22

QUESTION PAPER SERIES CODE
A

Registration No. : | | | | | |

Centre of Exam. : ______________________

Name of Candidate : ______________________

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Signature of Invigilator

ENTRANCE EXAMINATION, 2013

MASTER OF COMPUTER APPLICATIONS

[Field of Study Code : MCAM (224)]

Time Allowed : 3 hours

Maximum Marks : 480
Weightage : 100

INSTRUCTIONS FOR CANDIDATES

Candidates must read carefully the following instructions before attempting the Question Paper :

(i) Write your Name and Registration Number in the space provided for the purpose on the top of this Question Paper and in the Answer Sheet.

(ii) **Please darken the appropriate Circle of Question Paper Series Code on the Answer Sheet.**

(iii) All questions are compulsory.

(iv) Answer all the 120 questions in the Answer Sheet provided for the purpose by darkening the correct choice, i.e., (a) or (b) or (c) or (d) with BALLPOINT PEN only against the corresponding circle. Any overwriting or alteration will be treated as wrong answer.

(v) Each correct answer carries 4 marks. **There will be negative marking and 1 mark will be deducted for each wrong answer.**

(vi) Answer written by the candidates inside the Question Paper will not be evaluated.

(vii) Calculators and Log Tables may be used.

(viii) Pages at the end have been provided for Rough Work.

(ix) Return the Question Paper and Answer Sheet to the Invigilator at the end of the Entrance Examination. **DO NOT FOLD THE ANSWER SHEET.**

INSTRUCTIONS FOR MARKING ANSWERS

1. Use only Blue/Black Ballpoint Pen (do not use pencil) to darken the appropriate Circle.
2. Please darken the whole Circle.
3. Darken <u>ONLY ONE CIRCLE</u> for each question as shown in the example below :

Wrong	Wrong	Wrong	Wrong	Correct
● ⓑ ⓒ ●	ⓐ̸ ⓑ ⓒ ⓓ	ⓐ̸ ⓑ ⓒ ⓓ̸	⦿ ⓑ ⓒ ●	ⓐ ⓑ ⓒ ●

4. Once marked, no change in the answer is allowed.
5. Please do not make any stray marks on the Answer Sheet.
6. Please do not do any rough work on the Answer Sheet.
7. Mark your answer only in the appropriate space against the number corresponding to the question.
8. **Ensure that you have darkened the appropriate Circle of Question Paper Series Code on the Answer Sheet.**

1. For $a, b \in R$, define $A = \begin{bmatrix} 1 & a \\ 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ a & b \end{bmatrix}$.

Statement I : For any a, A is a diagonalizable matrix.

Statement II : For any a and $b \neq 1$, B is a diagonalizable matrix.

(a) Statement I is true, statement II is false

(b) Statement I is false, statement II is true

(c) Both the statements are false

(d) Both the statements are true

2. If @ means triple of, # means double of and ^ means half of, then the value of @#^@^5 + @#@^2 is

(a) 39·5

(b) 40·5

(c) 39·74

(d) None of the above

3. A ray of light incident at the point (–2, –1) gets reflected from the tangent at (0, –1) to the circle $x^2 + y^2 = 1$. The refracted ray touches the circle. The equation of the line along which the incident ray moved is

(a) $4x - 3y + 11 = 0$

(b) $4x + 3y + 11 = 0$

(c) $3x + 4y + 11 = 0$

(d) None of the above

4. The approximate value of $\int_0^1 f(x)\,dx$, where $f(x) = \frac{1}{1+x+x^3}$, using Taylor's linear approximation of $f(x)$ at $x = 0$ is

(a) 0·095

(b) 0·105

(c) 0·09

(d) 0·11

5. What is the next term in the sequence 49, 121, 225, 361, ...?

(a) 400

(b) 441

(c) 481

(d) 529

6. The number of pairs (a, b), for which $a(x+1)^2 = b(x^2 - 3x + 2) + x + 1 = 0\ \forall x \in R$, is

(a) 0

(b) 1

(c) 2

(d) infinite

7. A function $y = f(x)$ is defined parametrically as $y = t^2 + t|t|$, $x = 2t - |t|$, $t \in R$. Then at $x = 0$, $f(x)$ is

