AIEEE–2012

PART A: MATHEMATICS

1. The equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has
(1) infinite number of real roots (2) no real roots
(3) exactly one real root (4) exactly four real roots

1. 2

Sol. $e^{\sin x} - e^{-\sin x} = 4 \quad \Rightarrow e^{\sin x} = t$

$t - \frac{1}{t} = 4$

$t^2 - 4t - 1 = 0 \quad \Rightarrow t = \frac{4 \pm \sqrt{16+4}}{2}$

$\Rightarrow t = \frac{4 \pm 2\sqrt{5}}{2} \quad \Rightarrow t = 2 \pm \sqrt{5}$

$e^{\sin x} = 2 \pm \sqrt{5} \quad -1 \le \sin x \le 1 \quad \frac{1}{e} \le e^{\sin x} \le e$

$e^{\sin x} = 2 + \sqrt{5}$ not possible

$e^{\sin x} = 2 - \sqrt{5}$ not possible $\quad \therefore$ hence no solution

2. Let $\hat{a}$ and $\hat{b}$ be two unit vectors. If the vectors $\vec{c} = \hat{a} + 2\hat{b}$ and $\vec{d} = 5\hat{a} - 4\hat{b}$ are perpendicular to each other, then the angle between $\hat{a}$ and $\hat{b}$ is
(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{2}$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{4}$

2. 3

Sol. $\vec{c} \cdot \vec{d} = \vec{0} \quad \Rightarrow 5|\vec{a}|^2 + 6\vec{a} \cdot \vec{b} - 8|\vec{b}|^2 = 0$

$\Rightarrow 6\vec{a} \cdot \vec{b} = 3 \quad \Rightarrow \vec{a} \cdot \vec{b} = \frac{1}{2} \quad \Rightarrow (\vec{a} \cdot \vec{b}) = \frac{\pi}{3}$

3. A spherical balloon is filled with 4500π cubic meters of helium gas. If a leak in the balloon causes the gas to escape at the rate of 72π cubic meters per minute, then the rate (in meters per minute) at which the radius of the balloon decreases 49 minutes after the leakage began is
(1) $\frac{9}{7}$ (2) $\frac{7}{9}$ (3) $\frac{2}{9}$ (4) $\frac{9}{2}$

3. 3

Sol. $v = \frac{4}{3}\pi r^2$

After 49 minutes volume = $4500\pi - 49\,(72\pi) = 972\pi$

$\frac{4}{3}\pi r^3 = 972\pi \quad \Rightarrow r^3 = 729 \quad \Rightarrow r = 9$

$v = \frac{4}{3}\pi r^3 \quad \frac{dv}{dt} = \frac{4}{3}\pi 3r^2 \frac{dr}{dt} \quad 72\pi = 4\pi r^2 \frac{dr}{dt} \quad \frac{dr}{dt} = \frac{72}{4 \cdot 9 \cdot 9} = \frac{2}{9}$

4. **Statement 1:** The sum of the series 1 + (1 + 2 + 4) + (4 + 6 + 9) + (9 + 12 + 16) + ... + (361 + 380 + 400) is 8000.

Statement 2: $\sum_{k=1}^{n}\left(k^3 - (k-1)^3\right) = n^3$ for any natural number n.

(1) Statement 1 is false, statement 2 is true
(2) Statement 1 is true, statement 2 is true; statement 2 is a correct explanation for statement 1
(3) Statement 1 is true statement 2 is true; statement 2 is not a correct explanation for statement 1
(4) Statement 1 is true, statement 2 is false

4. 2

Sol. Statement 1 has 20 terms whose sum is 8000

And statement 2 is true and supporting statement 1.

$\because$ k^{th} bracket is $(k-1)^2 + k(k-1) + k^2 = 3k^2 - 3k + 1$.

