Class 11 Maths NCERT Solutions Chapter - 2

Relations and Functions Class 11

Chapter 2 Relations and Functions Exercise 2.1, 2.2, 2.3, miscellaneous Solutions

Exercise 2.1 : Solutions of Questions on Page Number : **33**

Q1 :

If $\left(\dfrac{x}{3}+1, \; y-\dfrac{2}{3}\right) = \left(\dfrac{5}{3}, \; \dfrac{1}{3}\right)$, **find the values of** *x* **and** *y.*

Answer :

It is given that $\left(\dfrac{x}{3}+1, \; y-\dfrac{2}{3}\right) = \left(\dfrac{5}{3}, \; \dfrac{1}{3}\right)$.

Since the ordered pairs are equal, the corresponding elements will also be equal.

Therefore, $\dfrac{x}{3}+1=\dfrac{5}{3}$ and $y-\dfrac{2}{3}=\dfrac{1}{3}$.

$$\dfrac{x}{3}+1=\dfrac{5}{3}$$

$$\Rightarrow \dfrac{x}{3}=\dfrac{5}{3}-1 \qquad y-\dfrac{2}{3}=\dfrac{1}{3}$$

$$\Rightarrow \dfrac{x}{3}=\dfrac{2}{3} \qquad \Rightarrow y=\dfrac{1}{3}+\dfrac{2}{3}$$

$$\Rightarrow x=2 \qquad \Rightarrow y=1$$

∴ *x*= 2 and *y*= 1

Q2 :

If the set A has 3 elements and the set B = {3, 4, 5}, then find the number of elements in (A x B)?

Answer :

It is given that set A has 3 elements and the elements of set B are 3, 4, and 5.

⇒ Number of elements in set B = 3

Number of elements in (A x B)

= (Number of elements in A) x (Number of elements in B)

= 3 x 3 = 9

Thus, the number of elements in (A x B) is 9.

Q3 :

If G = {7, 8} and H = {5, 4, 2}, find G x H and H x G.

Answer :

G = {7, 8} and H = {5, 4, 2}

We know that the Cartesian product P x Q of two non-empty sets P and Q is defined as

P x Q = {(p, q): p∈P, q ∈Q}

∴G x H = {(7, 5), (7, 4), (7, 2), (8, 5), (8, 4), (8, 2)}

H x G = {(5, 7), (5, 8), (4, 7), (4, 8), (2, 7), (2, 8)}

Q4 :

State whether each of the following statement are true or false. If the statement is false, rewrite the given statement correctly.

(i) If P = {m, n} and Q = {n, m}, then P x Q = {(m, n), (n, m)}.

(ii) If A and B are non-empty sets, then A x B is a non-empty set of ordered pairs (x, y) such that x ∈ A and y ∈ B.

(iii) If A = {1, 2}, B = {3, 4}, then A x (B ∩ Φ) = Φ.

Answer :

(i) False

If P = {m, n} and Q = {n, m}, then

P x Q = {(m, m), (m, n), (n, m), (n, n)}

(ii) True

(iii) True

Q5 :

If A = {-1, 1}, find A x A x A.

Answer :

It is known that for any non-empty set A, A x A x A is defined as

A x A x A = {(a, b, c): a, b, c ∈ A}

It is given that A = {-1, 1}

∴ A x A x A = {(-1, -1, -1), (-1, -1, 1), (-1, 1, -1), (-1, 1, 1),

(1, -1, -1), (1, -1, 1), (1, 1, -1), (1, 1, 1)}

Q6 :

If A x B = {(a, x), (a, y), (b, x), (b, y)}. Find A and B.

Answer :

It is given that A x B = {(a, x), (a, y), (b, x), (b, y)}

We know that the Cartesian product of two non-empty sets P and Q is defined as P x Q = {(p, q): p ∈P, q ∈Q}

∴ A is the set of all first elements and B is the set of all second elements.