(a) continuous but non-differentiable

(b) differentiable

(c) discontinuous

(d) None of the above

8. The value of $\tan^{-1}(1) + \tan^{-1}(2) + \tan^{-1}(3)$ is equal to

(a) π

(b) $\frac{\pi}{4}$

(c) $\frac{\pi}{2}$

(d) None of the above

9. The number of positive integral solutions of $\tan^{-1}(x) + \cot^{-1}(y) = \tan^{-1}(3)$ is

(a) one

(b) two

(c) three

(d) four

10. The interval, in which $\cos^{-1}(x) > \sin^{-1}(x)$, is

(a) $(-\infty, 1)$

(b) $(-1, 1)$

(c) $[-1, 1/\sqrt{2})$

(d) $[-1, 1]$

11. If $(1, -1, -1)^T$ is an eigenvector of the matrix

$$\begin{bmatrix} 4 & 1 & 1 \\ -5 & 0 & -3 \\ -1 & -1 & 2 \end{bmatrix}$$

then the corresponding eigenvalue is

(a) 2

(b) –2

(c) –1

(d) 3

12. A father's age is 4 times the age of his elder son and 5 times the age of his younger son. When the elder son lived to three times his present age, then the father's age will exceed his younger son's age by 3 years. What is the age of the father?

(a) 40 years

(b) 32 years

(c) 30 years

(d) None of the above

13. The perimeter of a triangle ABC is 6 times the arithmetic mean of the sines of its angles. If the side a is 1, then angle A is

(a) 30°

(b) 60°

(c) 90°

(d) 120°

14. Out of the 18 points in a plane, no three points are in the straight line except 5 points which are collinear. The number of straight lines that can be formed joining them is

(a) 155

(b) 153

(c) 144

(d) 143

15. The orthocentre of the triangle formed by the lines $x + y = 1$, $2x + 3y = 6$ and $4x - y + 4 = 0$ lies in

(a) I quadrant

(b) II quadrant

(c) III quadrant

(d) IV quadrant

16. Given the system of straight lines

$$a(2x + y - 3) + b(3x + 2y - 5) = 0$$

the line of the system situated farthest from the point (4, −3) has the equation

(a) $4x + 11y - 15 = 0$

(b) $7x + y - 8 = 0$

(c) $4x + 3y - 7 = 0$

(d) $3x - 4y + 1 = 0$

17. If $A + B = \frac{\pi}{3}$, where $A, B > 0$, then the minimum value of $\sec A + \sec B$ is equal to

(a) $4/\sqrt{3}$

(b) $8/\sqrt{3}$

(c) 6

(d) None of the above

18. If a, b, c and d are distinct real numbers such that

$$(a^2 + b^2 + c^2)x^2 - 2x(ab + bc + cd) + b^2 + c^2 + d^2 \leq 0$$

then they satisfy

(a) AP

(b) GP

(c) HP

(d) $ab = cd$

19. The equation $y^2 - x^2 + 2x - 1 = 0$ represents

(a) a pair of straight lines

(b) a circle

(c) a parabola

(d) an ellipse

20. The dimension of a subspace of R^4 spanned by the vectors {(2, –1, 0, 1), (1, 2, –3, 2), (1, –3, 2, 0), (0, 0, 1, –1)} is

(a) 1

(b) 2

(c) 3

(d) 4

21. In what ratio should wheat at ₹ 4·5 per kg be mixed with another variety at ₹ 5·25 per kg so that the mixture is worth of ₹ 5 per kg?

(a) 1 : 2

(b) 2 : 1

(c) 1 : 3

(d) 3 : 1

22. The values of p, for which the roots of the equation $(p-3)x^2 - 2px + 5p = 0$ are real and positive, are

(a) $p \in (3, 15/4]$

(b) $p \in (3, 15/4)$

(c) $p \in [3, 15/4)$

(d) $p \in [3, 15/4]$

23. $\lim_{x \to \infty} \dfrac{(1+x+x^2)}{x(\ln x)^3}$ is equal to

(a) 2

(b) e^2

(c) e^{-2}

(d) None of the above

24. The value of $2\sin^2\theta + 4\cos(\theta+\varphi)\sin\alpha\sin\theta + \cos(2\alpha+2\theta)$ is

(a) $\cos\theta + \cos\alpha$

(b) independent of θ

(c) independent of α

(d) None of the above

25. Which of the following is rational number?

(a) $\sin 15°$

(b) $\cos 15°$

(c) $\sin 15° \cos 15°$

(d) $\sin 15° \cos 75°$

26. The value of $\int_C \left(\frac{z+1}{z-1}\right)^3 dz$ around a circular contour $C = \{z : |z| < 2\}$ is

(a) 0

(b) $3\pi i$

(c) $6\pi i$

(d) $12\pi i$

27. Two taps can fill a tank in 18 minutes and 24 minutes respectively. When both the taps are opened, find when the first tap is turned off so that the tank may be filled in 12 minutes.