5. The negation of the statement "If I become a teacher, then I will open a school" is

(1) I will become a teacher and I will not open a school

(2) Either I will not become a teacher or I will not open a school

(3) Neither I will become a teacher nor I will open a school

(4) I will not become a teacher or I will open a school

5. 1

Sol. $\sim(\sim p \vee q) = p \wedge \sim q$

6. If the integral $\int \frac{5\tan x}{\tan x - 2}\,dx = x + a \ln|\sin x - 2\cos x| + k$, then a is equal to

(1) –1 (2) –2 (3) 1 (4) 2

6. 4

Sol. $\int \frac{5\tan x}{\tan x - 2}\,dx = \int \frac{5\sin x}{\sin x - 2\cos x}\,dx \quad \Rightarrow \int\left[\frac{2(\cos x + 2\sin x) + (\sin x - 2\cos x)}{\sin x - 2\cos x}\right]dx$

$= 2\int\left(\frac{\cos x + 2\sin x}{\sin x - 2\cos x}\right)dx + \int dx + k \quad = 2\log|\sin x - 2\cos x| + x + k \quad \therefore a = 2$

7. **Statement 1:** An equation of a common tangent to the parabola $y^2 = 16\sqrt{3}x$ and the ellipse $2x^2 + y^2 = 4$ is $y = 2x + 2\sqrt{3}$.

Statement 2: If the line $y = mx + \frac{4\sqrt{3}}{m}$, $(m \neq 0)$ is a common tangent to the parabola $y^2 = 16\sqrt{3}x$ and the ellipse $2x^2 + y^2 = 4$, then m satisfies $m^4 + 2m^2 = 24$.

(1) Statement 1 is false, statement 2 is true

(2) Statement 1 is true, statement 2 is true; statement 2 is a correct explanation for statement 1

(3) Statement 1 is true, statement 2 is true; statement 2 is not a correct explanation for statement 1

(4) Statement 1 is true, statement 2 is false

7. 2

Sol. $y^2 = 16\sqrt{3}x \qquad \frac{x^2}{2} + \frac{y^2}{4} = 1$

$y = mx + \frac{4\sqrt{3}}{m}$ is tangent to parabola

which is tangent to ellipse $\quad \Rightarrow c^2 = a^2m^2 + b^2$

$\Rightarrow \frac{48}{m^2} = 2m^2 + 4 \qquad \Rightarrow m^4 + 2m^2 = 24 \qquad \Rightarrow m^2 = 4$

8. Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}$. If u_1 and u_2 are column matrices such that $Au_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and $Au_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, then $u_1 + u_2$ is equal to

(1) $\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$ (2) $\begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$ (3) $\begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix}$ (4) $\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$

8. 4

Sol. $A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}$

Let $u_1 = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$; $u_2 = \begin{bmatrix} d \\ e \\ f \end{bmatrix}$

$Au_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \Rightarrow u_1 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$

$Au_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad \Rightarrow u_2 = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix} \quad \Rightarrow u_1 + u_2 = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$

9. If n is a positive integer, then $\left(\sqrt{3}+1\right)^{2n} - \left(\sqrt{3}-1\right)^{2n}$ is

(1) an irrational number
(2) an odd positive integer
(3) an even positive integer
(4) a rational number other than positive integers

9. 1

Sol. $\left(\sqrt{3}+1\right)^{2n} - \left(\sqrt{3}-1\right)^{2n} = \left[\left(\sqrt{3}+1\right)^{2}\right]^{n} - \left[\left(\sqrt{3}-1\right)^{2}\right]^{n} = \left(4+2\sqrt{3}\right)^{n} - \left(4-2\sqrt{3}\right)^{n}$

$= 2^n\left[\left(2+\sqrt{3}\right)^{n} - \left(2-\sqrt{3}\right)^{n}\right]$

$= 2^n\left\{\left[{}^nC_0 2^n + {}^nC_1 2^{n-1}\sqrt{3} + {}^nC_2 2^{n-2}3 + \cdots\cdots\right] - \left[{}^nC_0 2^n - {}^nC_1 2^{n-1}\sqrt{3} + {}^nC_2 2^{n-2}3 - \cdots\cdots\right]\right\}$

$= 2^{n+1}\left[{}^nC_1 2^{n-1}\sqrt{3} + {}^nC_3 2^{n-3}3\sqrt{3} + \cdots\cdot\right] = 2^{n+1}\sqrt{3}$ (some integer)