Thus, A = {a, b} and B = {x, y}

Q7 :

Let A = {1, 2}, B = {1, 2, 3, 4}, C = {5, 6} and D = {5, 6, 7, 8}. Verify that

(i) A x (B ∩C) = (A x B) ∩(A x C)

(ii) A x C is a subset of B x D

Answer :

(i) To verify: A x (B ∩C) = (A x B) ∩(A x C)

We have B ∩C = {1, 2, 3, 4} ∩{5, 6} = Φ

∴L.H.S. = A x (B ∩C) = A x Φ = Φ

A x B = {(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4)}

A x C = {(1, 5), (1, 6), (2, 5), (2, 6)}

∴ R.H.S. = (A x B) ∩(A x C) = Φ

∴L.H.S. = R.H.S

Hence, A x (B ∩C) = (A x B) ∩(A x C)

(ii) To verify: A x C is a subset of B x D

A x C = {(1, 5), (1, 6), (2, 5), (2, 6)}

B x D = {(1, 5), (1, 6), (1, 7), (1, 8), (2, 5), (2, 6), (2, 7), (2, 8), (3, 5), (3, 6), (3, 7), (3, 8), (4, 5), (4, 6), (4, 7), (4, 8)}

We can observe that all the elements of set A x C are the elements of set B x D.

Therefore, A x C is a subset of B x D.

Q8 :

Let A = {1, 2} and B = {3, 4}. Write A x B. How many subsets will A x B have? List them.

Answer :

A = {1, 2} and B = {3, 4}

∴A x B = {(1, 3), (1, 4), (2, 3), (2, 4)}

⇒ n(A x B) = 4

We know that if C is a set with n(C) = m, then n[P(C)] = 2^m.

Therefore, the set A x B has 2^4= 16 subsets. These are

Φ, {(1, 3)}, {(1, 4)}, {(2, 3)}, {(2, 4)}, {(1, 3), (1, 4)}, {(1, 3), (2, 3)},

{(1, 3), (2, 4)}, {(1, 4), (2, 3)}, {(1, 4), (2, 4)}, {(2, 3), (2, 4)},

{(1, 3), (1, 4), (2, 3)}, {(1, 3), (1, 4), (2, 4)}, {(1, 3), (2, 3), (2, 4)},

{(1, 4), (2, 3), (2, 4)}, {(1, 3), (1, 4), (2, 3), (2, 4)}

Q9 :

Let A and B be two sets such that n(A) = 3 and n (B) = 2. If (x, 1), (y, 2), (z, 1) are in A x B, find A and B, where$x$, y and z are distinct elements.

Answer :

It is given that n(A) = 3 and n(B) = 2; and (x, 1), (y, 2), (z, 1) are in A x B.

We know that A = Set of first elements of the ordered pair elements of A x B

B = Set of second elements of the ordered pair elements of A x B.

∴ x, y, and $z$are the elements of A; and 1 and 2 are the elements of B.

Since n(A) = 3 and n(B) = 2, it is clear that A = {x, y, z} and B = {1, 2}.

Q10 :

The Cartesian product A x A has 9 elements among which are found (-1, 0) and (0, 1). Find the set A and the remaining elements of A x A.

Answer :

We know that if n(A) = p and n(B) = q, then n(A x B) = pq.

∴ n(A x A) = n(A) x n(A)

It is given that n(A x A) = 9

∴ n(A) x n(A) = 9

⇒ n(A) = 3

The ordered pairs (-1, 0) and (0, 1) are two of the nine elements of A x A.

We know that A x A = {(a, a): a ∈A}. Therefore, -1, 0, and 1 are elements of A.

Since n(A) = 3, it is clear that A = {-1, 0, 1}.

The remaining elements of set A x A are (-1, -1), (-1, 1), (0, -1), (0, 0),

(1, -1), (1, 0), and (1, 1)

Exercise 2.2 : Solutions of Questions on Page Number : **35**

Q1 :

Let A = {1, 2, 3, ... , 14}. Define a relation R from A to A by R = {(x, y): 3x - y = 0, where x, y ∈ A}. Write down its domain, codomain and range.

Answer :

The relation R from A to A is given as

R = {(x, y): 3x - y = 0, where x, y ∈ A}

i.e., R = {(x, y): 3x = y, where x, y ∈ A}

∴R = {(1, 3), (2, 6), (3, 9), (4, 12)}

The domain of R is the set of all first elements of the ordered pairs in the relation.