(a) After 6 minutes

(b) After 10 minutes

(c) After 9 minutes

(d) After 12 minutes

28. If $(\cos x)/a = (\sin x)/b$, then $|a\cos 2x + b\sin 2x|$ is

(a) $\sqrt{a^3 b}$

(b) $a^2/|b|$

(c) $b^2/|a|$

(d) $|a|$

29. The area of the triangle formed by the lines $y = ax$, $x + y - a = 0$ and the y-axis is equal to

(a) $1/2|1+a|$

(b) $a^2/|1+a|$

(c) $(1/2)|1/(1+a)|$

(d) $a^2/2|1+a|$

30. A variable point $(1 + (\alpha/\sqrt{2}), 2 + (\alpha/\sqrt{2}))$ lies in between two parallel lines $x + 2y = 1$ and $2x + 4y = 15$. Then the range of α is given by

(a) $0 < \alpha < 5\sqrt{2}/6$

(b) $-4\sqrt{2}/3 < \alpha < 5\sqrt{2}/6$

(c) $-4\sqrt{2}/3 < \alpha < 0$

(d) None of the above

31. A and B are two fixed points. The vertex C of a triangle ABC moves such that $\cot A + \cot B$ = constant. The locus of C is a straight line

(a) perpendicular to AB

(b) parallel to AB

(c) inclined at an angle $(A - B)$ to AB

(d) None of the above

32. A straight line L with negative slope passes through the point (8, 2) and cuts the positive coordinate axes at points P and Q. As L varies the absolute minimum value of $OP + OQ$ (O is the origin) is

(a) 10

(b) 18

(c) 16

(d) 12

33. If a circle passes through the points of intersection of the lines $2x - y + 1 = 0$ and $x + ay - 3 = 0$ with the axes of the reference, then the value of a is

(a) 0·5

(b) 2

(c) 1

(d) −2

34. A foot of the normal from the point (4, 3) to a circle is (2, 1) and the diameter of the circle has the equation $2x - y = 2$. Then the equation of the circle is

(a) $x^2 + y^2 + 2x - 1 = 0$

(b) $x^2 + y^2 - 2x - 1 = 0$

(c) $x^2 + y^2 + 2y - 1 = 0$

(d) None of the above

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(a) 10

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33. If a circle passes through the points of intersection of the lines $2x - y + 1 = 0$ and $x + ay - 3 = 0$ with the axes of the reference, then the value of a is

(a) 0·5

(b) 2

(c) 1

(d) –2

34. A foot of the normal from the point (4, 3) to a circle is (2, 1) and the diameter of the circle has the equation $2x - y = 2$. Then the equation of the circle is

(a) $x^2 + y^2 + 2x - 1 = 0$

(b) $x^2 + y^2 - 2x - 1 = 0$

(c) $x^2 + y^2 + 2y - 1 = 0$

(d) None of the above

35. The radius of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{(n!)^3}{3n!} z^n$$

is

(a) 1/27

(b) 27

(c) 1/3

(d) 3

36. The average minimum temperature for Monday, Tuesday and Wednesday was 4 degree and that for Tuesday, Wednesday and Thursday was 5·5 degree. If the temperature on Monday was 2·6 degree, what was the temperature on Thursday?

(a) 4·1 degree

(b) 12·1 degree

(c) 7·1 degree

(d) 11·1 degree

37. The equation of the circle touching the line $|y| = x$ at a distance $\sqrt{2}$ units from the origin is

(a) $x^2 + y^2 - 4x + 2 = 0$

(b) $x^2 + y^2 + 4x - 2 = 0$

(c) $x^2 + y^2 + 4x + 2 = 0$

(d) None of the above

38. Circles with radii 3, 4 and 5 touch each other externally. P is the point of intersection of tangents to these circles at their points of contact. The distance of P from the point of contact is

(a) $\sqrt{3}$

(b) $\sqrt{4}$

(c) $\sqrt{5}$

(d) 12

39. The angle between the circles

$$C1: x^2 + y^2 - 4x + 6y + 11 = 0$$

$$C2: x^2 + y^2 - 2x + 8y + 13 = 0$$

is

(a) 15°

(b) 30°

(c) 45°

(d) 60°

40. For $1 < |z| < 4$, the coefficient of z^2 in the Laurent series expansion of $\frac{1}{z^2 - 5z + 4}$ is equal to

(a) 1/192

(b) 1/48

(c) −1/48

(d) −1/192

41. A sum of ₹ 3,310 is to be paid back in 3 equal annual installments. What is the total interest charged if the interest is compounded annually at 10%?

