cos\left(\frac{7x+5x}{2}\right)\cdot\cos\left(\frac{7x-5x}{2}\right)\right] + \left[2\cos\left(\frac{9x+3x}{2}\right)\cdot\cos\left(\frac{9x-3x}{2}\right)\right]}$$

$$= \frac{[2\sin 6x\cdot\cos x] + [2\sin 6x\cdot\cos 3x]}{[2\cos 6x\cdot\cos x] + [2\cos 6x\cdot\cos 3x]}$$

$$= \frac{2\sin 6x[\cos x + \cos 3x]}{2\cos 6x[\cos x + \cos 3x]}$$

= tan 6*x*

= R.H.S.

Q7 :

Prove that: $\sin 3x + \sin 2x - \sin x = 4\sin x\cos\frac{x}{2}\cos\frac{3x}{2}$

Answer :

L.H.S. = $\sin 3x + \sin 2x - \sin x$

$$= \sin 3x + (\sin 2x - \sin x)$$

$$= \sin 3x + \left[2\cos\left(\frac{2x+x}{2}\right)\sin\left(\frac{2x-x}{2}\right)\right] \qquad \left[\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\right]$$

$$= \sin 3x + \left[2\cos\left(\frac{3x}{2}\right)\sin\left(\frac{x}{2}\right)\right]$$

$$= \sin 3x + 2\cos\frac{3x}{2}\sin\frac{x}{2}$$

$$= 2\sin\frac{3x}{2}\cdot\cos\frac{3x}{2} + 2\cos\frac{3x}{2}\sin\frac{x}{2} \qquad [\sin 2A = 2\sin A\cdot\cos B]$$

$$= 2\cos\left(\frac{3x}{2}\right)\left[\sin\left(\frac{3x}{2}\right) + \sin\left(\frac{x}{2}\right)\right]$$

$$= 2\cos\left(\frac{3x}{2}\right)\left[2\sin\left\{\frac{\left(\frac{3x}{2}\right)+\left(\frac{x}{2}\right)}{2}\right\}\cos\left\{\frac{\left(\frac{3x}{2}\right)-\left(\frac{x}{2}\right)}{2}\right\}\right] \quad \left[\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$$

$$= 2\cos\left(\frac{3x}{2}\right).2\sin x\cos\left(\frac{x}{2}\right)$$

$$= 4\sin x\cos\left(\frac{x}{2}\right)\cos\left(\frac{3x}{2}\right) = \text{R.H.S.}$$

Q8 :

$\tan x = -\frac{4}{3}$ **, *x* in quadrant II**

Answer :

Here, *x* is in quadrant II.

i.e., $\frac{\pi}{2} < x < \pi$

$$\Rightarrow \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$$

Therefore, $\sin\frac{x}{2}$, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$ are all positive.

It is given that $\tan x = -\frac{4}{3}$.

$$\sec^2 x = 1 + \tan^2 x = 1 + \left(\frac{-4}{3}\right)^2 = 1 + \frac{16}{9} = \frac{25}{9}$$

$$\therefore \cos^2 x = \frac{9}{25}$$

$$\Rightarrow \cos x = \pm \frac{3}{5}$$

As x is in quadrant II, $\cos x$ is negative.

$$\therefore \cos x = \frac{-3}{5}$$

Now, $\cos x = 2\cos^2 \frac{x}{2} - 1$

$$\Rightarrow \frac{-3}{5} = 2\cos^2 \frac{x}{2} - 1$$

$$\Rightarrow 2\cos^2 \frac{x}{2} = 1 - \frac{3}{5}$$

$$\Rightarrow 2\cos^2 \frac{x}{2} = \frac{2}{5}$$

$$\Rightarrow \cos^2 \frac{x}{2} = \frac{1}{5}$$

$$\Rightarrow \cos \frac{x}{2} = \frac{1}{\sqrt{5}} \qquad \left[\because \cos \frac{x}{2} \text{ is positive}\right]$$

$$\therefore \cos \frac{x}{2} = \frac{\sqrt{5}}{5}$$

$$\sin^2 \frac{x}{2} + \cos^2 \frac{x}{2} = 1$$

$$\Rightarrow \sin^2 \frac{x}{2} + \left(\frac{1}{\sqrt{5}}\right)^2 = 1$$

$$\Rightarrow \sin^2 \frac{x}{2} = 1 - \frac{1}{5} = \frac{4}{5}$$

$$\Rightarrow \sin \frac{x}{2} = \frac{2}{\sqrt{5}} \qquad \left[\because \sin \frac{x}{2} \text{ is positive}\right]$$

$$\therefore \sin \frac{x}{2} = \frac{2\sqrt{5}}{5}$$

$$\tan\frac{x}{2} = \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} = \frac{\left(\frac{2}{\sqrt{5}}\right)}{\left(\frac{1}{\sqrt{5}}\right)} = 2$$

Thus, the respective values of $\sin\frac{x}{2},\ \cos\frac{x}{2}$ and $\tan\frac{x}{2}$ are $\frac{2\sqrt{5}}{5},\ \frac{\sqrt{5}}{5},$ and 2.

Q9 :

Find $\sin\frac{x}{2},\ \cos\frac{x}{2}$ and $\tan\frac{x}{2}$ **for** $\cos x = -\frac{1}{3}$**,** ***x*** **in quadrant III**

Answer :

Here, *x* is in quadrant III.

$$\text{i.e.,}\ \pi < x < \frac{3\pi}{2}$$

$$\Rightarrow \frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4}$$

Therefore, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$ are negative, whereas $\sin\frac{x}{2}$ is positive.