∴Domain of R = {1, 2, 3, 4}

The whole set A is the codomainof the relation R.

∴Codomain of R = A = {1, 2, 3, ..., 14}

The range of R is the set of all second elements of the ordered pairs in the relation.

∴Range of R = {3, 6, 9, 12}

Q2 :

Define a relation R on the set Nof natural numbers by R = {(x, y): y= x + 5, x is a natural number less than 4; x,y ∈ N}. Depict this relationship using roster form. Write down the domain and the range.

Answer :

R = {(x, y): y= x+ 5, xis a natural number less than 4, x, y ∈ **N**}

The natural numbers less than 4 are 1, 2, and 3.

∴R = {(1, 6), (2, 7), (3, 8)}

The domain of R is the set of all first elements of the ordered pairs in the relation.

∴ Domain of R = {1, 2, 3}

The range of R is the set of all second elements of the ordered pairs in the relation.

∴ Range of R = {6, 7, 8}

Q3 :

A = {1, 2, 3, 5} and B = {4, 6, 9}. Define a relation R from A to B by R = {(x, y): the difference between x and y is odd; x ∈A, y ∈B}. Write R in roster form.

Answer :

A = {1, 2, 3, 5} and B = {4, 6, 9}

R = {(x, y): the difference between x and y is odd; x ∈ A, y ∈B}

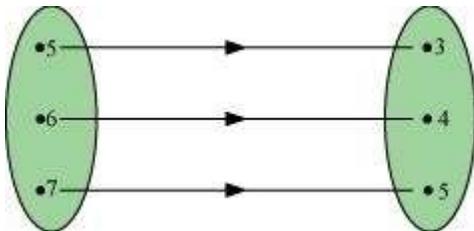∴R = {(1, 4), (1, 6), (2, 9), (3, 4), (3, 6), (5, 4), (5, 6)}

Q4 :

The given figure shows a relationship between the sets P and Q. write this relation

(i) in set-builder form (ii) in roster form.

What is its domain and range?

Answer :

According to the given figure, P = {5, 6, 7}, Q = {3, 4, 5}

(i) R = {(x, y): y = x- 2; x ∈ P} or R = {(x, y): y = x- 2 for x= 5, 6, 7}

(ii) R = {(5, 3), (6, 4), (7, 5)}

Domain of R = {5, 6, 7}

Range of R = {3, 4, 5}

Q5 :

Let A = {1, 2, 3, 4, 6}. Let R be the relation on A defined by

{(a, b): a, b ∈A, b is exactly divisible by a}.

(i) Write R in roster form

(ii) Find the domain of R

(iii) Find the range of R.

Answer :

A = {1, 2, 3, 4, 6}, R = {(a, b): a, b ∈A, b is exactly divisible by a}

(i) R = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 6), (2, 2), (2, 4), (2, 6), (3, 3), (3, 6), (4, 4), (6, 6)}

(ii) Domain of R = {1, 2, 3, 4, 6}

(iii) Range of R = {1, 2, 3, 4, 6}

Q6 :

Determine the domain and range of the relation R defined by R = {(x, x + 5): x ∈{0, 1, 2, 3, 4, 5}}.

Answer :

R = {(x, x+ 5): x ∈{0, 1, 2, 3, 4, 5}}

∴ R = {(0, 5), (1, 6), (2, 7), (3, 8), (4, 9), (5, 10)}

∴Domain of R = {0, 1, 2, 3, 4, 5}

Range of R = {5, 6, 7, 8, 9, 10}

Q7 :

Write the relation R = {(x, x^3): x is a prime number less than 10} in roster form.

Answer :

R = {(x, x^3): x is a prime number less than 10}

The prime numbers less than 10 are 2, 3, 5, and 7.

∴R = {(2, 8), (3, 27), (5, 125), (7, 343)}

Q8 :

Let A = {x, y, z} and B = {1, 2}. Find the number of relations from A to B.

Answer :

It is given that A = {x, y, z} and B = {1, 2}.

∴ A x B = {(x, 1), (x, 2), (y, 1), (y, 2), (z, 1), (z, 2)}

Since n(A x B) = 6, the number of subsets of A x B is 2^6.