(a) ₹ 1,331

(b) ₹ 683

(c) ₹ 331

(d) ₹ 993

42. The greatest integer which divides the number $101^{100} - 1$ is

(a) 100

(b) 1000

(c) 10000

(d) None of the above

43. The sum of the coefficients of all the integral powers of x in the expansion of $(1+2\sqrt{x})^{40}$ is

(a) $3^{40}+1$

(b) $3^{40}-1$

(c) $(1/2)(3^{40}-1)$

(d) $(1/2)(3^{40}+1)$

44. If $(1+x)^n = C_0 + C_1 x + \ldots + C_n x^n$, then the value of

$$\sum_{r=0}^{n}\sum_{s=0}^{n}(C_r + C_s)$$

is equal to

(a) $(n+1)2^{n+1}$

(b) $(n-1)2^{n+1}$

(c) $(n+1)2^{n}$

(d) None of the above

45. Which of the following is ***not*** the property of ${}^nC_r$?

(a) ${}^nC_1 = {}^nC_{n-1}$

(b) ${}^nC_r = {}^nC_{n-r}$

(c) $r\,{}^nC_r = n\,{}^{n-1}C_{r-1}$

(d) $(r-1)\,{}^nC_r = (n-1)\,{}^{n-1}C_{r-1}$

46. In triangle ABC, the value of $\sum_{r=0}^{n} {}^nC_r a^r b^{n-r}\cos(rB-(n-r)A)$ is equal to

(a) c^n

(b) b^n

(c) a^n

(d) 0

47. A city has 12 gates. In how many ways can a person enter the city through one gate and come out through a different gate?

(a) 144

(b) 132

(c) 12

(d) None of the above

48. Let z be a complex variable and $P(z)$ be a monic polynomial with real coefficients such that $P(0) = -1$. If $P(z) = 0$ has no real roots in the open disc $\{z : |z| < 1\}$, then

(a) $P(1) > 1$

(b) $0 < P(1) < 1$

(c) $P(1) = 1$

(d) $P(1) = 0$

49. A bag contains 5 paisa coins, 10 paisa coins and 20 paisa coins in the ratio of 3 : 2 : 1. If their total value is ₹ 11, what is the number of 5 paisa coins?

(a) 50

(b) 60

(c) 120

(d) 200

50. In how many ways can 5 letters be posted in 4 letter boxes?

(a) 20

(b) 32

(c) 4^5

(d) 5^4

55. If the lengths of the sides of a triangle are 3, 4 and 5 units, then the circum-radius is

(a) 2·0 units

(b) 2·5 units

(c) 3·0 units

(d) 3·5 units

56. The sides of a triangle are 17, 25 and 28. The greatest altitude of length is

(a) 15

(b) 84/5

(c) 420/17

(d) None of the above

57. The straight line $y = mx + c$ touches the parabola $y^2 = 4a(x + a)$ if

(a) $c = am - a/m$

(b) $c = m - a/m$

(c) $c = am + a/m$

(d) None of the above

58. P is any point on the ellipse $81x^2 + 144y^2 = 1944$, whose foci are S and S'. Then $SP + S'P$ is equal to

(a) 3

(b) $4\sqrt{6}$

(c) 36

(d) 324

59. The locus of a variable point, whose distance from $(-2, 0)$ is $2/3$ times its distance from the line $x = -9/2$, is

(a) ellipse

(b) parabola

(c) hyperbola

(d) None of the above

60. If $0 < a < b$, then $\lim_{n \to \infty} (b^n + a^n)^{1/n}$ is equal to

(a) e

(b) a

(c) b

(d) None of the above

61. $\lim_{x \to 0} [\sin x / x]$, where $[\cdot]$ denotes the greatest integer function

(a) is equal to 1

(b) is equal to zero

(c) does not exist

(d) None of the above

62. $\lim_{n \to \infty} \int_0^1 \frac{nx^{n-1}}{1+x} dx$

(a) does not exist

(b) is equal to 1

(c) is equal to 0

(d) is equal to 0·5

63. A shopkeeper marks his goods 20% above his cost price but allows a discount of 8% for cash. What is his profit percent?