Which is irrational

10. If 100 times the 100^{th} term of an AP with non zero common difference equals the 50 times its 50^{th} term, then the 150^{th} term of this AP is

(1) –150
(2) 150 times its 50^{th} term
(3) 150
(4) zero

10. 4

Sol. $100(T_{100}) = 50(T_{50}) \quad \Rightarrow 2[a + 99d] = a + 49d \quad \Rightarrow a + 149d = 0 \quad \Rightarrow T_{150} = 0$

11. In a ΔPQR, if 3 sin P + 4 cos Q = 6 and 4 sin Q + 3 cos P = 1, then the angle R is equal to

(1) $\frac{5\pi}{6}$ (2) $\frac{\pi}{6}$ (3) $\frac{\pi}{4}$ (4) $\frac{3\pi}{4}$

11. 2

Sol. 3 sin P + 4 cos Q = 6 ...... (1)

4 sin Q + 3 cos P = 1 ...... (2)

From (1) and (2) $\angle P$ is obtuse.

$(3 \sin P + 4 \cos Q)^2 + (4 \sin Q + 3 \cos P)^2 = 37$

$\Rightarrow 9 + 16 + 24 (\sin P \cos Q + \cos P \sin Q) = 37$

$\Rightarrow 24 \sin (P + Q) = 12$

$\Rightarrow \sin (P + Q) = \frac{1}{2} \quad \Rightarrow P + Q = \frac{5\pi}{6} \quad \Rightarrow R = \frac{\pi}{6}$

12. An equation of a plane parallel to the plane x – 2y + 2z – 5 = 0 and at a unit distance from the origin is

(1) x – 2y + 2z – 3 = 0
(2) x – 2y + 2z + 1 = 0

(3) $x - 2y + 2z - 1 = 0$ (4) $x - 2y + 2z + 5 = 0$

12. 1

Sol. Equation of plane parallel to $x - 2y + 2z - 5 = 0$ is $x - 2y + 2z + k = 0$ ...... (1)

perpendicular distance from O(0, 0, 0) to (1) is 1

$\frac{|k|}{\sqrt{1+4+4}} = 1$ $\Rightarrow |k| = 3$ $\Rightarrow k = \pm 3$ $\therefore x - 2y + 2z - 3 = 0$

13. If the line $2x + y = k$ passes through the point which divides the line segment joining the points (1, 1) and (2, 4) in the ratio 3 : 2, then k equals

(1) $\frac{29}{5}$ (2) 5 (3) 6 (4) $\frac{11}{5}$

13. 3

Sol. Point p = $\left(\frac{6+2}{5}, \frac{12+2}{5}\right)$

p = $\left(\frac{8}{5}, \frac{14}{5}\right)$

$p\left(\frac{8}{5}, \frac{14}{5}\right)$ lies on $2x + y = k$ $\Rightarrow \frac{16}{5} + \frac{14}{5} = k$ $\Rightarrow k = \frac{30}{5} = 6$

14. Let $x_1, x_2, ..., x_n$ be n observations, and let $\bar{x}$ be their arithematic mean and σ^2 be their variance.

Statement 1: Variance of $2x_1, 2x_2, ..., 2x_n$ is $4\sigma^2$.

Statement 2: Arithmetic mean of $2x_1, 2x_2, ..., 2x_n$ is $4\bar{x}$.

(1) Statement 1 is false, statement 2 is true

(2) Statement 1 is true, statement 2 is true; statement 2 is a correct explanation for statement 1

(3) Statement 1 is true, statement 2 is true; statement 2 is not a correct explanation for statement 1

(4) Statement 1 is true, statement 2 is false

14. 4

Sol. $\sigma^2 = \sum \frac{x_i^2}{n} - \left(\sum \frac{x_i}{n}\right)^2$

Variance of $2x_1, 2x_2, ..., 2x_n$ = $\sum \frac{(2x_i)^2}{n} - \left(\sum \frac{2x_i}{n}\right)^2 = 4\left[\sum \frac{x_i^2}{n} - \left(\sum \frac{x_i}{n}\right)^2\right] = 4\sigma^2$

Statement 1 is true.