Therefore, the number of relations from A to B is 2^6

Q9 :

Let R be the relation on Zdefined by R = {(*a*, *b*): *a*, *b* ∈ Z, *a* - *b*is an integer}. Find the domain and range of R.

Answer :

R = {(*a*, *b*): *a*, *b* ∈ **Z**, *a* - *b*is an integer}

It is known that the difference between any two integers is always an integer.

∴Domain of R = **Z**

Range of R = **Z**

Exercise 2.3 : Solutions of Questions on Page Number : **44**
Q1 :

Which of the following relations are functions? Give reasons. If it is a function, determine its domain and range.

(i) {(2, 1), (5, 1), (8, 1), (11, 1), (14, 1), (17, 1)}

(ii) {(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6), (14, 7)}

(iii) {(1, 3), (1, 5), (2, 5)}

Answer :

(i) {(2, 1), (5, 1), (8, 1), (11, 1), (14, 1), (17, 1)}

Since 2, 5, 8, 11, 14, and 17 are the elements of the domain of the given relation having their unique images, this relation is a function.

Here, domain = {2, 5, 8, 11, 14, 17} and range = {1}

(ii) {(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6), (14, 7)}

Since 2, 4, 6, 8, 10, 12, and 14 are the elements of the domain of the given relation having their unique images, this relation is a function.

Here, domain = {2, 4, 6, 8, 10, 12, 14} and range = {1, 2, 3, 4, 5, 6, 7}

(iii) {(1, 3), (1, 5), (2, 5)}

Since the same first element i.e., 1 corresponds to two different images i.e., 3 and 5, this relation is not a function.

Q2 :

Find the domain and range of the following real function:

(i) $f(x) = -|x|$ (ii) $f(x) = \sqrt{9-x^2}$

Answer :

(i) $f(x) = -|x|$, $x \in R$

We know that $|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

$\therefore f(x) = -|x| = \begin{cases} -x, & x \geq 0 \\ x, & x < 0 \end{cases}$

Since $f(x)$ is defined for $x \in$ **R**, the domain of f is **R.**

It can be observed that the range of $f(x) = -|x|$ is all real numbers except positive real numbers.

$\therefore$The range of f is $(-\infty, 0]$.

(ii) $f(x) = \sqrt{9-x^2}$

Since $\sqrt{9-x^2}$ is defined for all real numbers that are greater than or equal to -3 and less than or equal to 3, the domain of $f(x)$ is $\{x : -3 \leq x \leq 3\}$ or $[-3, 3]$.

For any value of x such that $-3 \leq x \leq 3$, the value of $f(x)$ will lie between 0 and 3.

$\therefore$The range of $f(x)$ is $\{x: 0 \leq x \leq 3\}$ or $[0, 3]$.

Q3 :

A function f is defined by $f(x) = 2x- 5$. Write down the values of

(i) $f(0)$, (ii) $f(7)$, (iii) $f(-3)$

Answer :

The given function is $f(x) = 2x- 5$.

Therefore,

(i) $f(0) = 2 \times 0 - 5 = 0 - 5 = -5$

(ii) $f(7) = 2 \times 7 - 5 = 14 - 5 = 9$

(iii) $f(-3) = 2 \times (-3) - 5 = -6 - 5 = -11$

Q4 :

The function 't' which maps temperature in degree Celsius into temperature in degree Fahrenheit is defined

by $t(C) = \dfrac{9C}{5} + 32$.

Find (i) $t(0)$ (ii) $t(28)$ (iii) $t(-10)$ (iv) The value of C, when $t(C) = 212$

Answer :

The given function is $t(C) = \dfrac{9C}{5} + 32$.

Therefore,

(i) $t(0) = \dfrac{9 \times 0}{5} + 32 = 0 + 32 = 32$

(ii) $t(28) = \dfrac{9 \times 28}{5} + 32 = \dfrac{252 + 160}{5} = \dfrac{412}{5}$

(iii) $t(-10) = \dfrac{9 \times (-10)}{5} + 32 = 9 \times (-2) + 32 = -18 + 32 = 14$

(iv) It is given that $t(C) = 212$

$\therefore 212 = \dfrac{9C}{5} + 32$

$\Rightarrow \dfrac{9C}{5} = 212 - 32$

$\Rightarrow \dfrac{9C}{5} = 180$

$\Rightarrow 9C = 180 \times 5$

$\Rightarrow C = \dfrac{180 \times 5}{9} = 100$

Thus, the value of t, when $t(C) = 212$, is 100.