(a) 10·6

(b) 11·6

(c) 9·6

(d) 10·4

64. If $A+B+C=\pi$, then the greatest value of $\cos A+\cos B+\cos C$ is

(a) 2

(b) 3

(c) 3/2

(d) 1

65. If $f,\ g:[0,\ 1]\rightarrow R$ are defined as

$$f(x)=\begin{cases}1, & x=0, \dfrac{1}{n!}, \dfrac{2}{n!}, \ldots, \dfrac{n}{n!}, n\in N\\ 0, & \text{otherwise}\end{cases}$$

$$g(x)=\begin{cases}1, & x=\dfrac{1}{n}, n\in N\\ 0, & \text{otherwise}\end{cases}$$

then

(a) both f and g are Riemann integrable on [0, 1]

(b) f is Riemann integrable on [0, 1] but g is not

(c) g is Riemann integrable on [0, 1] but f is not

(d) both f and g are not Riemann integrable on [0, 1]

66. At what time between 6 and 7 do the hands of the clock coincide?

(a) 6:13

(b) 6:36

(c) 6:32

(d) 6:35

67. A lamp of negligible height is placed on the ground t metre away from a wall. A man m metre tall is walking at a speed of $0 \cdot 1t$ metre/sec from the lamp to the nearest point on the wall. When he is midway between the lamp and the wall, the rate of change in the length of his shadow on the wall is

(a) $-2 \cdot 5t$ metre/sec

(b) $-0 \cdot 4m$ metre/sec

(c) $-0 \cdot 2t$ metre/sec

(d) $-0 \cdot 2m$ metre/sec

68. The total number of critical points of $f(x) = \max(\sin x, \cos x)$ $\forall x \in (-2\pi, 2\pi)$ is equal to

(a) 3

(b) 4

(c) 5

(d) 7

69. A group of 7 students with average weight of 66 kg is joined by another 5 students with average weight of 64 kg. What is the average weight of the total group?

(a) 64·8 kg

(b) 64·5 kg

(c) 65·3 kg

(d) 65·17 kg

70. Let $P(x)$ be a non-constant cubic polynomial with real coefficients and $Q(x) = P(x) - \sqrt{2}P'(x)$. Then $Q(x) = 0$ has

(a) all roots real

(b) all roots complex

(c) exactly one real root

(d) exactly one complex root

71. A rectangular tank is 4 m long, 3 m wide and 1·5 m high and is filled with water up to 0·6 m. How many stones of 15 cm × 10 cm × 8 cm are to be dropped to take the water to the top of the tank?

(a) 9000

(b) 10000

(c) 1000

(d) 900

72. Let $f(x) = 2x - \tan^{-1} x - \ln(x + \sqrt{1+x^2})$ $\forall x \in R$. Then

(a) $f(x)$ is non-increasing in $(-\infty, \infty)$

(b) $f(x)$ is non-decreasing in $(-\infty, \infty)$

(c) $f(x)$ is increasing in $(-\infty, \infty)$

(d) $f(x)$ is decreasing in $(-\infty, \infty)$

73. If α, β be the roots of $x^2 - x - 1 = 0$ and $A_n = \alpha^n + \beta^n$, then the arithmetic mean of A_n and A_{n-1} is

(a) $2A_{n+1}$

(b) $\frac{1}{2}A_{n+1}$

(c) $2A_{n-2}$

(d) None of the above

74. The slope of the normal at the point with abscissa $x = -2$ of the graph of the function $f(x) = |x^2 - x|$ is

(a) $-\frac{1}{6}$

(b) $-\frac{1}{3}$

(c) $\frac{1}{6}$

(d) None of the above

75. 10·9876543210 is divisible by

(a) 5, 9 and 11

(b) 5 and 9, but not by 11

(c) 9 and 11, but not by 5

(d) 11 and 5, but not by 9

76. What is the digit in the units place of the product $23^{49} * 51^{36}$?

(a) 1

(b) 3

(c) 7

(d) 9

77. If $P(x) = ax^2 + bx + c$ and $Q(x) = -ax^2 + bx + c$, where $ac \neq 0$, then the equation $P(x)Q(x) = 0$ has

(a) one real root

(b) two real roots

(c) at least two real roots

(d) at most two real roots

78. If $\{a_n\}$ is a real sequence such that $a_0 > 0$, $a_1 > 0$ and $a_n = \sqrt{a_{n-1}} + \sqrt{a_{n-2}}$, $n \geq 2$, then $\{a_n\}$