A.M. of $2x_1, 2x_2, ..., 2x_n$ = $\frac{2x_1 + 2x_2 + \cdots + 2x_n}{n} = 2\left(\frac{x_1 + x_2 + \cdots + x_n}{n}\right) = 2\bar{x}$

Statement 2 is false.

15. The population p(t) at time t of a certain mouse species satisfies the differential equation $\frac{dp(t)}{dt} = 0.5\,p(t) - 450$. If p(0) = 850, then the time at which the population becomes zero is

(1) 2 ln 18 (2) ln 9 (3) $\frac{1}{2}\ln 18$ (4) ln 18

15. 1

Sol. $\frac{d(p(t))}{dt} = \frac{1}{2}p(t) - 450$

$\frac{d(p(t))}{dt} = \frac{p(t) - 900}{2}$

$2\int \frac{d(p(t))}{p(t) - 900} = \int dt$

$2 \ln |p(t) - 900| = t + c$

$t = 0 \quad \Rightarrow 2 \ln 50 = 0 + c \quad \Rightarrow c = 2 \ln 50$

$\therefore\ 2 \ln |p(t) - 900| = t + 2 \ln 50$

$P(t) = 0 \quad \Rightarrow 2 \ln 900 = t + 2 \ln 50$

$t = 2 (\ln 900 - \ln 50) = 2\ln\left(\frac{900}{50}\right) = 2 \ln 18.$

16. Let a, b $\in$ R be such that the function f given by $f(x) = \ln |x| + bx^2 + ax$, $x \neq 0$ has extreme values at $x = -1$ and $x = 2$.

Statement 1: f has local maximum at $x = -1$ and at $x = 2$.

Statement 2: $a = \frac{1}{2}$ and $b = \frac{-1}{4}$

(1) Statement 1 is false, statement 2 is true
(2) Statement 1 is true, statement 2 is true; statement 2 is a correct explanation for statement 1
(3) Statement 1 is true, statement 2 is true; statement 2 is not a correct explanation for statement 1
(4) Statement 1 is true, statement 2 is false

16. 2

Sol. $f'(x) = \frac{1}{x} + 2b\,x + a$

f has extremevalues and differentiable

$\Rightarrow f'(-1) = 0 \quad \Rightarrow a - 2b = 1$

$f'(2) = 0 \quad \Rightarrow a + 4b = -\frac{1}{2} \quad \Rightarrow a = \frac{1}{2},\ b = -\frac{1}{4}$

$f''(-1)$, $f''(2)$ are negative. f has local maxima at -1, 2

17. The area bounded between the parabolas $x^2 = \frac{y}{4}$ and $x^2 = 9y$, and the straight line $y = 2$ is

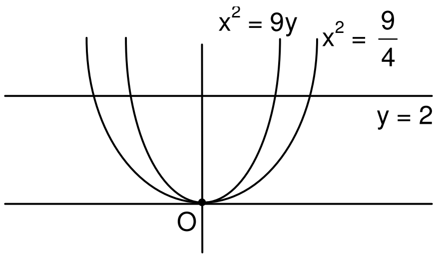
(1) $20\sqrt{2}$ (2) $\frac{10\sqrt{2}}{3}$ (3) $\frac{20\sqrt{2}}{3}$ (4) $10\sqrt{2}$

17. 3

Sol. Required area

$$A = 2\left[\int_0^2 \left(3\sqrt{y} - \frac{\sqrt{y}}{2}\right) dy\right] = 2\int_0^2 \frac{5\sqrt{y}}{2}\,dy$$

$$= 5\left[\frac{y^{3/2}}{3/2}\right]_0^2 = \frac{10}{3}\left[2^{3/2} - 0\right] = \frac{20\sqrt{2}}{3}$$



18. Assuming the balls to be identical except for difference in colours, the number of ways in which one or more balls can be selected from 10 white, 9 green and 7 black balls is

(1) 880 (2) 629 (3) 630 (4) 879

18. 4

Sol. Number of ways of selecting one or more balls from 10 white, 9 green, and 7 black balls

