Solutions to JEE(Main) - 2017

PARTA - MATHEMATICS

1. If S is the set of distinct values of 'b' for which the following system of linear equations
$x + y + z = 1$
$x + ay + z = 1$
$ax + by + z = 0$
has no solution, then S is:
(1) an empty set (2) an infinite set
(3) a finite set containing two or more elements (4) a singleton

Sol. **(4)**
Here, $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a & 1 \\ a & b & 1 \end{vmatrix} = 0 \Rightarrow a = 1$
For a = 1, the equations become
$x + y + z = 1$
$x + y + z = 1$
$x + by + z = 0$
These equations give no solution for b = 1
$\Rightarrow$ S is singleton set

2. The following statement $(p \rightarrow q) \rightarrow [(\sim p \rightarrow q) \rightarrow q]$ is:
(1) a tautology (2) equivalent to $\sim p \rightarrow q$
(3) equivalent to $p \rightarrow \sim q$ (4) a fallacy

Sol. **(1)**
$(p \rightarrow q) \rightarrow ((\sim p \rightarrow q) \rightarrow q)$
$= (p \rightarrow q) \rightarrow ((p \vee q) \rightarrow q)$
$= (p \rightarrow q) \rightarrow ((\sim p \wedge \sim q) \vee q)$
$= (p \rightarrow q) \rightarrow ((\sim p \vee q) \wedge (\sim q \vee q))$
$= (p \rightarrow q) \rightarrow (\sim p \vee q)$
$= (p \rightarrow q) \rightarrow (p \rightarrow q)$
$= T$

3. If $5(\tan^2 x - \cos^2 x) = 2 \cos 2x + 9$, then the value of $\cos 4x$ is:
(1) $-\frac{3}{5}$ (2) $\frac{1}{3}$
(3) $\frac{2}{9}$ (4) $-\frac{7}{9}$

Sol. **(4)**
$5(\sec^2 x - 1 - \cos^2 x) = 2(2\cos^2 x - 1) + 9$
$5\left(\frac{1}{t} - 1 - t\right) = 2(2t - 1) + 9$
$5(1 - t - t^2) = 4t^2 + 7t$
$\Rightarrow 9t^2 + 12t - 5 = 0$
$t = \frac{1}{3}, -\frac{5}{3}$
$\Rightarrow \cos^2 x = \frac{1}{3}$

$\Rightarrow \cos 2x = \frac{2}{3} - 1 = -\frac{1}{3}$

$\Rightarrow \cos 4x = -\frac{7}{9}$

4. For three events A, B and C, P(Exactly one of A or B occurs) = P(Exactly one of B or C occurs) = P (Exactly one of C or A occurs) = $\frac{1}{4}$ and P(All the three events occur simultaneously) = $\frac{1}{16}$. Then the probability that at least one of the events occurs, is:

(1) $\frac{7}{32}$ (2) $\frac{7}{16}$

(3) $\frac{7}{64}$ (4) $\frac{3}{16}$

Sol. **(2)**

$P(A) + P(B) - 2P(A \cap B) = \frac{1}{4}$

$P(B) + P(C) - 2P(B \cap C) = \frac{1}{4}$

$P(A) + P(C) - 2P(A \cap C) = \frac{1}{4}$

$\Rightarrow \sum P(A) - \sum P(A \cap B) = \frac{3}{8}$

$\Rightarrow P(A \cup B \cup C) = \sum P(A) - \sum P(A \cap B) + P(A \cap B \cap C)$

$= \frac{3}{8} + \frac{1}{16} = \frac{7}{16}$.

5. Let ω be a complex number such that 2ω + 1 = z where $z = \sqrt{-3}$. If

$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2 - 1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = 3k$, then k is equal to:

(1) – z (2) z

(3) – 1 (4) 1

Sol. **(1)**

Determinant simplifies to $3k = \begin{vmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{vmatrix}$

$= 3\begin{vmatrix} 1 & 1 & 1 \\ 0 & \omega & \omega^2 \\ 0 & \omega^2 & \omega \end{vmatrix}$

$= -3z$

$\Rightarrow k = -z$

6. Let k be an integer such that the triangle with vertices (k, – 3k), (5, k) and (–k, 2) has area 28 sq. units. Then the orthocentre of this triangle is at the point:

(1) $\left(2, -\frac{1}{2}\right)$ (2) $\left(1, \frac{3}{4}\right)$

(3) $\left(1, -\frac{3}{4}\right)$ (4) $\left(2, \frac{1}{2}\right)$

Sol. **(4)**

$$\Delta = \frac{1}{2}\left\|\begin{matrix} k & -3k & 1 \\ 5 & k & 1 \\ -k & 2 & 1 \end{matrix}\right\| = 28$$

$\Rightarrow$ k = 2 (since $k \in I$)

$\Rightarrow$ Orthocentre is $\left(2, \frac{1}{2}\right)$

7. Twenty meters of wire is available for fencing off a flower–bed in the form of a circular sector. Then the maximum area (in sq. m) of the flower–bed, is:

(1) 12.5 (2) 10

(3) 25 (4) 30

Sol. **(3)**

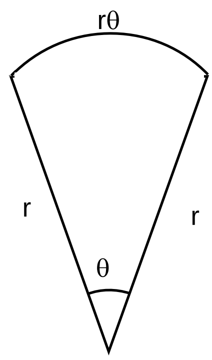
Length of wire = $r(\theta + 2) = 20$ m

Area A = $\frac{\theta}{2}r^2$

$\Rightarrow A(r) = 10r - r^2$

$\Rightarrow$ Area is maximum if r = 5.

Maximum area A = 25 sq. m



8. The area (in sq. units) of the region $\{(x, y) : x \geq 0, x + y \leq 3, x^2 \leq 4y \text{ and } y \leq 1 + \sqrt{x}\}$ is:

(1) $\frac{59}{12}$ (2) $\frac{3}{2}$

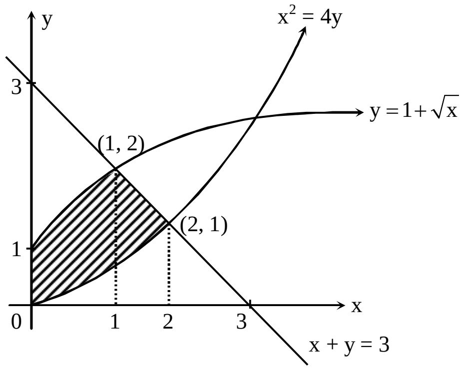
(3) $\frac{7}{3}$ (4) $\frac{5}{2}$

Sol. **(4)**

Required area

$$= \int_0^1 \left(1+\sqrt{x}\right)dx + \frac{1}{2}(3\times 1) - \int_0^2 \frac{x^2}{4}dx$$

$= \frac{5}{2}$ sq. units



9. If the image of the point P(1, –2, 3) in the plane, 2x + 3y – 4z + 22 = 0 measured parallel to the line, $\frac{x}{1}=\frac{y}{4}=\frac{z}{5}$ is Q, then PQ is equal to:

(1) $3\sqrt{5}$
(2) $2\sqrt{42}$
(3) $\sqrt{42}$
(4) $6\sqrt{5}$

Sol. **(2)**

$\frac{x-1}{1}=\frac{y+2}{4}=\frac{z-3}{5}=\lambda$

Let midpoint of PQ be M which lines on the plane

$\Rightarrow$ M(x, y, z) = (1 + λ, 4λ – 2, 5λ + 3)

2 (1 + λ) + 3 (4λ – 2) – 4 (5λ + 3) + 22 = 0

$\Rightarrow$ – 6λ + 6 = 0 $\Rightarrow$ λ = 1

$\Rightarrow$ M (2, 2, 8), P (1, – 2, 3)

PM = $\sqrt{1+16+25}=\sqrt{42}$

PQ = $2\sqrt{42}$.

10. If for $x \in \left(0, \frac{1}{4}\right)$, the derivative of $\tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right)$ is $\sqrt{x}$. g(x), then g(x) equals:

(1) $\frac{9}{1+9x^3}$
(2) $\frac{3x\sqrt{x}}{1-9x^3}$
(3) $\frac{3x}{1-9x^3}$
(4) $\frac{3}{1+9x^3}$

Sol. **(1)**

Here, $y = 2\tan^{-1} 3x^{\frac{3}{2}}$

$\frac{dy}{dx}=\frac{9x^{\frac{1}{2}}}{1+9x^3}=\sqrt{x}g(x)$

$\Rightarrow g(x)=\frac{9}{1+9x^3}$

11. If (2 + sinx) $\frac{dy}{dx}$ + (y + 1)cosx = 0 and y(0) = 1, then $y\left(\frac{\pi}{2}\right)$ is equal to:

(1) $\frac{1}{3}$
(2) $-\frac{2}{3}$
(3) $-\frac{1}{3}$
(4) $\frac{4}{3}$

Sol. **(1)**

(2 + sin x)dy + cos x(y + 1)dx = 0

(y + 1)(2 + sin x) = C

$\Rightarrow$ (1 + 1)(2 + 0) = C = 4

(y + 1)·(2 + sin x) = 4

Put $x=\frac{\pi}{2}$

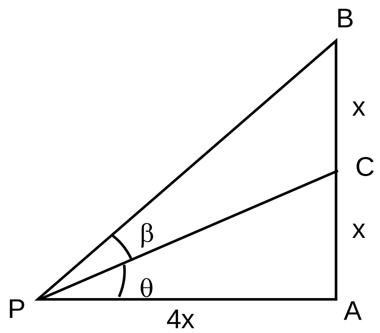
$y=\frac{1}{3}$

12. Let a vertical tower AB have its end A on the level ground. Let C be the mid–point of AB and P be a point on the ground such that AP = 2AB. If $\angle BPC = \beta$, then $\tan\beta$ is equal to:

(1) $\frac{6}{7}$ (2) $\frac{1}{4}$

(3) $\frac{2}{9}$ (4) $\frac{4}{9}$

Sol. **(3)**



$\tan(\theta+\beta)=\frac{1}{2}$

and $\tan\theta = \frac{1}{4}$

$\Rightarrow \dfrac{\frac{1}{4}+\tan\beta}{1-\frac{1}{4}\tan\beta}=\frac{1}{2}$

$\Rightarrow 9\tan\beta = 2$

$\Rightarrow \tan\beta = \frac{2}{9}$

13. If $A=\begin{bmatrix}2 & -3\\ -4 & 1\end{bmatrix}$, then adj $(3A^2 + 12A)$ is equal to:

(1) $\begin{bmatrix}72 & -84\\ -63 & 51\end{bmatrix}$ (2) $\begin{bmatrix}51 & 63\\ 84 & 72\end{bmatrix}$

(3) $\begin{bmatrix}51 & 84\\ 63 & 72\end{bmatrix}$ (4) $\begin{bmatrix}72 & -63\\ -84 & 51\end{bmatrix}$

Sol. **(2)**

$A^2=\begin{bmatrix}2 & -3\\ -4 & 1\end{bmatrix}\begin{bmatrix}2 & -3\\ -4 & 1\end{bmatrix}$

$=\begin{bmatrix}16 & -9\\ -12 & 13\end{bmatrix}$

$(3A^2 + 12A) = \begin{bmatrix}72 & -63\\ -84 & 51\end{bmatrix}$

Adj $(3A^2 + 12A) = \begin{bmatrix}51 & 63\\ 84 & 72\end{bmatrix}$.