It is given that $\cos x = -\frac{1}{3}$.

$$\cos x = 1 - 2\sin^2\frac{x}{2}$$

$$\Rightarrow \sin^2\frac{x}{2} = \frac{1-\cos x}{2}$$

$$\Rightarrow \sin^2\frac{x}{2} = \frac{1-\left(-\frac{1}{3}\right)}{2} = \frac{\left(1+\frac{1}{3}\right)}{2} = \frac{\frac{4}{3}}{2} = \frac{2}{3}$$

$$\Rightarrow \sin\frac{x}{2} = \frac{\sqrt{2}}{\sqrt{3}} \qquad \left[\because\ \sin\frac{x}{2}\ \text{is positive}\right]$$

$$\therefore \sin\frac{x}{2} = \frac{\sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{6}}{3}$$

Now, $\cos x = 2\cos^2\frac{x}{2} - 1$

$$\Rightarrow \cos^2\frac{x}{2} = \frac{1+\cos x}{2} = \frac{1+\left(-\frac{1}{3}\right)}{2} = \frac{\left(\frac{3-1}{3}\right)}{2} = \frac{\left(\frac{2}{3}\right)}{2} = \frac{1}{3}$$

$$\Rightarrow \cos\frac{x}{2} = -\frac{1}{\sqrt{3}} \qquad \left[\because \ \cos\frac{x}{2} \text{ is negative}\right]$$

$$\therefore \cos\frac{x}{2} = -\frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{-\sqrt{3}}{3}$$

$$\tan\frac{x}{2} = \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} = \frac{\left(\frac{\sqrt{2}}{\sqrt{3}}\right)}{\left(\frac{-1}{\sqrt{3}}\right)} = -\sqrt{2}$$

Thus, the respective values of $\sin\frac{x}{2},\ \cos\frac{x}{2}$ and $\tan\frac{x}{2}$ are $\frac{\sqrt{6}}{3},\ \frac{-\sqrt{3}}{3}$, and $-\sqrt{2}$.

Q10 :

Find $\sin\frac{x}{2},\ \cos\frac{x}{2}$ and $\tan\frac{x}{2}$ for $\sin x = \frac{1}{4}$, *x* in quadrant II

Answer :

Here, *x* is in quadrant II.

$$\text{i.e., } \frac{\pi}{2} < x < \pi$$

$$\Rightarrow \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$$

Therefore, $\sin\frac{x}{2}, \cos\frac{x}{2}$, and $\tan\frac{x}{2}$ are all positive.

It is given that $\sin x = \frac{1}{4}$.

$$\cos^2 x = 1 - \sin^2 x = 1 - \left(\frac{1}{4}\right)^2 = 1 - \frac{1}{16} = \frac{15}{16}$$

$\Rightarrow \cos x = -\frac{\sqrt{15}}{4}$ [cos*x* is negative in quadrant II]

$$\sin^2\frac{x}{2}=\frac{1-\cos x}{2}=\frac{1-\left(-\frac{\sqrt{15}}{4}\right)}{2}=\frac{4+\sqrt{15}}{8}$$

$$\Rightarrow \sin\frac{x}{2}=\sqrt{\frac{4+\sqrt{15}}{8}} \qquad \left[\because\ \sin\frac{x}{2}\ \text{is positive}\right]$$

$$=\sqrt{\frac{4+\sqrt{15}}{8}\times\frac{2}{2}}$$

$$=\sqrt{\frac{8+2\sqrt{15}}{16}}$$

$$=\frac{\sqrt{8+2\sqrt{15}}}{4}$$

$$\cos^2\frac{x}{2}=\frac{1+\cos x}{2}=\frac{1+\left(-\frac{\sqrt{15}}{4}\right)}{2}=\frac{4-\sqrt{15}}{8}$$

$$\Rightarrow \cos\frac{x}{2}=\sqrt{\frac{4-\sqrt{15}}{8}} \qquad \left[\because\ \cos\frac{x}{2}\ \text{is positive}\right]$$

$$=\sqrt{\frac{4-\sqrt{15}}{8}\times\frac{2}{2}}$$

$$=\sqrt{\frac{8-2\sqrt{15}}{16}}$$

$$=\frac{\sqrt{8-2\sqrt{15}}}{4}$$

$$\tan\frac{x}{2}=\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}=\frac{\left(\frac{\sqrt{8+2\sqrt{15}}}{4}\right)}{\left(\frac{\sqrt{8-2\sqrt{15}}}{4}\right)}=\frac{\sqrt{8+2\sqrt{15}}}{\sqrt{8-2\sqrt{15}}}$$

$$=\sqrt{\frac{8+2\sqrt{15}}{8-2\sqrt{15}}\times\frac{8+2\sqrt{15}}{8+2\sqrt{15}}}$$

$$=\sqrt{\frac{\left(8+2\sqrt{15}\right)^2}{64-60}}=\frac{8+2\sqrt{15}}{2}=4+\sqrt{15}$$

Thus, the respective values of $\sin\frac{x}{2}$, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$ are $\frac{\sqrt{8+2\sqrt{15}}}{4}$, $\frac{\sqrt{8-2\sqrt{15}}}{4}$, and $4+\sqrt{15}$.