$= (10 + 1)(9 + 1)(7 + 1) - 1 = 11 \times 10 \times 8 - 1 = 879.$

19. If f: R $\rightarrow$ R is a function defined by $f(x) = [x] \cos\left(\frac{2x-1}{2}\right)\pi$, where [x] denotes the greatest integer function, then f is

(1) continuous for every real x
(2) discontinuous only at x = 0
(3) discontinuous only at non-zero integral values of x
(4) continuous only at x = 0

19. 1

Sol. $f(x) = [x]\cos\left(\frac{2x-1}{2}\right)\pi = [x]\cos\left(x-\frac{1}{2}\right)\pi$

$= [x] \sin \pi x$ is continuous for every real x.

20. If the lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect, then k is equal to

(1) –1 (2) $\frac{2}{9}$ (3) $\frac{9}{2}$ (4) 0

20. 3

Sol. Any point on $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4} = t$ is $(2t + 1, 3t - 1, 4t + 1)$

And any point on $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1} = s$ is $(s + 3, 2s + k, s)$

Given lines are intersecting $\Rightarrow t = -\frac{3}{2}$ and $s = -5$ $\therefore k = \frac{9}{2}$

21. Three numbers are chosen at random without replacement from {1, 2, 3, ... 8}. The probability that their minimum is 3, given that their maximum is 6, is

(1) $\frac{3}{8}$ (2) $\frac{1}{5}$ (3) $\frac{1}{4}$ (4) $\frac{2}{5}$

21. 2

Sol. Let A be the event that maximum is 6.
B be event that minimum is 3

$P(A) = \frac{{}^5C_2}{{}^8C_3}$ (the numbers < 6 are 5)

$P(B) = \frac{{}^5C_2}{{}^8C_3}$ (the numbers > 3 are 5)

$P(A \cap B) = \frac{{}^2C_1}{{}^8C_3}$

Required probability is $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{{}^2C_1}{{}^5C_2} = \frac{2}{10} = \frac{1}{5}$.

22. If $z \neq 1$ and $\frac{z^2}{z-1}$ is real, then the point represented by the complex number z lies

(1) either on the real axis or on a circle passing through the origin
(2) on a circle with centre at the origin
(3) either on the real axis or on a circle not passing through the origin
(4) on the imaginary axis

22. 1

Sol. Let $z = x + iy$ ($\because x \neq 1$ as $z \neq 1$)

$z^2 = (x^2 - y^2) + i(2xy)$

$\frac{z^2}{z-1}$ is real $\Rightarrow$ its imaginary part = 0

$\Rightarrow 2xy\,(x - 1) - y(x^2 - y^2) = 0$
$\Rightarrow y(x^2 + y^2 - 2x) = 0$
$\Rightarrow y = 0;\ x^2 + y^2 - 2x = 0$

$\therefore$ z lies either on real axis or on a circle through origin.

23. Let P and Q be 3×3 matrices with $P \neq Q$. If $P^3 = Q^3$ and $P^2Q = Q^2P$, then determinant of $(P^2 + Q^2)$ is equal to

(1) –2 (2) 1 (3) 0 (4) –1

23. 3

Sol. $P^3 = Q^3$

$P^3 - P^2Q = Q^3 - Q^2P$

$P^2(P - Q) = Q^2 (Q - P)$

$P^2(P - Q) + Q^2 (P - Q) = O$

$(P^2 + Q^2)(P - Q) = O \quad \Rightarrow |P^2 + Q^2| = 0$

24. If $g(x) = \int_0^x \cos 4t \, dt$, then $g(x + \pi)$ equals

(1) $\frac{g(x)}{g(\pi)}$ (2) $g(x) + g(\pi)$ (3) $g(x) - g(\pi)$ (4) $g(x) . g(\pi)$

24. 2 or 4

Sol. $g(x) = \int_0^x \cos 4t \, dt$

$\Rightarrow g'(x) = \cos 4x \quad \Rightarrow g(x) = \frac{\sin 4x}{4} + k \quad \Rightarrow g(x) = \frac{\sin 4x}{4} \ [\because g(0) = 0]$