14. For any three positive real numbers a, b and c, $9(25a^2 + b^2) + 25(c^2 - 3ac) = 15b(3a + c)$. Then:

(1) b, c and a are in G.P. (2) b, c and a are in A.P.

(3) a, b and c are in A.P. (4) a, b and c are in G.P.

Sol. **(2)**

$(15a - 3b)^2 + (15a - 5c)^2 + (3b - 5c)^2 = 0$

Let, $15a = 3b = 5c = 45\lambda$

$\Rightarrow$ $a = 3\lambda$; $b = 15\lambda$; $c = 9\lambda$

$\Rightarrow$ $2c = a + b$

b, c, a are in A.P.

15. The distance of the point (1, 3, – 7) from the plane passing through the point (1, –1, – 1), having normal perpendicular to both the lines $\frac{x-1}{1}=\frac{y+2}{-2}=\frac{z-4}{3}$ and $\frac{x-2}{2}=\frac{y+1}{-1}=\frac{z+7}{-1}$, is:

(1) $\frac{20}{\sqrt{74}}$
(2) $\frac{10}{\sqrt{83}}$
(3) $\frac{5}{\sqrt{83}}$
(4) $\frac{10}{\sqrt{74}}$

Sol. **(2)**

Equation of plane is $\begin{vmatrix} x-1 & y+1 & z+1 \\ 1 & -2 & 3 \\ 2 & -1 & -1 \end{vmatrix} = 0$

5x + 7y + 3z + 5 = 0

Distance from (1, 3, –7) = $\frac{|5+21-21+5|}{\sqrt{83}} = \frac{10}{\sqrt{83}}$

16. Let $I_n = \int \tan^n x\, dx, (n > 1)$. If $I_4 + I_6 = a \tan^5 x + bx^5 + C$, where C is a constant of integration, then the ordered pair (a, b) is equal to:

(1) $\left(-\frac{1}{5}, 1\right)$
(2) $\left(\frac{1}{5}, 0\right)$
(3) $\left(\frac{1}{5}, -1\right)$
(4) $\left(-\frac{1}{5}, 0\right)$

Sol. **(2)**

$I_n = \int \tan^n x dx$, n > 1

$= \int \tan^{n-2} x(\sec^2 x - 1)\, dx$

$= \frac{\tan^{n-1} x}{n-1} - I_{n-2} + C$

$\Rightarrow \quad I_n + I_{n-2} = \frac{\tan^{n-1} x}{n-1} + C$

$\Rightarrow \quad I_6 + I_4 = \frac{\tan^5 x}{5} + C$

Given, $I_4 + I_6 = a \tan^5 x + bx^3 + C$

$\Rightarrow \quad a = \frac{1}{5}$, b = 0

17. The eccentricity of an ellipse whose centre is at the origin is $\frac{1}{2}$. If one of its directrices is x = – 4, then the equation of the normal to it at $\left(1, \frac{3}{2}\right)$ is:

(1) 2y – x = 2
(2) 4x – 2y = 1
(3) 4x + 2y = 7
(4) x + 2y = 4

Sol. **(2)**

Eccentricity, $e=\frac{1}{2}$

Let 2a be the length of major axis and 2b be the length of minor axis

$\Rightarrow \frac{a}{e}=4$

$\Rightarrow a = 2$

Also, $b=\sqrt{3}$, as $e=\frac{1}{2}$

$\Rightarrow$ Equation of ellipse is $\frac{x^2}{4}+\frac{y^2}{3}=1$

$\Rightarrow$ Equation of normal at $\left(1,\frac{3}{2}\right)$ is $4x - 2y = 1$

18. A hyperbola passes through the point $P\left(\sqrt{2},\sqrt{3}\right)$ and has foci at $\left(\pm 2, 0\right)$. Then the tangent to this hyperbola at P also passes through the point:

(1) $\left(3\sqrt{2}, 2\sqrt{3}\right)$
(2) $\left(2\sqrt{2}, 3\sqrt{3}\right)$
(3) $\left(\sqrt{3}, \sqrt{2}\right)$
(4) $\left(-\sqrt{2}, -\sqrt{3}\right)$

Sol. **(2)**

$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

$\frac{x^2}{a^2}-\frac{y^2}{4-a^2}=1$

$\Rightarrow a^2 = 8, 1, (a^2 \neq 8)$

$\Rightarrow \frac{x^2}{1}-\frac{y^2}{3}=1.$

Hence equation of tangent at $P\left(\sqrt{2},\sqrt{3}\right)$ is $\frac{x\sqrt{2}}{1}-\frac{y\sqrt{3}}{3}=1$

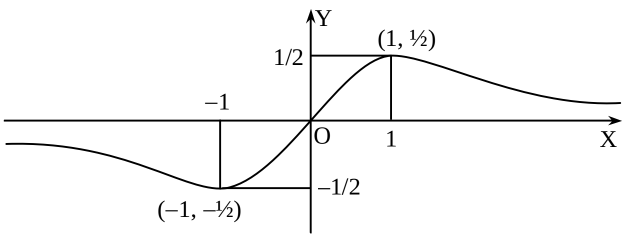
$\Rightarrow \sqrt{6}x - y = \sqrt{3}$

19. The function $f:R\rightarrow\left[-\frac{1}{2},\frac{1}{2}\right]$ defined as $f\left(x\right)=\frac{x}{1+x^2}$, is:

(1) invertible.
(2) injective but not surjective.
(3) surjective but not injective.
(4) neither injective nor surjective.

Sol. **(3)**

For, $f\left(x\right)=\frac{x}{1+x^2}$ the curve has graph as shown



Which is onto but not one-one for, $f:R\rightarrow\left[-\frac{1}{2},\frac{1}{2}\right]$

20. $\lim_{x\to\frac{\pi}{2}} \frac{\cot x - \cos x}{(\pi - 2x)^3}$ equals:

(1) $\frac{1}{24}$ (2) $\frac{1}{16}$

(3) $\frac{1}{8}$ (4) $\frac{1}{4}$

Sol. **(2)**

$\lim_{x\to\frac{\pi}{2}} \frac{\cos x - \cot x}{8\left(x-\frac{\pi}{2}\right)^3}$

Put $x - \frac{\pi}{2} = t$; $x = t + \frac{\pi}{2}$

$\lim_{t\to 0} \frac{1}{8}\cdot\frac{-\sin t + \tan t}{t^3}$

$\lim_{t\to 0} \frac{1}{8}\cdot\frac{\sin t(1-\cos t)}{t\cdot\cos t\cdot t^2}$

$= \frac{1}{8}\cdot 1\cdot\frac{1}{2} = \frac{1}{16}$

21. Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$. Let $\vec{c}$ be a vector such that $|\vec{c} - \vec{a}| = 3$, $|(\vec{a}\times\vec{b})\times\vec{c}| = 3$ and the angle between $\vec{c}$ and $\vec{a}\times\vec{b}$ be 30º. Then $\vec{a}\cdot\vec{c}$ is equal to:

(1) $\frac{25}{8}$ (2) 2

(3) 5 (4) $\frac{1}{8}$

Sol. **(2)**

$|\vec{c}|^2 + |\vec{a}|^2 - 2\vec{a}\cdot\vec{c} = 9$

$\Rightarrow |\vec{c}|^2 - 2\vec{a}\cdot\vec{c} = 0$

and $|\vec{a}\times\vec{b}|\cdot|\vec{c}|\cdot\sin 30^\circ = 3$

$\Rightarrow 3\times|\vec{c}|\times\frac{1}{2} = 3$

$\Rightarrow |\vec{c}| = 2$

$\therefore\ \vec{a}\cdot\vec{c} = 2$

22. The normal to the curve y(x – 2)(x – 3) = x + 6 at the point where the curve intersects the y–axis passes through the point:

(1) $\left(-\frac{1}{2}, -\frac{1}{2}\right)$ (2) $\left(\frac{1}{2}, \frac{1}{2}\right)$

(3) $\left(\frac{1}{2}, -\frac{1}{3}\right)$ (4) $\left(\frac{1}{2}, \frac{1}{3}\right)$

Sol. **(2)**

$\frac{dy}{dx}$ at x = 0 is 1

$\Rightarrow$ Slope of normal at (0, 1) is –1

$\Rightarrow$ Equation of normal is x + y = 1

23. If two different numbers are taken from the set {0, 1, 2, 3, …, 10}; then the probability that their sum as well as absolute difference are both multiple of 4, is:

(1) $\frac{6}{55}$ (2) $\frac{12}{55}$

(3) $\frac{14}{45}$ (4) $\frac{7}{55}$

Sol. **(1)**

Consider two sequences : 0, 4, 8 and 2, 6, 10

Take both numbers from either of these sequences.