Q5 :

Find the range of each of the following functions.

(i) $f(x) = 2 - 3x,\ x \in R,\ x > 0$.

(ii) $f(x) = x^2 + 2,\ x,$ is a real number.

(iii) $f(x) = x,\ x$ is a real number

Answer :

(i) $f(x) = 2 - 3x,\ x \in \mathbf{R},\ x > 0$

The values of $f(x)$ for various values of real numbers $x > 0$ can be written in the tabular form as

x	0.01	0.1	0.9	1	2	2.5	4	5	...
$f(x)$	1.97	1.7	-0.7	-1	-4	-5.5	-10	-13	...

Thus, it can be clearly observed that the range of f is the set of all real numbers less than 2.

i.e., range of $f = (-\infty, 2)$

Alter:

Let $x > 0$

$\Rightarrow 3x > 0$

$\Rightarrow 2 - 3x < 2$

$\Rightarrow f(x) < 2$

$\therefore$ Range of $f = (-\infty, 2)$

(ii) $f(x) = x^2 + 2$, x, is a real number

The values of $f(x)$ for various values of real numbers x can be written in the tabular form as

x	0	±0.3	±0.8	±1	±2	±3		...
$f(x)$	2	2.09	2.64	3	6	11		.....

Thus, it can be clearly observed that the range of f is the set of all real numbers greater than 2.

i.e., range of $f = [2, \infty)$

Alter:

Let x be any real number.

Accordingly,

$x^2 \geq 0$

$\Rightarrow x^2 + 2 \geq 0 + 2$

$\Rightarrow x^2 + 2 \geq 2 \Rightarrow f(x) \geq 2$

$\therefore$ Range of $f = [2, \infty)$

(iii) $f(x) = x$, x is a real number

It is clear that the range of f is the set of all real

numbers. $\therefore$ Range of $f = \mathbf{R}$

Q1 :

The relation *f* is defined by $f(x) = \begin{cases} x^2, & 0 \le x \le 3 \\ 3x, & 3 \le x \le 10 \end{cases}$

The relation *g* is defined by $g(x) = \begin{cases} x^2, & 0 \le x \le 2 \\ 3x, & 2 \le x \le 10 \end{cases}$

Show that *f* is a function and *g* is not a function.

Answer :

The relation *f* is defined as $f(x) = \begin{cases} x^2, & 0 \le x \le 3 \\ 3x, & 3 \le x \le 10 \end{cases}$

It is observed that for

$0 \le x < 3$, $f(x) = x^2$

$3 < x \le 10$, $f(x) = 3x$

Also, at $x = 3$, $f(x) = 3^2 = 9$ or $f(x) = 3 \times 3 = 9$

i.e., at $x = 3$, $f(x) = 9$

Therefore, for $0 \le x \le 10$, the images of $f(x)$ are unique.

Thus, the given relation is a function.

The relation *g* is defined as $g(x) = \begin{cases} x^2, & 0 \le x \le 2 \\ 3x, & 2 \le x \le 10 \end{cases}$

It can be observed that for $x = 2$, $g(x) = 2^2 = 4$ and $g(x) = 3 \times 2 = 6$

Hence, element 2 of the domain of the relation *g* corresponds to two different images i.e., 4 and 6. Hence, this relation is not a function

Q2 :

If *f*(x) = x², find $\dfrac{f(1.1) - f(1)}{(1.1 - 1)}$.

Answer :

$f(x) = x^2$

$\therefore \dfrac{f(1.1) - f(1)}{(1.1 - 1)} = \dfrac{(1.1)^2 - (1)^2}{(1.1 - 1)} = \dfrac{1.21 - 1}{0.1} = \dfrac{0.21}{0.1} = 2.1$

Q3 :

Find the domain of the function $f(x) = \dfrac{x^2 + 2x + 1}{x^2 - 8x + 12}$

Answer :

The given function is $f(x) = \dfrac{x^2 + 2x + 1}{x^2 - 8x + 12}$.

$$f(x) = \frac{x^2 + 2x + 1}{x^2 - 8x + 12} = \frac{x^2 + 2x + 1}{(x - 6)(x - 2)}$$

It can be seen that function f is defined for all real numbers except at $x = 6$ and $x = 2$.