$g(x + \pi) = g(x) + g(\pi) = g(x) - g(\pi) \ (\because g(\pi) = 0)$

25. The length of the diameter of the circle which touches the x-axis at the point (1, 0) and passes through the point (2, 3) is

(1) $\frac{10}{3}$ (2) $\frac{3}{5}$ (3) $\frac{6}{5}$ (4) $\frac{5}{3}$

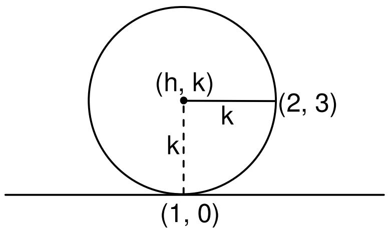
25. 1

Sol. Let (h, k) be centre.

$(h - 1)^2 + (k - 0)^2 = k^2 \quad \Rightarrow h = 1$

$(h - 2)^2 + (k - 3)^2 = k^2 \quad \Rightarrow k = \frac{5}{3}$

$\therefore$ diameter is $2k = \frac{10}{3}$



26. Let X = {1, 2, 3, 4, 5}. The number of different ordered pairs (Y, Z) that can be formed such that $Y \subseteq X$, $Z \subseteq X$ and $Y \cap Z$ is empty, is

(1) 5^2 (2) 3^5 (3) 2^5 (4) 5^3

26. 2

Sol. $Y \subseteq X$, $Z \subseteq X$

Let $a \in X$, then we have following chances that

(1) $a \in Y, \quad a \in Z$

(2) $a \notin Y, \quad a \in Z$

(3) $a \in Y, \quad a \notin Z$

(4) $a \notin Y, \quad a \notin Z$

We require $Y \cap Z = \phi$

Hence (2), (3), (4) are chances for 'a' to satisfy $Y \cap Z = \phi$.

$\therefore Y \cap Z = \phi$ has 3 chances for a.

Hence for five elements of X, the number of required chances is $3 \times 3 \times 3 \times 3 \times 3 = 3^5$

27. An ellipse is drawn by taking a diameter of the circle $(x - 1)^2 + y^2 = 1$ as its semiminor axis and a diameter of the circle $x^2 + (y - 2)^2 = 4$ as its semi-major axis. If the centre of the ellipse is the origin and its axes are the coordinate axes, then the equation of the ellipse is

(1) $4x^2 + y^2 = 4$ (2) $x^2 + 4y^2 = 8$ (3) $4x^2 + y^2 = 8$ (4) $x^2 + 4y^2 = 16$

27. 4

Sol. Semi minor axis b = 2

Semi major axis a = 4

Equation of ellipse = $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$ $\Rightarrow \frac{x^2}{16}+\frac{y^2}{4} = 1$

$\Rightarrow x^2 + 4y^2 = 16$.

28. Consider the function $f(x) = |x - 2| + |x - 5|$, $x \in R$.

Statement 1: $f'(4) = 0$

Statement 2: f is continuous in [2, 5], differentiable in (2, 5) and f(2) = f(5).

(1) Statement 1 is false, statement 2 is true

(2) Statement 1 is true, statement 2 is true; statement 2 is a correct explanation for statement 1

(3) Statement 1 is true, statement 2 is true; statement 2 is not a correct explanation for statement 1

(4) Statement 1 is true, statement 2 is false

28. 2

Sol. $f(x) = 7 - 2x;\ x < 2$

$= 3;\quad 2 \le x \le 5$

$= 2x - 7;\ x > 5$

f(x) is constant function in [2, 5]

f is continuous in [2, 5] and differentiable in (2, 5) and f(2) = f(5)

by Rolle's theorem $f'(4) = 0$

$\therefore$ Statement 2 and statement 1 both are true and statement 2 is correct explanation for statement 1.