Hence, probability = $\frac{^3C_2 + {}^3C_2}{^{11}C_2} = \frac{6}{55}$.

24. A man X has 7 friends, 4 of them are ladies and 3 are men. His wife Y also has 7 friends, 3 of them are ladies and 4 are men. Assume X and Y have no common friends. Then the total number of ways in which X and Y together can throw a party inviting 3 ladies and 3 men, so that 3 friends of each of X and Y are in this party, is:

(1) 485 (2) 468

(3) 469 (4) 484

Sol. **(1)**

X : 4L, 3M; Y : 3L, 4M

Possible combinations

	(1)	(2)	(3)	(4)
X	3L	2L, 1M	1L, 2M	3M
Y	3M	1L, 2M	2L, 1M	3L

$\therefore$ Number of ways = $^4C_3 \cdot {}^4C_3 + {}^4C_2 \cdot {}^3C_1 \cdot {}^3C_1 \cdot {}^4C_2 + {}^4C_1 \cdot {}^3C_2 \cdot {}^3C_2 \cdot {}^4C_1 + {}^3C_3 \cdot {}^3C_3$

= 485

25. The value of $\left({}^{21}C_1 - {}^{10}C_1\right) + \left({}^{21}C_2 - {}^{10}C_2\right) + \left({}^{21}C_3 - {}^{10}C_3\right) + \left({}^{21}C_4 - {}^{10}C_4\right) + ... + \left({}^{21}C_{10} - {}^{10}C_{10}\right)$ is:

(1) $2^{21} - 2^{11}$ (2) $2^{21} - 2^{10}$

(3) $2^{20} - 2^{9}$ (4) $2^{20} - 2^{10}$

Sol. **(4)**

Let S = $\left({}^{21}C_1 - {}^{10}C_1\right) + \left({}^{21}C_2 - {}^{10}C_2\right) + ... + \left({}^{21}C_{10} - {}^{10}C_{10}\right)$

$\Rightarrow$ S = $\left({}^{21}C_0 + {}^{21}C_1 + ... + {}^{21}C_{10}\right) - \left({}^{10}C_0 + {}^{10}C_1 + ... + {}^{10}C_{10}\right)$

$\Rightarrow$ S = $2^{20} - 2^{10}$.

26. A box contains 15 green and 10 yellow balls. If 10 balls are randomly drawn, one–by–one, with replacement, then the variance of the number of green balls drawn is:

(1) $\frac{12}{5}$ (2) 6

(3) 4 (4) $\frac{6}{25}$

Sol. **(1)**

$p = \frac{15}{25}$, $q = \frac{10}{25}$, n = 10

$\sigma^2 = npq = 10 \cdot \frac{3}{5} \cdot \frac{2}{5} = \frac{12}{5}$.

27. Let a, b, c ∈ R. If $f(x) = ax^2 + bx + c$ is such that a + b + c = 3 and f(x + y) = f(x) + f(y) + xy, ∀ x, y ∈ R, then $\sum_{n=1}^{10} f(n)$ is equal to:

(1) 330 (2) 165
(3) 190 (4) 255

Sol. **(1)**

Partially differentiating, we get

$f'(x) - x = \text{constant} = \lambda$

$\Rightarrow f(x) = \frac{x^2}{2} + \lambda x + k$

$f(0) = 0 \Rightarrow k = 0$

$\frac{1}{2} + \lambda = 3 \Rightarrow \lambda = \frac{5}{2}$

$\sum_{n=1}^{10} f(n) = a\sum_{n=1}^{10} n^2 + b\sum_{n=1}^{10} n$

$= \frac{1}{2}\frac{n(n+1)(2n+1)}{6} + \frac{5}{2} \times \frac{n(n+1)}{2}$

$= \frac{1}{2} \cdot \frac{10 \times 11 \times 21}{6} + \frac{5}{2} \times \frac{10 \times 11}{2} = 330$.

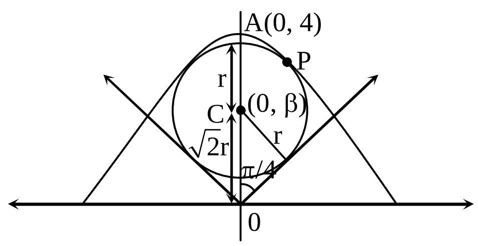
*28. The radius of a circle, having minimum area, which touches the curve $y = 4 - x^2$ and the lines, y = |x| is:

(1) $2(\sqrt{2}+1)$ (2) $2(\sqrt{2}-1)$
(3) $4(\sqrt{2}-1)$ (4) $4(\sqrt{2}+1)$

Sol. **(3)**

Let $P\left(\frac{t}{2}, 4 - \frac{t^2}{4}\right)$ be point where circle touches the parabola $y = 4 - x^2$

$\Rightarrow$ Normal at P: $ty - x + \frac{t^3}{4} - \frac{7t}{2} = 0$ to the parabola passes through centre (c) of the circle (0, β).



$\Rightarrow t^3 - 14t + 4\beta t = 0$ ..... (1)

Also, radius $r = \frac{|\beta|}{\sqrt{2}}$

$\Rightarrow t^4 + 4t^2 + 8\beta t^2 - 32t^2 - 128\beta + 256 + 16\beta^2 = 16r^2$

$\Rightarrow t^4 + (8\beta - 28)t^2 - 128\beta + 256 + 8\beta^2 = 0$ ..... (2)

From equation (1) and (2), we get

Either $\beta = 8 \pm 4\sqrt{2}$ for t = 0

or $\beta = \frac{-\sqrt{2} \pm \sqrt{17}}{\sqrt{2}}$ for $t^2 = 14 - 4\beta$

As, $r = \frac{|\beta|}{\sqrt{2}} \Rightarrow r = 4\sqrt{2} \pm 4, \frac{\sqrt{17}-\sqrt{2}}{2}$

$\Rightarrow$ Minimum possible radius, $r = \frac{\sqrt{17}-\sqrt{2}}{2}$

[But of the given options r = $4\left(\sqrt{2}-1\right)$ is minimum]

29. If, for a positive integer n, the quadratic equation, x(x + 1) + (x + 1)(x + 2) + … + $\left(x+\overline{n-1}\right)$ (x + n) = 10n has two consecutive integral solutions, then n is equal to:

(1) 12 (2) 9
(3) 10 (4) 11

Sol. **(4)**

$x^2 + nx + \frac{n^2-31}{3} = 0$

Let I and I + 1 be the roots of the equation

$2I + 1 = -n$ … (1)

$I(I+1) = \frac{n^2-31}{3}$ … (2)

Eliminating I from (1) and (2), we get

$\frac{n^2-1}{4} = \frac{n^2-31}{3}$

$\Rightarrow n^2 = 121$

$\Rightarrow n = 11.$

30. The integral $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1+\cos x}$ is equal to:

(1) – 2 (2) 2
(3) 4 (4) – 1

Sol. **(2)**

$I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \left(\text{cosec}^2 x - \text{cosec}\, x \cdot \cot x\right) dx$

= 2

PART B-PHYSICS

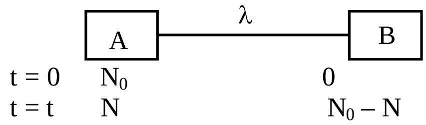
(ALL THE GRAPHS/DIAGRAMS GIVEN ARE SCHEMATIC AND NOT DRAWN TO SCALE.)

31. A radioactive nucleus A with a half life T, decays into a nucleus B. At t = 0, there is no nucleus B. At sometime t, the ratio of the number of B to that of A is 0.3. Then, t is given by :

(1) $t = \frac{T}{\log(1.3)}$ (2) $t = \frac{T}{2}\frac{\log 2}{\log 1.3}$

(3) $t = T\frac{\log 1.3}{\log 2}$ (4) $t = T\log(1.3)$

Sol. **(3)**



$\frac{N_0 - N}{N} = 0.3$

$\Rightarrow N = \frac{N_0}{1.3}$

$N = N_0\, e^{-\lambda t}$

$\Rightarrow \frac{1}{1.3} = e^{-\lambda t}$

$\Rightarrow t = \frac{\ell n(1.3)}{\lambda} = T\frac{\ell n(1.3)}{\ell n(2)}$

$\because \lambda = \frac{\ell n 2}{T}$.

32. The following observations were taken for determining surface tension T of water by capillary method:
diameter of capillary, $D = 1.25 \times 10^{-2}$ m
rise of water, $h = 1.45 \times 10^{-2}$m
Using $g = 9.80$ m/s^2 and the simplified relation $T = \frac{rhg}{2}\times 10^3\, N/m$, the possible error in surface tension is closest to :

(1) 10 % (2) 0.15 %
(3) 1.5 % (4) 2.4 %

Sol. **(3)**

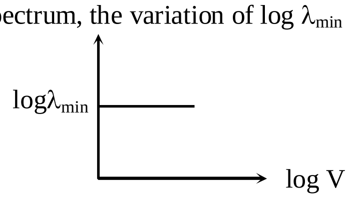
$T = \frac{rhg}{2}\times 10^3\, N/m$

$\frac{\Delta T}{T} = \left|\frac{\Delta r}{r}\right| + \left|\frac{\Delta h}{h}\right| = \frac{0.01}{1.25} + \frac{0.01}{1.45}$

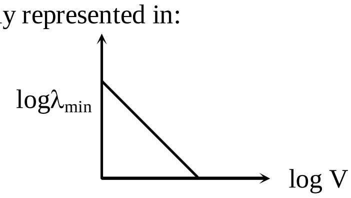
% error = $\frac{\Delta T}{T}\times 100 = \frac{1}{1.25} + \frac{1}{1.45} = 0.8 + 0.69 \simeq 1.5\%$

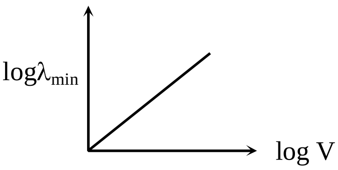
33. An electron beam is accelerated by a potential difference V to hit a metallic target to produce X –rays. It produces continuous as well as characteristic X-rays. If λ_{min} is the smallest possible wavelength of X-ray in the spectrum, the variation of log λ_{min} with log V is correctly represented in:

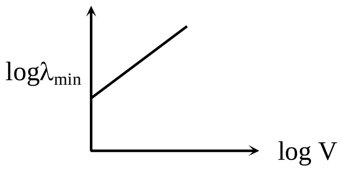
(1)



(2)



(3) 

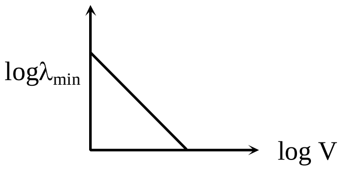
(4) 

Sol. **(2)**

$$\frac{hc}{\lambda_{min}} = eV$$

$$\log\frac{hc}{e} - \log\lambda_{min} = \log V$$

$$\Rightarrow \log\lambda_{min} = k - \log V$$



34. The moment of inertia of a uniform cylinder of length ℓ and radius R about its perpendicular bisector is I. What is the ratio ℓ/R such that the moment of inertia is minimum ?