Hence, the domain of f is **R** - {2, 6}.

Q4 :

Find the domain and the range of the real function f defined by $f(x) = \sqrt{(x - 1)}$.

Answer :

The given real function is $f(x) = \sqrt{x - 1}$.

It can be seen that $\sqrt{x - 1}$ is defined for $(x - 1) \geq 0$.

i.e., $f(x) = \sqrt{(x - 1)}$ is defined for $x \geq 1$.

Therefore, the domain of f is the set of all real numbers greater than or equal to 1 i.e., the domain of $f = [1, \infty)$.

As $x \geq 1 \Rightarrow (x - 1) \geq 0 \Rightarrow \sqrt{x - 1} \geq 0$

Therefore, the range of f is the set of all real numbers greater than or equal to 0 i.e., the range of $f = [0, \infty)$.

Q5 :

Find the domain and the range of the real function f defined by $f(x) = |x - 1|$.

Answer :

The given real function is $f(x) = |x - 1|$.

It is clear that $|x - 1|$ is defined for all real numbers.

∴ Domain of $f = $ **R**

Also, for $x \in \mathbf{R}$, $|x-1|$ assumes all real numbers.

Hence, the range of f is the set of all non-negative real numbers.

Q6 :

Let $f = \left\{\left(x, \dfrac{x^2}{1+x^2}\right) : x \in \mathbf{R}\right\}$ be a function from R into R. Determine the range of f.

Answer :

$$f = \left\{\left(x, \dfrac{x^2}{1+x^2}\right) : x \in \mathbf{R}\right\}$$

$$= \left\{(0,\ 0),\ \left(\pm 0.5,\ \dfrac{1}{5}\right),\ \left(\pm 1,\ \dfrac{1}{2}\right),\ \left(\pm 1.5,\ \dfrac{9}{13}\right),\ \left(\pm 2,\ \dfrac{4}{5}\right),\ \left(3,\ \dfrac{9}{10}\right),\ \left(4,\ \dfrac{16}{17}\right),\ \ldots\right\}$$

The range of f is the set of all second elements. It can be observed that all these elements are greater than or equal to 0 but less than 1.

[Denominator is greater numerator]

Thus, range of $f = [0, 1)$

Q7 :

Let f, g: R → R be defined, respectively by $f(x) = x + 1$, $g(x) = 2x - 3$. Find $f + g$, $f - g$ and $\dfrac{f}{g}$.

Answer :

f, g: R → R is defined as $f(x) = x + 1$, $g(x) = 2x - 3$

$(f + g) (x) = f(x) + g(x) = (x + 1) + (2x - 3) = 3x - 2$

$\therefore (f + g) (x) = 3x - 2$

$(f - g) (x) = f(x) - g(x) = (x + 1) - (2x - 3) = x + 1 - 2x + 3 = -x+4$

$\therefore (f - g) (x) = -x+ 4$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, \ g(x) \neq 0, \ x \in \mathbf{R}$$

$$\therefore \left(\frac{f}{g}\right)(x) = \frac{x+1}{2x-3}, \ 2x - 3 \neq 0 \text{ or } 2x \neq 3$$

$$\therefore \left(\frac{f}{g}\right)(x) = \frac{x+1}{2x-3}, \ x \neq \frac{3}{2}$$

Q8 :

Let f = {(1, 1), (2, 3), (0, -1), (-1, -3)} be a function from Z to Z defined by $f(x)$ = ax+ b, for some integers a, b. Determine a, b.

Answer :

f = {(1, 1), (2, 3), (0, -1), (-1, -3)}

$f(x)$ = ax+ b

(1, 1) ∈ f

⇒ f(1) = 1

⇒ a x 1 + b= 1

⇒ a+ b= 1

(0, -1) ∈ f

⇒ f(0) = -1

⇒ a x 0 + b= -1

⇒ b= -1

On substituting b= -1 in a+ b= 1, we obtain a+ (-1) = 1 ⇒ a= 1 + 1 = 2.