(a) does not converge

(b) converges to $\sqrt{2}$

(c) converges to 2

(d) converges to 4

79. The population of a town was 24000. If males increase by 6% and females by 9%, it becomes 25620. The number of males and females were

(a) 19000, 5000

(b) 16000, 8000

(c) 20000, 4000

(d) 18000, 6000

80. If $\sin^6\theta + \cos^6\theta + k\cos^2 2\theta = 1$, then k is equal to

(a) $(1/2)\tan^2 2\theta$

(b) $(1/4)\tan^2 2\theta$

(c) $4\cot^2 2\theta$

(d) $(3/4)\tan^2 2\theta$

81. A sum of money amounts to ₹ 6,050 in 2 years and to ₹ 6,655 in 3 years at compound interest, being compounded annually. Find the sum and the rate.

(a) ₹ 4,000, 12%

(b) ₹ 5,000, 10%

(c) ₹ 5,500, 10%

(d) None of the above

82. Today is Tuesday. What day of the week was it 124 days before?

(a) Wednesday

(b) Thursday

(c) Tuesday

(d) Friday

83. The Fourier series expansion of the function

$$f(x) = \begin{cases} -x, & -4 \le x \le 0 \\ x, & 0 \le x \le 4 \end{cases}$$

in the interval [–4, 4] involves

(a) no cosine terms

(b) no sine terms

(c) no constant term, but both sine and cosine terms

(d) a non-zero constant term and both sine and cosine terms

84. Let $(C[0, 5], d_\infty)$ be a metric space, where $C[0, 5]$ is the set of all real-valued continuous functions on [0, 5] and $d_\infty(f, g) = \sup\{|f(x) - g(x)| : x \in [0, 5]\}$. If $f(x) = x^2 - 4x$ and $g(x) = 3x - 6$, then $d_\infty(f, g)$ is equal to

(a) 6·0

(b) 6·25

(c) 6·15

(d) 6·35

85. Three solid cubes of edges 3 cm, 4 cm and 5 cm are melted to form a new cube. The edge of the new cube is

(a) 7·7 cm

(b) 6·0 cm

(c) 6·5 cm

(d) 8·0 cm

86. If BATCH is coded as ABSDG, then what is the code for TERM?

(a) PFSL

(b) SFQL

(c) SFQN

(d) None of the above

87. If $g(a+b-x)=g(x)$, then $\int_a^b xg(x)\,dx$ is equal to

(a) $\frac{(a+b)}{2}\int_a^b g(b-x)\,dx$

(b) $\frac{(a+b)}{2}\int_a^b g(b+x)\,dx$

(c) $\frac{(b-a)}{2}\int_a^b g(x)\,dx$

(d) $\frac{(a+b)}{2}\int_a^b g(x)\,dx$

88. Suppose all 5 men at a party throw their hats in the centre of the room. Each man then randomly selects a hat. The probability that none of the five men selects his own hat is

(a) $\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\frac{1}{5!}$

(b) $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}$

(c) $1-\frac{1}{2!}+\frac{1}{3!}-\frac{1}{4!}+\frac{1}{5!}$

(d) None of the above

89. A particular solution of the differential equation $(x-y)(dx+dy)=dx-dy$, given that $y=-1$, when $x=0$, is

(a) $\log|x+y|=x-y+1$

(b) $\log|x+y|=x+y+1$

(c) $\log|x^2+y^2|=x-y+1$

(d) None of the above

90. The planes $2x-3y+4z-5=0$ and $5x-2\cdot5y+10z-6=0$

(a) are perpendicular

(b) are parallel

(c) intersect y-axis

(d) pass through (0, 0, 1·25)

91. Let $n \in N$, the set of natural numbers. Define a sequence of functions on (–1, 1) as

$$f_n(x) = \begin{cases} \dfrac{2-(1+x)^n}{n}, & -1 < x < 0 \\ \dfrac{(1-x)^n}{n}, & 0 < x < 1 \end{cases}$$