(1) $\frac{3}{\sqrt{2}}$ (2) $\sqrt{\frac{3}{2}}$

(3) $\frac{\sqrt{3}}{2}$ (4) 1

Sol. **(2)**

$$I = \frac{m}{12}\left[3R^2 + \ell^2\right] \qquad \left(R^2 = \frac{m}{\pi\ell\rho}\right)$$

$$= \frac{m}{12}\left[\frac{3m}{\pi\rho}\ell^{-1} + \ell^2\right]$$

$$\frac{dI}{d\ell} = \frac{m}{12}\left[-\frac{3m}{\pi\rho\ell^2} + 2\ell\right]$$

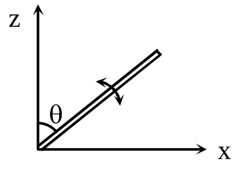
For minima

$$0 = \frac{-3m}{\pi\rho\ell^2} + 2\ell$$

$$\Rightarrow \frac{3\pi R^2\ell\rho}{\pi\rho\ell^2} = 2\ell$$

$$\Rightarrow \frac{\ell}{R} = \sqrt{\frac{3}{2}}$$

35. A slender uniform rod of mass M and length ℓ is pivoted at one end so that it can rotate in a vertical plane (see figure). There is negligible friction at the pivot. The free end is held vertically above the pivot and then released. The angular acceleration of the rod when it makes an angle θ with the vertical is:



(1) $\frac{2g}{3\ell}\cos\theta$ (2) $\frac{3g}{2\ell}\sin\theta$

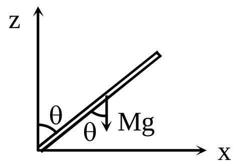
(3) $\frac{2g}{3\ell}\sin\theta$ (4) $\frac{3g}{2\ell}\cos\theta$

Sol. **(2)**

$\tau = I\alpha$

$\Rightarrow Mg\ \frac{\ell}{2}\sin\theta = \frac{M\ell^2}{3}\alpha$

$\Rightarrow\ \alpha = \frac{3}{2}\frac{g}{\ell}\sin\theta$



36. C_p and C_v are specific heats at constant pressure and constant volume respectively. It is observed that
$C_P - C_v = a$ for hydrogen gas
$C_P - C_v = b$ for nitrogen gas
The correct relation between a and b is

(1) a = 28 b
(2) $a = \frac{1}{14}b$
(3) a = b
(4) a = 14 b

Sol. **(4)**
For ideal gas
$C_P - C_v = R/M$
If C_P and C_v are specific heats (J/kg – ^{0}C)
M = molar mass of gas
$\Rightarrow a = R/2$ and $b = R/28$
$\Rightarrow a = 14b$

37. A copper ball of mass 100 gm is at a temperature T. It is dropped in a copper calorimeter of mass 100 gm, filled with 170 gm of water at room temperature. Subsequently, the temperature of the system is found to be 75^{0}C. T is given by: (Given : room temperature = 30^{0}C, specific heat of copper = 0.1 cal/gm^{0}C)
(1) 825^{0}C
(2) 800^{0}C
(3) 885^{0}C
(4) 1250^{0}C

Sol. **(3)**
Final temperature of calorimeter and its contents is given, $T_0 = 75^0C$
$\Rightarrow 100 \times 0.1 \times (75 - T) + 100 \times 0.1\ (75 - 30) + 170 \times 1 \times (75 - 30) = 0$
$\Rightarrow 75 - T + 45 + 765 = 0$
$\Rightarrow T = 885^0C$

38. In amplitude modulation, sinusoidal carrier frequency used is denoted by ω_c and the signal frequency is denoted by ω_m. The bandwidth $(\Delta\omega_m)$ of the signal is such that $\Delta\omega_m << \omega_c$. Which of the following frequencies is **not** contained in the modulated wave?
(1) $\omega_c - \omega_m$
(2) ω_m
(3) ω_c
(4) $\omega_m + \omega_c$

Sol. **(2)**
Modulated signal can be written as
$C_m(t) = (A_c + A_m \sin \omega_m t) \sin \omega_c t$
$\Rightarrow C_m(t) = A_c \sin\omega_c t + \frac{\mu A_c}{2}\cos(\omega_c - \omega_m)t - \frac{\mu A_c}{2}\cos(\omega_c + \omega_m)t$
where $\mu = \frac{A_m}{A_c}$

39. The temperature of an open room of volume 30 m^3 increases from 17^{0}C to 27^{0}C due to the sunshine. The atmospheric pressure in the room remains 1×10^5 Pa. If n_i and n_f are the number of molecules in the room before and after heating, then $n_f - n_i$ will be :
(1) -2.5×10^{25}
(2) -1.61×10^{23}
(3) 1.38×10^{23}
(4) 2.5×10^{25}

Sol. **(1)**

Using, $n = \left(\frac{PV}{RT}\right)$

$$n_f - n_i = \frac{PV}{R}\left(\frac{1}{T_f} - \frac{1}{T_i}\right)\text{moles}$$

$$= \frac{1\times10^5\times30}{8.32}\left(\frac{1}{300} - \frac{1}{290}\right)\times6.023\times10^{23}\text{ molecules}$$

$$= -2.5\times10^{25}\text{ molecules}$$

40. In a Young's double slit experiment, slits are separated by 0.5 mm, and the screen is placed 150 cm away. A beam of light consisting of two wavelengths, 650 nm and 520 nm, is used to obtain interference fringes on the screen. The least distance from the common central maximum to the point where the bright fringes due to both the wavelengths coincide is

(1) 15.6 mm
(2) 1.56 mm
(3) 7.8 mm
(4) 9.75 mm

Sol. **(3)**

$$y = \frac{m\times650\times10^{-9}\times D}{d} = \frac{n\times520\times10^{-9}\times D}{d}$$

$\Rightarrow \frac{m}{n} = \frac{4}{5}$ $\Rightarrow$ minimum values of m and n will be 4 and 5 respectively.

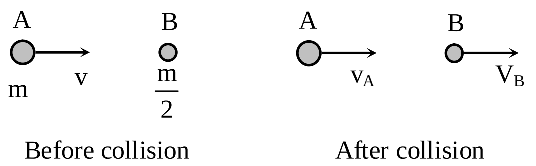
$$y = \frac{4\times650\times10^{-9}\times1.5}{5\times10^{-4}}\text{ meter}$$

$= 7.8$ mm

41. A particle A of mass m and initial velocity v collides with a particle B of mass $\frac{m}{2}$ which is at rest. The collision is head on, and elastic. The ratio of the de-Broglie wavelengths λ_A to λ_B after the collision is:

(1) $\frac{\lambda_A}{\lambda_B} = \frac{1}{2}$
(2) $\frac{\lambda_A}{\lambda_B} = \frac{1}{3}$
(3) $\frac{\lambda_A}{\lambda_B} = 2$
(4) $\frac{\lambda_A}{\lambda_B} = \frac{2}{3}$

Sol. **(3)**



$mv = mv_A + \frac{m}{2}v_B$ (conservation of linear momentum)

$\therefore\ v = v_A + \frac{v_B}{2} = v_B - v_A$ (elastic collision)

$\therefore\ \frac{v_B}{v_A} = 4$

$\therefore\ \frac{\lambda_A}{\lambda_B} = \frac{m_B v_B}{m_A v_A} = 2$

42. A magnetic needle of magnetic moment 6.7×10^{-2} Am^2 and moment of inertia 7.5×10^{-6} kg m^2 is performing simple harmonic oscillations in a magnetic field of 0.01 T. Time taken for 10 complete oscillations is:

(1) 8.76 s
(2) 6.65 s
(3) 8.89 s
(4) 6.98 s

Sol. **(2)**

$$T = 2\pi\sqrt{\frac{I}{MB}}$$

$$= 2\pi\sqrt{\frac{7.5\times10^{-6}}{6.7\times10^{-2}\times0.01}} = 0.665 \text{ sec}$$

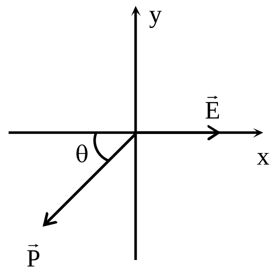
So, time of 10 oscillations = 6.65 sec

43. An electric dipole has a fixed dipole moment $\vec{p}$, which makes angle θ with respect to x-axis. When subjected to an electric field $\vec{E}_1 = E\hat{i}$, it experiences a torque $\vec{T}_1 = \tau\hat{k}$. When subjected to another electric field $\vec{E}_2 = \sqrt{3}E_1\hat{j}$ it experiences a torque $\vec{T}_2 = -\vec{T}_1$. The angle θ is:

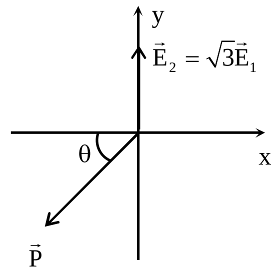
(1) 90° (2) 30°
(3) 45° (4) 60°

Sol. **(4)**

From the given information



Case - I



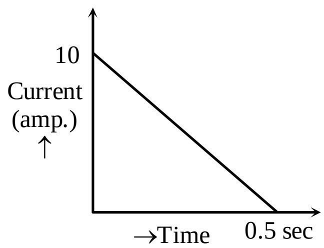
Case - II

$\therefore$ $PE_1 \sin\theta = \sqrt{3}PE_1 \sin\left(\frac{\pi}{2}+\theta\right)$

$\therefore$ $\theta = 60°$

44. In a coil of resistance 100 Ω, a current is induced by changing the magnetic flux through it as shown in the figure. The magnitude of change in flux through the coil is:

(1) 275 Wb
(2) 200 Wb
(3) 225 Wb
(4) 250 Wb



Sol. **(4)**

Change in flux = $R\int idt$ = 250 Wb

45. A time dependent force F = 6t acts on a particle of mass 1 kg. If the particle starts from rest, the work done by the force during the first 1 sec. will be:

(1) 18 J (2) 4.5 J
(3) 22 J (4) 9 J

Sol. **(2)**

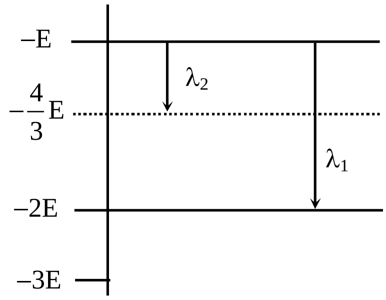
From impulse momentum theorem

$$\int_0^1 6t\,dt = mv$$

$\therefore$ v = 3 m/s

So, work done by the force = $\Delta K.E. = \frac{1}{2}(1)(3)^2 = 4.5J$

46. Some energy levels of a molecule are shown in the figure. The ratio of the wavelengths $r = \lambda_1/\lambda_2$, is given by:



(1) $r = \frac{1}{3}$ (2) $r = \frac{4}{3}$

(3) $r = \frac{2}{3}$ (4) $r = \frac{3}{4}$

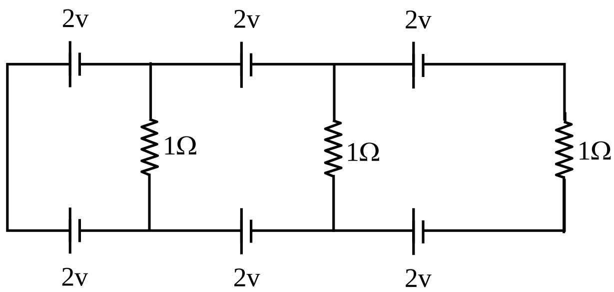
Sol. **(1)**

$\Delta E \propto \frac{1}{\lambda}$

$\Rightarrow \frac{\lambda_1}{\lambda_2} = \frac{\Delta E_2}{\Delta E_1} = \frac{1}{3}$

47. In the given circuit, the current in each resistance is:

(1) 0 A
(2) 1 A
(3) 0.25 A
(4) 0.5 A

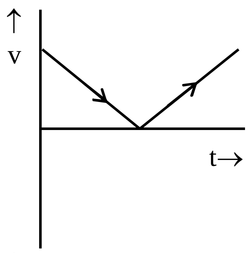


Sol. **(1)**

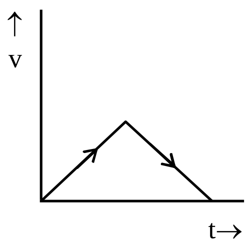
Potential difference across each resistor is zero.

48. A body is thrown vertically upwards. Which one of the following graphs correctly represent the velocity vs time?

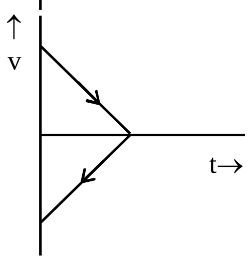
(1)



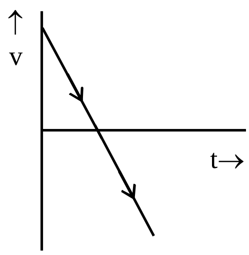
(2)



(3)



(4)



Sol. **(4)**

$v = u - gt$

49. A capacitance of 2 μF is required in an electrical circuit across a potential difference of 1.0 kV. A large number of 1 μF capacitors are available which can withstand a potential difference of not more than 300 V. The minimum number of capacitors required to achieve this is:

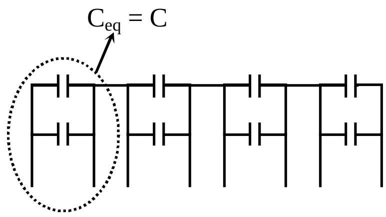
(1) 32 (2) 2
(3) 16 (4) 24

Sol. **(1)**

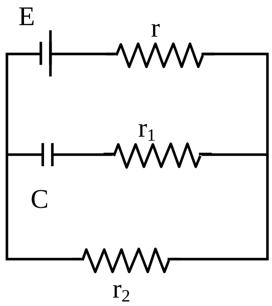
$\frac{C}{4} = 2 \Rightarrow C = 8\ \mu F$

Which requires eight 1 μF capacitors in parallel.

$\Rightarrow$ Minimum number of capacitors required is 32.



50. In the given circuit diagram when the current reaches steady state in the circuit, the charge on the capacitor of capacitance C will be:



(1) $CE\frac{r_1}{(r_1 + r)}$ (2) CE
(3) $CE\frac{r_1}{(r_2 + r)}$ (4) $CE\frac{r_2}{(r + r_2)}$

Sol. **(4)**

$q = CV$

$= \frac{CEr_2}{r + r_2}$

51. In a common emitter amplifier circuit using an n-p-n transistor, the phase difference between the input and the output voltages will be:

(1) 180° (2) 45°
(3) 90° (4) 135°

Sol. **(1)**

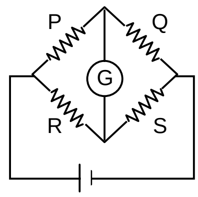
In common emitter amplifier circuit the output voltage is out of phase w.r.t. input voltage.

52. Which of the following statements is **false**?

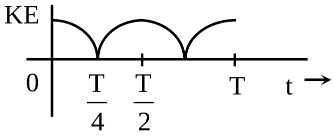
(1) Kirchhoff's second law represents energy conservation.
(2) Wheatstone bridge is the most sensitive when all the four resistances are of the same order of magnitude.
(3) In a balanced wheatstone bridge if the cell and the galvanometer are exchanged, the null point is disturbed.
(4) A rheostat can be used as a potential divider.

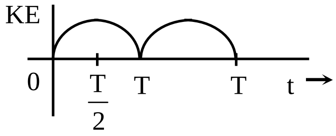
Sol. **(3)**

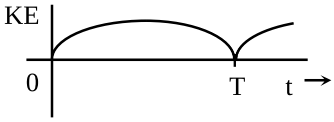
The balanced condition is given by $\frac{P}{Q} = \frac{R}{S}$; When battery and Galvanometer are exchanged, it become $\frac{P}{R} = \frac{Q}{S}$; which is same as previous

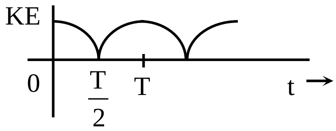


53. A particle is executing simple harmonic motion with a time period T. At time t = 0, it is at its position of equilibrium. The kinetic energy – time graph of the particle will look like:

(1)


(2)


(3)


(4)


Sol. **(1)**

For given SHM x = A sinωt

$$v = \frac{dx}{dt} = A\omega\cos\omega t$$

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}mA^2\omega^2\cos^2\omega t = KE_{max}\left(\frac{1+\cos 2\omega t}{2}\right)$$

54. An observer is moving with half the speed of light towards a stationary microwave source emitting waves at frequency 10 GHz. What is the frequency of the microwave measured by the observer? (speed of light = 3×10^8 ms^{-1})

(1) 15.3 GHz (2) 10.1 GHz
(3) 12.1 GHz (4) 17.3 GHz

Sol. **(4)**

$$f' = \left(\sqrt{\frac{1+\beta}{1-\beta}}\right)f, \text{ where } \beta = \frac{v}{c}$$

So, f' = 17.3 GHz

55. A man grows into a giant such that his linear dimensions increase by a factor of 9. Assuming that his density remains same, the stress in the leg will change by a factor of:

(1) $\frac{1}{81}$ (2) 9
(3) $\frac{1}{9}$ (4) 81

Sol. **(2)**

$$\text{Stress} = \frac{F}{A} = \frac{mg}{A} = \frac{\rho\ell Ag}{A} = \rho\ell g$$

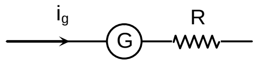
$$\text{So, } \frac{\text{Stress}_f}{\text{Stress}_i} = 9$$

56. When a current of 5 mA is passed through a galvanometer having a coil of resistance 15Ω, it shows full scale deflection. The value of the resistance to be put in series with the galvanometer to convert it into a voltmeter of range 0 – 10V is:

(1) 4.005×10^3 Ω (2) 1.985×10^3 Ω
(3) 2.045×10^3 Ω (4) 2.535×10^3 Ω

Sol. **(2)**

$i_g\left(R+R_g\right)=V$

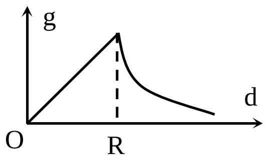


$R=\frac{V}{i_g}-R_g$

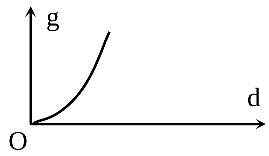
$R=\frac{10}{5\times10^{-3}}-15=1.985\times10^{3}\,\Omega$