Thus, the respective values of $a$and $b$are 2 and -1.

Q9 :

Let R be a relation from N to N defined by R = {(a, b): a, b ∈ N and a = b^2}. Are the following true?

(i) (a, a) ∈ R, for all a ∈ N
(ii) (a, b) ∈ R, implies (b, a) ∈ R

(iii) (a, b) ∈ R, (b, c) ∈ R implies (a, c) ∈ R.

Justify your answer in each case.

Answer :

R = {(a, b): a, b ∈ **N** and a = b²}

(i) It can be seen that 2 ∈ **N**;however, 2 ≠ 2² = 4.

Therefore, the statement "(a, a) ∈ R, for all a ∈ **N**" is not true.

(ii) It can be seen that (9, 3) ∈ **N** because 9, 3 ∈ **N** and 9 = 3².

Now, 3 ≠ 9² = 81; therefore, (3, 9) ∉ **N**

Therefore, the statement "(a, b) ∈ R, implies (b, a) ∈ R" is not true.

(iii) It can be seen that (16, 4) ∈ R, (4, 2) ∈ R because 16, 4, 2 ∈ **N** and 16 = 4² and 4 = 2².

Now, 16 ≠ 2² = 4; therefore, (16, 2) ∉ **N**

Therefore, the statement "(a, b) ∈ R, (b, c) ∈ R implies (a, c) ∈ R" is not true.

Q10 :

Let A = {1, 2, 3, 4}, B = {1, 5, 9, 11, 15, 16} and f = {(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)}. Are the following true?

(i) f is a relation from A to B (ii) f is a function from A to B.

Justify your answer in each case.

Answer :

A = {1, 2, 3, 4} and B = {1, 5, 9, 11, 15, 16}

∴A x B = {(1, 1), (1, 5), (1, 9), (1, 11), (1, 15), (1, 16), (2, 1), (2, 5), (2, 9), (2, 11), (2, 15), (2, 16), (3, 1), (3, 5), (3, 9), (3, 11), (3, 15), (3, 16), (4, 1), (4, 5), (4, 9), (4, 11), (4, 15), (4, 16)}

It is given that f = {(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)}

(i) A relation from a non-empty set A to a non-empty set B is a subset of the Cartesian product A x B.

It is observed that f is a subset of A x B.

Thus, f is a relation from A to B.

(ii) Since the same first element i.e., 2 corresponds to two different images i.e., 9 and 11, relation f is not a function.

Q11 :

Let f be the subset of Z x Z defined by f = {(ab, a+ b): a, b ∈ Z}. Is f a function from Z to Z: justify your answer.

Answer :

The relation f is defined as f = {(ab, a+ b): a, b ∈ **Z**}

We know that a relation f from a set A to a set B is said to be a function if every element of set A has unique images inset B.

Since 2, 6, -2, -6 ∈ **Z**, (2 x 6, 2 + 6), (-2 x -6, -2 + (-6)) ∈ f

i.e., (12, 8), (12, -8) ∈ f

It can be seen that the same first element i.e., 12 corresponds to two different images i.e., 8 and -8. Thus, relation *f* is not a function.

Q12 :

Let A = {9, 10, 11, 12, 13} and let *f*: A → N be defined by *f*(*n*) = the highest prime factor of *n*. Find the range of *f*.

Answer :

A = {9, 10, 11, 12, 13}

f: A → **N** is defined as

f(*n*) = The highest prime factor of *n*

Prime factor of 9 = 3

Prime factors of 10 = 2, 5

Prime factor of 11 = 11

Prime factors of 12 = 2, 3

Prime factor of 13 = 13

∴*f*(9) = The highest prime factor of 9 = 3

f(10) = The highest prime factor of 10 = 5

f(11) = The highest prime factor of 11 = 11

f(12) = The highest prime factor of 12 = 3

f(13) = The highest prime factor of 13 = 13

The range of *f* is the set of all *f*(*n*), where *n* ∈A.

∴Range of *f* = {3, 5, 11, 13}