Then $\{f_n(x)\}$

(a) does not converge on (–1, 1)

(b) converges only for $x = 0$

(c) converges on (–1, 1)

(d) converges on (–1, 1), except for $x = 0$

92. A number is multiplied by 9 and 9 is added to it. If the result is divisible by 17, the number is

(a) 12

(b) 15

(c) 17

(d) 16

93. The cumulative distribution function of a random variable is

$$F(x) = \begin{cases} 0, & x \le \alpha \\ \dfrac{x-\alpha}{\beta-\alpha}, & \alpha < x < \beta \\ 1, & x \ge \beta \end{cases}$$

The random variate follows which of the following probability distributions?

(a) Uniform

(b) Exponential

(c) Binomial

(d) Poisson

94. The maximum value of xyz, when $x^2 + y^2 + z^2 = 1$, is

(a) $\dfrac{1}{3\sqrt{3}}$

(b) 1

(c) $\dfrac{1}{27}$

(d) None of the above

95. $\int \frac{dx}{\sqrt{\sin^3 x \sin(x+\alpha)}}$ is equal to

(a) $\frac{2}{\sin\alpha}\sqrt{\frac{\sin(x+\alpha)}{\sin x}}+C$

(b) $-\frac{1}{2\sin\alpha}\sqrt{\frac{\sin(x+\alpha)}{\sin x}}+C$

(c) $-\frac{2}{\sin\alpha}\sqrt{\frac{\sin(x+\alpha)}{\sin x}}+C$

(d) None of the above

96. Two numbers x and y are chosen at random from interval (0, 1). The probability that $|x-y|<\frac{1}{6}$ is

(a) $\frac{1}{3}$

(b) $\frac{1}{36}$

(c) $\frac{5}{36}$

(d) $\frac{11}{36}$

97. The value of

$$1+\frac{2^3}{2!}+\frac{3^3}{3!}+\frac{4^3}{4!}+\dots$$

is

(a) ∞

(b) π

(c) e

(d) None of the above

98. If 830 is divided into three parts such that 4 times the first part is equal to 5 times the second and 7 times the third, the first part is

(a) 350

(b) 280

(c) 200

(d) 230

99. In a row of 21 girls, when *M* was shifted by four places towards right, she became 12th from the left end. What was her earlier position from the right end?

(a) 11th

(b) 12th

(c) 13th

(d) 14th

100. A solid cube is painted black on two of its opposite faces and is cut into 343 equal pieces. How many small pieces have no paint?

(a) 245

(b) 313

(c) 294

(d) None of the above

101. The area of the circle $x^2 + y^2 = 16$ exterior to the parabola $y^2 = 6x$ is

(a) $\frac{4(4\pi - \sqrt{3})}{3}$

(b) $\frac{4(4\pi + \sqrt{3})}{3}$

(c) $\frac{4(8\pi - \sqrt{3})}{3}$

(d) $\frac{4(8\pi + \sqrt{3})}{3}$

102. The line $y - ax + 1 = 0$ is a tangent to the curve $y^2 - 4x = 0$, if the value of a is

(a) 1

(b) 2

(c) 3

(d) $\frac{1}{2}$

103. A fair coin is tossed $2n$ times. Let p_{2n} be the probability of getting the same number of heads as tails. It can be shown as

$$p_{2n} \sim \frac{1}{\sqrt{\pi}} \frac{1}{n^s} \text{ as } n \to \infty$$

The constant s is

(a) 1

(b) 2

(c) $\frac{1}{2}$

(d) 0

104. Let X, Y and Z be three independent normal (0, 1) random variables. Calculate

$$E[(X+Y+Z)^2]$$

(a) 0

(b) 1

(c) 3

(d) 9

105. A fair coin is tossed 100 times. The probability of getting exactly 50 heads is close to

(a) 0·001

(b) 0·1

(c) 0·5

(d) 0·9

106. A number is randomly chosen from the interval (0, 1). What is the probability that its second decimal digit will be 5?