*57. The variation of acceleration due to gravity g with distance d from centre of the earth is best represented by (R = Earth's radius):

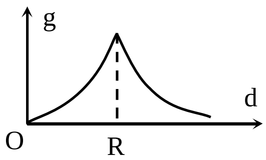
(1)



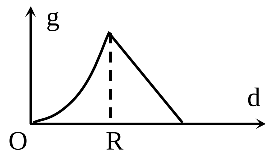
(2)



(3)



(4)



Sol. **(1)**

The variation of magnitude of acceleration due to gravity is given by

$g=\left(\frac{GM}{R^3}\right)d$, where $0\le d\le R$

$=\frac{GM}{d^2}$, where $d>R$

58. An external pressure P is applied on a cube at 0°C so that it is equally compressed from all sides. K is the bulk modulus of the material of the cube and α is its coefficient of linear expansion. Suppose we want to bring the cube to its original size by heating. The temperature should be raised by:

(1) $3PK\alpha$

(2) $\frac{P}{3\alpha K}$

(3) $\frac{P}{\alpha K}$

(4) $\frac{3\alpha}{PK}$

Sol. **(2)**

By applying pressure, $\Delta P=-\frac{B\Delta V}{V}$

$\Rightarrow -\frac{\Delta V}{V}=\frac{\Delta P}{B}=\frac{P}{K}\left(\text{given } B=K\right)$

By increasing temperature, fractional increase in volume

$-\frac{\Delta V}{V}=3\alpha\Delta\theta$

$\frac{P}{K}=3\alpha\Delta\theta$

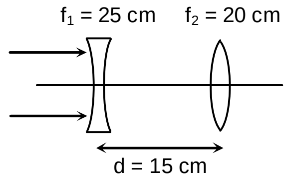
$\Delta\theta=\frac{P}{3\alpha K}$

59. A diverging lens with magnitude of focal length 25 cm is placed at a distance of 15cm from a converging lens of magnitude of focal length 20cm. A beam of parallel light falls on the diverging lens. The final image formed is:

(1) real and at a distance of 6 cm from the convergent lens.
(2) real and at a distance of 40 cm from convergent lens.
(3) virtual and at a distance of 40 cm from convergent lens.
(4) real and at a distance of 40 cm from the divergent lens.

Sol. **(2)**

For diverging lens,



$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}\Rightarrow\frac{1}{v}-\frac{1}{\infty}=\frac{1}{-25}$$

$$\Rightarrow v=-25\text{ cm}$$

First image is formed at a distance 25cm left to the diverging lens.

For the converging lens.

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{v}-\frac{1}{-40}=\frac{1}{20}\Rightarrow\frac{1}{v}=\frac{1}{20}-\frac{1}{40}$$

$$\Rightarrow v=+40\text{ cm}$$

60. A body of mass m = 10^{-2} kg is moving in a medium and experiences a frictional force F = $-kv^2$. Its initial speed is v_0 = 10 ms^{-1}. If, after 10 s, its energy is $\frac{1}{8}mv_0^2$, the value of k will be:

(1) 10^{-1}kg $m^{-1}s^{-1}$
(2) 10^{-3}kg m^{-1}
(3) 10^{-3}kg s^{-1}
(4) 10^{-4}kg m^{-1}

Sol. **(4)**

$$\frac{1}{2}mv_f^2=\frac{1}{8}mv_0^2\Rightarrow v_f=\frac{v_0}{2}$$

Now, $\frac{mdv}{dt}=-kv^2$

$$\Rightarrow m\int_{v_0}^{\frac{v_0}{2}}\frac{dv}{v^2}=-k\int_0^{10}dt$$

$$\Rightarrow m\left[-\frac{1}{v}\right]_{v_0}^{\frac{v_0}{2}}=-k\left[t\right]_0^{10}$$

$$\Rightarrow m\left(\frac{2}{v_0}-\frac{1}{v_0}\right)=10k$$

$$\Rightarrow\frac{m}{v_0}=10k$$

$$\Rightarrow k=\frac{m}{10v_0}=\frac{10^{-2}}{10\times10}=10^{-4}\text{kg m}^{-1}$$

PART C - CHEMISTRY

61 . 1 gram of a carbonate (M_2CO_3) on treatment with excess HCl produces 0.01186 mole of CO_2. The molar mass of M_2CO_3 in g mol^{-1} is:

(1) 84.3
(2) 118.6
(3) 11.86
(4) 1186

Sol. **(1)**

$$M_2CO_3 + 2HCl \longrightarrow CO_2 + 2MCl + H_2O$$

Moles of M_2CO_3 = Moles of CO_2 produced.

$$\text{moles of } M_2CO_3 = \frac{w}{\text{molar mass}} = 0.01186$$

$\therefore$ Molar mass = 84.3 g mol^{-1}

So, option (1) is correct.

62 . Given $C_{(graphite)} + O_2(g) \longrightarrow CO_2(g);$

$\Delta_r H^0 = -393.5 \text{kJ mol}^{-1}$

$H_2(g) + \frac{1}{2}O_2(g) \longrightarrow H_2O(1);$

$\Delta_r H^0 = -285.8 \text{ kJ mol}^{-1}$

$CO_2(g) + 2H_2O(1) \longrightarrow CH_4(g) + 2O_2(g);$

$\Delta_r H^0 = +890.3 \text{kJ mol}^{-1}$

Based on the above thermochemical equations, the value of $\Delta_r H^0$ at 298 K for the reaction

$C_{(graphite)} + 2H_2(g) \longrightarrow CH_4(g)$ will be:

(1) +144.0 kJ mol^{-1}
(2) – 74.8 kJ mol^{-1}
(3) –144.0 kJ mol^{-1}
(4) +74.8 kJ mol^{-1}

Sol. **(2)**

$C_{(graphite)} + O_2(g) \longrightarrow CO_2(g); \quad \Delta_r H^o = -393.5 \text{ kJ / mol}^{-1}$ … (1)

$H_2(g) + \frac{1}{2}O_2(g) \longrightarrow H_2O(\ell); \quad \Delta_r H^o = -285.8 \text{ kJ / mol}^{-1}$ … (2)

$CO_2(g) + 2H_2O(\ell) \longrightarrow CH_4(g) + 2O_2(g); \Delta H_r^o = +890.3 \text{ kJ / mol}^{-1}$ … (3)

$C_{(graphite)} + 2H_2(g) \longrightarrow CH_4(g); \quad \Delta H = ?$ … (4)

[Eq. (1) + Eq. (3)] + [2 × Eq. (2)] = Eq. (4)

$\therefore [\Delta H_1 + \Delta H_3] + [2 \times \Delta H_2] = \Delta H_4$

$[(-393.5) + (890.3)] + [2(-285.8)] = -74.8 \text{ kJ / mol}$

$= -74.8 \text{ kJ/mol}^{-1}$

63. The freezing point of benzene decreases by 0.45^0C when 0.2 g of acetic acid is added to 20g of benzene. If acetic acid associates to form a dimer in benzene, percentage association of acetic acid in benzene will be: (K_f for benzene = 5.12 K kg mol^{-1})

(1) 80.4 %
(2) 74.6 %
(3) 94.6 %
(4) 64.6 %

Sol. **(3)**

$$\Delta T_f = i \times K_f \times m$$

$$\Rightarrow 0.45 = i \times 5.12 \times \frac{0.2 \times 1000}{60 \times 20}$$

$i = 0.527$

$2CH_3COOH \rightleftharpoons (CH_3COOH)_2$

$1-\alpha \qquad \frac{\alpha}{2}$

$i = 1-\alpha+\frac{\alpha}{2}$

$\alpha = 0.946$

$\therefore$ % dissociation is 94.6%.

64 . The most abundant elements by mass in the body of a healthy human adult are: Oxygen (61.4%); Carbon (22.9%), Hydrogen (10.0%); and Nitrogen (2.6%). The weight which a 75 kg person would gain if all ^1H atoms are replaced by ^2H atoms is:

(1) 37.5 kg (2) 7.5 kg
(3) 10 kg (4) 15 kg

Sol. **(2)**

Total hydrogen ($_1H^1$) = $\frac{10}{100}\times 75 = 7.5$ kg

If it is replaced by $_1H^2$ then mass will be doubled so now hydrogen mass = 15 kg

So, mass of person will be increased by 7.5 kg.

65 . ΔU is equal to

(1) Isobaric work (2) Adiabatic work
(3) Isothermal work (4) Isochoric work

Sol. **(2)**

$\Delta U = q + w$

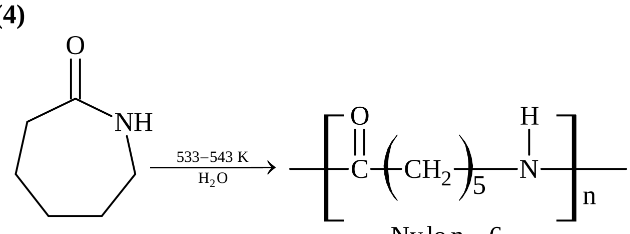
$q = 0$ in adiabatic process.

So, $\Delta U = w$

66. The formation of which of the following polymers involves hydrolysis reaction?

(1) Bakelite (2) Nylon 6, 6
(3) Terylene (4) Nylon 6

Sol. **(4)**



67. Given

$E^0_{Cl_2/Cl^-} = 1.36V, E^0_{Cr^{3+}/Cr} = -0.74$ V

$E^0_{Cr_2O_7^{2-}/Cr^{3+}} = 1.33$ V, $E^0_{MnO_4^-/Mn^{2+}} = 1.51V$.

Among the following, the strongest reducing agent is

(1) Mn^{2+} (2) Cr^{3+}
(3) Cl^- (4) Cr

Sol. **(4)**

Reduction potential of

$E^{o}_{Cr^{3+}/Cr} = -0.74\ V$

So, $E_{Cr/Cr^{3+}} = +0.74\ V$

$\therefore$ Cr would be strongest reducing agent.

68. The Tyndall effect is observed only when following conditions are satisfied:

(a) The diameter of the dispersed particles is much smaller than the wavelength of the light used.
(b) The diameter of the dispersed particle is not much smaller than the wavelength of the light used.
(c) The refractive indices of the dispersed phase and dispersion medium are almost similar in magnitude.
(d) The refractive indices of the dispersed phase and dispersion medium differ greatly in magnitude.

(1) (b) and (d) (2) (a) and (c)
(3) (b) and (c) (4) (a) and (d)

Sol. **(1)**

Tyndall effect is observed only when

(i) The diameter of the dispersed particle is not much smaller than the wavelength of the light used.
(ii) The refractive indices of the dispersed phase and dispersion medium differ greatly in magnitude. So, (b) and (d) are correct.

69 . In the following reactions, ZnO is respectively acting as a/an:

(a) $ZnO + Na_2O \longrightarrow Na_2ZnO_2$

(b) $ZnO + CO_2 \longrightarrow ZnCO_3$

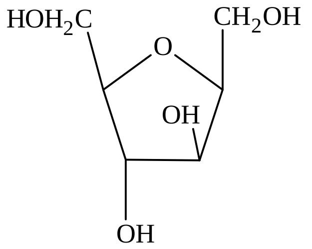
(1) base and base (2) acid and acid
(3) acid and base (4) base and acid

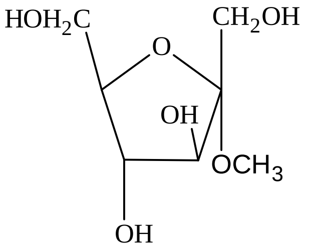
Sol. **(3)**

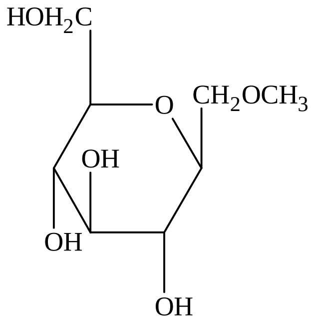
$ZnO + Na_2O \longrightarrow Na_2ZnO_2$; ZnO behaving as an acid.

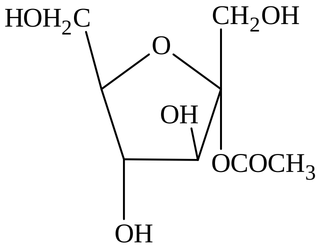
$ZnO + CO_2 \longrightarrow ZnCO_3$; ZnO behaving as a base.

70. Which of the following compounds will behave as a reducing sugar in an aqueous KOH solution?

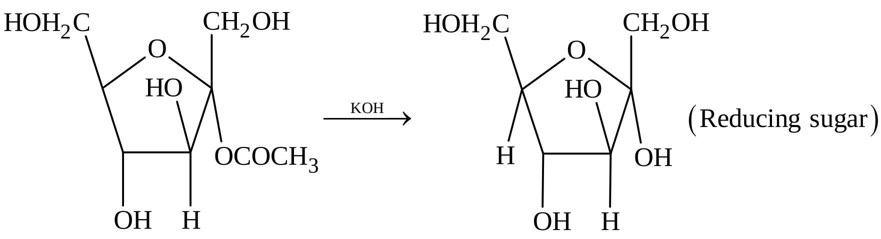
(1)


(2)


(3)


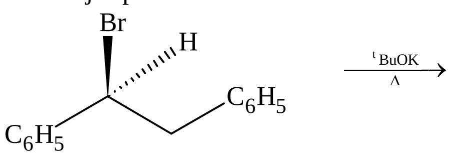
(4)


Sol. **(4)**



Hemi-acetal

71. The major product obtained in the following reaction is:



(+)

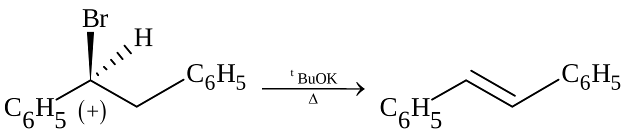
(1) $C_6H_5CH=CHC_6H_5$

(2) $(+)C_6H_5CH(O^tBu)CH_2C_6H_5$

(3) $(-)C_6H_5CH(O^tBu)CH_2C_6H_5$

(4) $(\pm)C_6H_5CH(O^tBu)CH_2C_6H_5$

Sol. **(1)**



Elimination major product

72 . Which of the following species is **not** paramagnetic?

(1) CO

(2) O_2

(3) B_2

(4) NO

Sol. **(1)**

(14) CO – diamagnetic
(16) O_2 – paramagnetic
(10) B_2 – paramagnetic
(15) NO – paramagnetic

73. On treatment of 100 mL of 0.1 M solution of $CoCl_3.6H_2O$ with excess $AgNO_3$; 1.2×10^{22} ions are precipitated. The complex is:

(1) $[Co(H_2O)_3Cl_3].3H_2O$

(2) $[Co(H_2O)_6]Cl_3$

(3) $[Co(H_2O)_5Cl]Cl_2.H_2O$

(4) $[Co(H_2O)_4Cl_2]Cl.2H_2O$

Sol. **(4)**

Moles of $CoCl_3.6H_2O \longrightarrow 100\text{ mL} \times 0.1\text{M} = 10 \times 10^{-3}$ moles

Ions $\rightarrow 6.023 \times 10^{23} \times 0.01$

$= 6.023 \times 10^{21}$ ions

Precipitated ions $= 1.2 \times 10^{22}$

$\therefore$ 1 Ag^+ ion and 1 Cl^- ion.

So $[Co(H_2O)_5Cl_2]Cl.H_2O$ is correct.

74 . pK_a of a weak acid (HA) and pK_b of a weak base (BOH) are 3.2 and 3.4, respectively. The pH of their salt (AB) solution is:

(1) 6.9 (2) 7.0

(3) 1.0 (4) 7.2

Sol. **(1)**

$$pH = 7 + \frac{pK_a}{2} - \frac{pK_b}{2}$$

$$= 7 + \frac{3.2}{2} - \frac{3.4}{2}$$

= 6.9

75. The increasing order of the reactivity of the following halides for the S_N1 reaction is:

$CH_3\underset{\underset{Cl}{|}}{C}HCH_2CH_3$ (I) $CH_3CH_2CH_2Cl$ (II) $p-H_3CO-C_6H_4-CH_2Cl$ (III)

(1) (II) < (I) < (III) (2) (I) < (III) < (II)

(3) (II) < (III) < (I) (4) (III) < (II) < (I)

Sol. **(1)**

Rate of $S_N1 \propto$ carbocation stability

$CH_3\underset{\oplus}{C}HCH_2CH_3$ (I) $CH_3CH_2\overset{\oplus}{C}H_2$ (II) $p—CH_3O—C_6H_5—\overset{\oplus}{C}H_2$ (III)

$\therefore$ II < I < III

76 . Both lithium and magnesium display several similar properties due to the diagonal relationship; however, the one which is incorrect, is:

(1) both form soluble bicarbonates

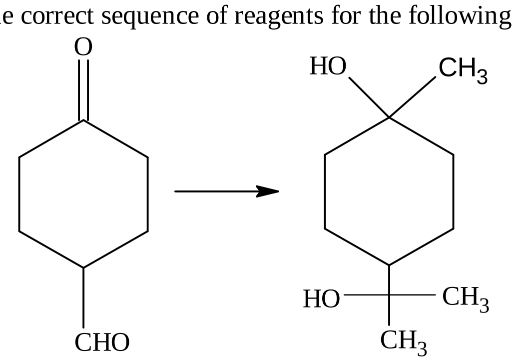
(2) both from nitrides

(3) nitrates of both Li and Mg yield NO_2 and O_2 on heating

(4) both form basic carbonates

Sol. **(1)**

77. The correct sequence of reagents for the following conversion will be:



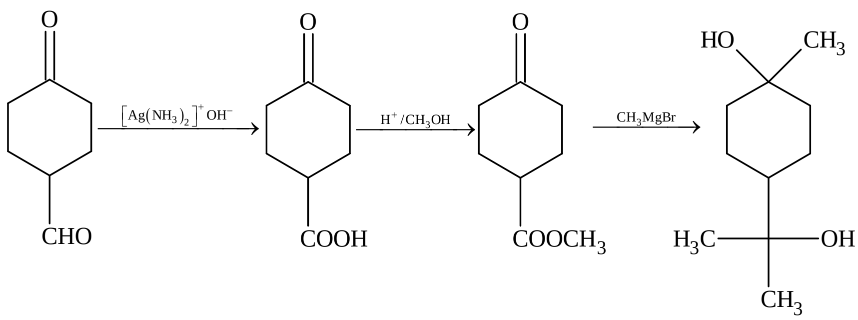
(1) $CH_3MgBr, H^+/CH_3OH, \left[Ag(NH_3)_2\right]^+ OH^-$

(2) $CH_3MgBr, \left[Ag(NH_3)_2\right]^+ OH^-, H^+/CH_3OH$

(3) $\left[Ag(NH_3)_2\right]^+ OH^-, CH_3MgBr, \ , H^+/CH_3OH$

(4) $\left[Ag(NH_3)_2\right]^+ OH^-, H^+/CH_3OH, CH_3MgBr$

Sol. **(4)**



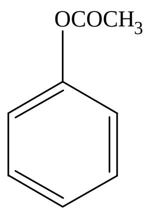
78. The products obtained when chlorine gas reacts with cold and dilute aqueous NaOH are:

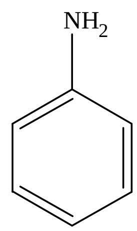
(1) ClO_2^- and ClO_3^-
(2) Cl^- and ClO^-
(3) Cl^- and ClO_2^-
(4) ClO^- and ClO_3^-