(a) 0·1

(b) 0·5

(c) 0·01

(d) 0·07

107. If $f(x) = \dfrac{4^x}{4^x + 2}$, then find

$$f\left(\frac{1}{2001}\right) + f\left(\frac{2}{2001}\right) + \ldots + f\left(\frac{2000}{2001}\right)$$

(a) 1000

(b) 2000

(c) $\dfrac{1000}{2001}$

(d) Cannot be found

108. Given

$$D_r = \begin{vmatrix} r & x & \dfrac{n(n+1)}{2} \\ 2r-1 & y & n^2 \\ 3r-2 & z & \dfrac{n(3n-1)}{2} \end{vmatrix}$$

Compute $\sum_{r=1}^{n} D_r$

(a) n

(b) $\dfrac{n(n+1)(2n+1)}{6}$

(c) 1

(d) 0

109. An equilateral triangle is drawn in a unit square such that one of its vertices coincides with a vertex of the square. What is the maximum possible area of the triangle?

(a) $2\sqrt{3}-3$

(b) $\sqrt{3}-1$

(c) $2\sqrt{3}$

(d) $3\sqrt{3}$

110. Let *A*, *B* and *C* be events which are mutually independent with probability α, β and γ. Let *N* be the random number of events which occur. What is $E[N]$?

(a) $\alpha+\beta+\gamma$

(b) $\alpha\beta(1-\gamma)+\beta\gamma(1-\alpha)+\gamma\alpha(1-\beta)$

(c) $\alpha\beta+\beta\gamma+\gamma\alpha$

(d) $\alpha(1-\alpha)+\beta(1-\beta)+\gamma(1-\gamma)$

111. The amount of bread (in hundreds of kilograms) that a bakery sells in a day is a random variable with density

$$f(x)=\begin{cases} kx, & 0\le x<3 \\ k(6-x), & 3\le x<6 \\ 0, & 0 \end{cases}$$

Find the value of *k* which makes *f* a probability density function.

(a) 3

(b) $\frac{1}{3}$

(c) 9

(d) $\frac{1}{9}$

112. $\int \frac{dx}{x\sqrt{ax-x^2}}$ is equal to

(a) $-\left(1+\frac{1}{x^4}\right)^{5/4}+C$

(b) $-\left(1+\frac{1}{x^3}\right)^{1/4}+C$

(c) $\left(1+\frac{1}{x^4}\right)^{1/4}+C$

(d) None of the above

113. The value of $\int_{\pi/6}^{\pi/3} \frac{(\sin x + \cos x)}{\sqrt{\sin 2x}} dx$ is

(a) $\sin^{-1} \frac{(\sqrt{3}-1)}{1}$

(b) $\tan^{-1} \frac{(\sqrt{3}-1)}{1}$

(c) $\sec^{-1} \frac{(\sqrt{3}-1)}{1}$

(d) None of the above

114. Equal quantities of 1 : 5 and 3 : 5 milk to water solutions are mixed together. What will be the ratio of milk to water in the new mixture?

(a) 13 : 35

(b) 2 : 5

(c) 5 : 8

(d) 35 : 13

115. In a km race, P beats Q by 25 metres or 5 seconds. Then find the time taken by P to complete the race.

(a) 3 minutes 15 seconds

(b) 4 minutes 20 seconds

(c) 2 minutes 30 seconds

(d) 5 minutes 10 seconds

116. The points on the curve $9y^2 - x^3 = 0$, where the normal to the curve makes equal intercepts with the axes, are

(a) $\left(4, \pm\frac{8}{3}\right)$

(b) $\left(4, -\frac{8}{3}\right)$

(c) $\left(4, \pm\frac{3}{8}\right)$

(d) $\left(\pm 4, \frac{3}{8}\right)$

117. The truncated Poisson distribution with the zero class missing has probability function

$$P(x = k) = \frac{\lambda^k}{(e^\lambda - 1)k!}, \; k = 1, 2, \ldots$$

Compute $E[x]$.

(a) λ

(b) $\frac{1}{(e^\lambda - 1)}$

(c) $\frac{\lambda}{(1 - e^{-\lambda})}$

(d) $\frac{\lambda^2}{(1 - e^{-\lambda})}$

118. A number when divided by 238 leaves a remainder of 79. What will be the remainder when the number is divided by 17?

(a) 8

(b) 9

(c) 10

(d) 11

119. A balance shows 900 gm for 1 kg. Find the profit of trader if he marks his goods up by 20% of CP.

(a) 7%

(b) 10%

(c) 11%

(d) 8%

120. Distance between the two planes $2x + 3y + 4z - 4 = 0$ and $4x + 6y + 8z - 12 = 0$ is

(a) 8

(b) 4

(c) 2

(d) None of the above

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