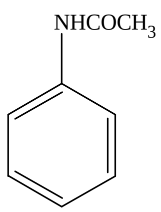
Sol. **(2)**

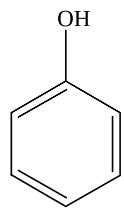
$$Cl_2 + \underset{\text{(cold \& dilute)}}{2NaOH} \longrightarrow NaCl + NaOCl + H_2O$$

79. Which of the following compounds will form significant amount of *meta* product during mono-nitration reaction?

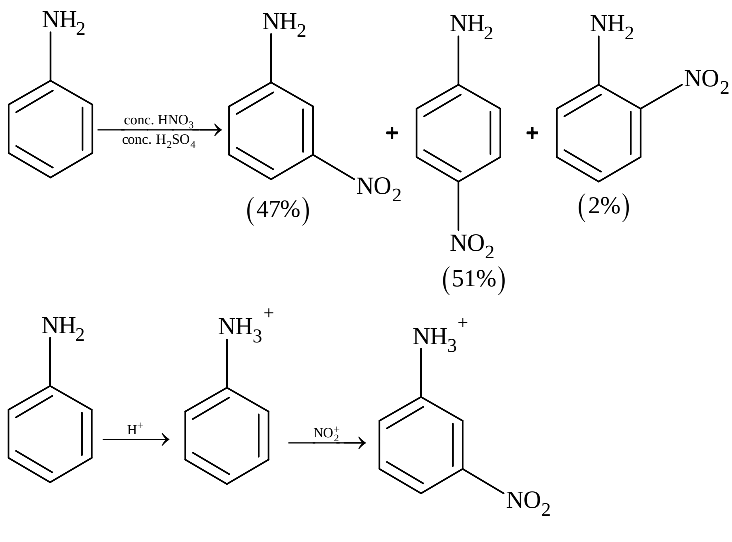
(1) 

(2) 

(3) 

(4) 

Sol. **(2)**

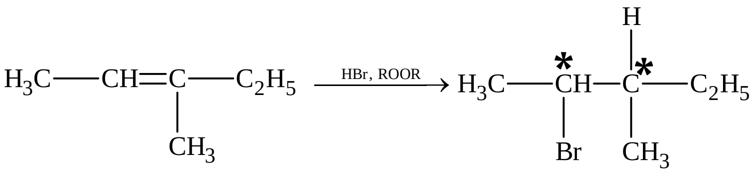


80 . 3-Methyl-pent-2-ene on reaction with HBr in presence of peroxide forms an addition product. The number of possible stereoisomers for the product is :

(1) Zero (2) Two

(3) Four (4) Six

Sol. **(3)**



Stereoisomers = $2^n = 2^2 = 4$

81. Two reactions R_1 and R_2 have identical pre-exponential factors. Activation energy of R_1 exceeds that of R_2 by 10 kJ mol^{-1}. If k_1 and k_2 are rate constants for reactions R_1 and R_2 respectively at 300 K, then $\ln(k_2 / k_1)$ is equal to :

($R = 8.314\ J\ mol^{-1}\ K^{-1}$)

(1) 12 (2) 6

(3) 4 (4) 8

Sol. **(3)**

$$k_1 = Ae^{-E_{a_1}/RT}$$

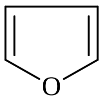
$$k_2 = Ae^{-E_{a_2}/RT}$$

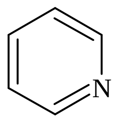
$$\frac{k_2}{k_1} = e^{-\frac{(E_{a_2} - E_{a_1})}{RT}}$$

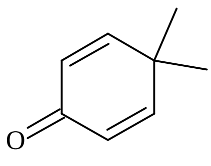
$$\frac{k_2}{k_1} = e^{+\frac{10\times10^3}{8.314\times300}}$$

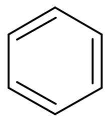
$$\ln\frac{k_2}{k_1} = 4$$

82 . Which of the following molecules is least resonance stabilized?

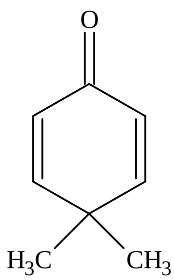
(1) 

(2) 

(3) 

(4) 

Sol. **(3)**

Except 

, all are aromatic in nature, so it has least resonance energy.

83 . The group having isoelectronic species is:

(1) O^{-}, F^{-}, Na, Mg^{+}

(2) $O^{2-}, F^{-}, Na, Mg^{2+}$

(3) $O^{-}, F^{-}, Na^{+}, Mg^{2+}$

(4) $O^{2-}, F^{-}, Na^{+}, Mg^{2+}$

Sol. **(4)**

O^{2-}, F^{-}, Na^{+} and Mg^{2+}, all have 10 electrons each.

84 . The radius of the second Bohr orbit for hydrogen atom is:

(Planck's Const. h = 6.6262×10^{-34} Js; mass of electron = 9.1091×10^{-31} kg; charge of electron e = 1.60210×10^{-19} C; permittivity of vacuum $\epsilon_0 = 8.854185 \times 10^{-12}$ $kg^{-1} m^{-3} A^{2}$)

(1) 4.76 Å

(2) 0.529 Å

(3) 2.12 Å

(4) 1.65 Å

Sol. **(3)**

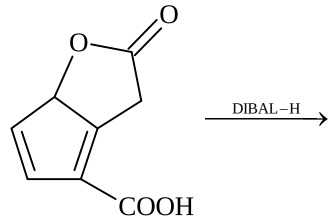
$$r_n = \frac{0.53n^2}{Z} \text{Å}$$

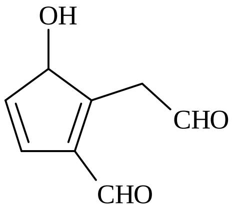
n = 2

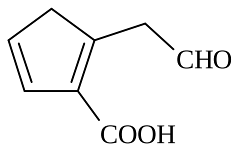
Z = 1

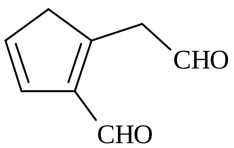
$$r_2 = 0.53 \times 4 \text{Å} = 2.12 \text{Å}$$

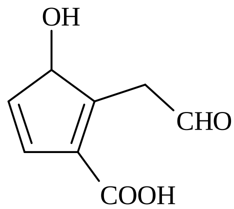
85. The major product obtained in the following reaction is:



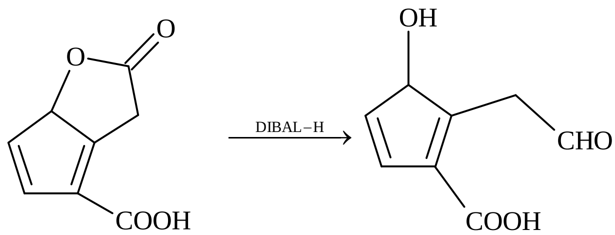
(1) 

(2) 

(3) 

(4) 

Sol. **(4)**



86 . Which of the following reactions is an example of a redox reaction?

(1) $XeF_2 + PF_5 \longrightarrow [XeF]^+ PF_6^-$ (2) $XeF_6 + H_2O \longrightarrow XeOF_4 + 2HF$

(3) $XeF_6 + 2H_2O \longrightarrow XeO_2F_2 + 4HF$ (4) $XeF_4 + O_2F_2 \longrightarrow XeF_6 + O_2$

Sol. **(4)**

$$\overset{+4}{X}eF_4 + \overset{+1}{O_2}F_2 \longrightarrow \overset{+6}{Xe}F_6 + \overset{0}{O_2}$$

Xenon oxidises and oxygen gets reduced.

87. A metal crystallises in a face centred cubic structure. If the edge length of its unit cell is ‘a’, the closest approach between two atoms in metallic crystal will be:

(1) $2\sqrt{2}\,a$ (2) $\sqrt{2}\,a$

(3) $\frac{a}{\sqrt{2}}$ (4) 2 a

Sol. **(3)**

In FCC structure

$4r = \sqrt{2}a$

$2r = \frac{a}{\sqrt{2}}$ = closest approach between two atoms.

88. Sodium salt of an organic acid ‘X’ produces effervescence with conc. H_2SO_4. ‘X’ reacts with the acidified aqueous $CaCl_2$ solution to give a white precipitate which decolourises acidic solution of $KMnO_4$. ‘X’ is:

(1) HCOONa (2) CH_3COONa

(3) $Na_2C_2O_4$ (4) C_6H_5COONa

Sol. **(3)**

$$(X)\,Na_2C_2O_4 \xrightarrow[H_2SO_4]{conc.} CO_2\uparrow + CO\uparrow + Na_2SO_4$$

$$Na_2C_2O_4 \xrightarrow{CaCl_2} \underset{\text{white ppt.}}{CaC_2O_4\downarrow} \xrightarrow[KMnO_4]{acidic} CO_2\uparrow$$

89 . A water sample has ppm level concentration of following anions

$F^- = 10;\ SO_4^{2-} = 100;\ NO_3^- = 50$

The anion/anions that make/makes the water sample unsuitable for drinking is/are:

(1) both SO_4^{2-} and NO_3^- (2) only F^-

(3) only SO_4^{2-} (4) only NO_3^-

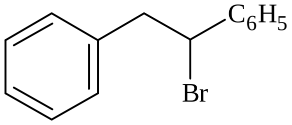
Sol. **(2)**

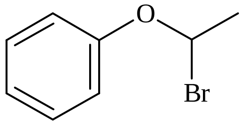
Permissible limit for SO_4^{2-} = 500 ppm

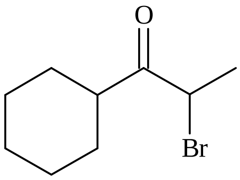
Permissible limit for NO_3^- = 50 ppm

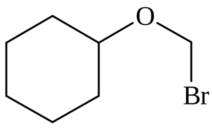
Permissible limit for F^- = 1 ppm

90. Which of the following, upon treatment with *tert*-BuONa followed by addition of bromine water, fails to decolourize the colour of bromine?

(1) 

(2) 

(3) 

(4) 

Sol. **(4)**