29. A line is drawn through the point (1, 2) to meet the coordinate axes at P and Q such that it forms a triangle OPQ, where O is the origin. If the area of the triangle OPQ is least, then the slope of the line PQ is

(1) $-\frac{1}{4}$ (2) –4 (3) –2 (4) $-\frac{1}{2}$

29. 3

Sol. Equation of line passing through (1, 2) with slope m is $y - 2 = m(x - 1)$

Area of $\Delta OPQ = \frac{(m-2)^2}{2|m|}$

$\Delta = \frac{m^2+4-4m}{2m}$ $\Delta = \frac{m}{2}+\frac{2}{m}-2$

Δ is least if $\frac{m}{2}=\frac{2}{m}$ $\Rightarrow m^2 = 4$ $\Rightarrow m = \pm 2$ $\Rightarrow m = -2$

30. Let ABCD be a parallelogram such that $\overrightarrow{AB}=\vec{q}, \overrightarrow{AD}=\vec{p}$ and $\angle BAD$ be an acute angle. If $\vec{r}$ is the vector that coincides with the altitude directed from the vertex B to the side AD, then $\vec{r}$ is given by

(1) $\vec{r} = 3\vec{q} - \frac{3(\vec{p}\cdot\vec{q})}{(\vec{p}\cdot\vec{p})}\vec{p}$ (2) $\vec{r} = -\vec{q} + \left(\frac{\vec{p}\cdot\vec{q}}{\vec{p}\cdot\vec{p}}\right)\vec{p}$

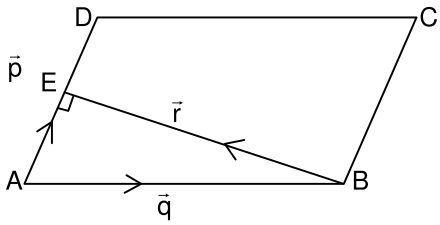
(3) $\vec{r} = \vec{q} - \left(\frac{\vec{p}\cdot\vec{q}}{\vec{p}\cdot\vec{p}}\right)\vec{p}$ (4) $\vec{r} = -3\vec{q} + \frac{3(\vec{p}\cdot\vec{q})}{(\vec{p}\cdot\vec{p})}\vec{p}$

30. 2

Sol. $\overrightarrow{AE}$ = vector component of $\vec{q}$ on $\vec{p}$

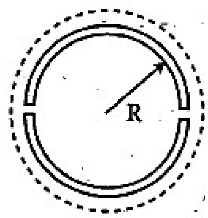
$\overrightarrow{AE} = \frac{(\vec{p}\cdot\vec{q})}{(\vec{p}\cdot\vec{q})}\vec{p}$ $\quad\therefore$ From ΔABE; $\overrightarrow{AB}+\overrightarrow{BE}=\overrightarrow{AE}$

$\Rightarrow \vec{q}+\vec{r} = \frac{(\vec{p}\cdot\vec{q})\vec{p}}{(\vec{p}\cdot\vec{q})}$ $\quad\Rightarrow \vec{r} = -\vec{q}+\frac{(\vec{p}\cdot\vec{q})}{(\vec{p}\cdot\vec{p})}\vec{p}$



PART B: PHYSICS

31. A wooden wheel of radius R is made of two semicircular parts (see figure); The two parts are held together by a ring made of a metal strip of cross sectional area S and length L. L is slightly less than $2\pi R$. To fit the ring on the wheel, it is heated so that its temperature rises by ΔT and it just steps over the wheel. As it cools down to surrounding temperature, it presses the semicircular parts together. If the coefficient of linear expansion of the metal is α, and its Youngs' modulus is Y, the force that one part of the wheel applies on the other part is :



(1) $2\pi SY\alpha\Delta T$ (2) $SY\alpha\Delta T$ (3) $\pi SY\alpha\Delta T$ (4) $2SY\alpha\Delta T$

31. 4

Sol. If temperature increases by ΔT,

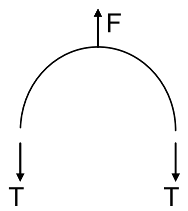
Increase in length L, $\Delta L = L\alpha\Delta T$

$\therefore \quad \frac{\Delta L}{L} = \alpha\Delta T$

Let tension developed in the ring is T.

$\therefore \quad \frac{T}{S} = Y\frac{\Delta L}{L} = Y\alpha\Delta T$

$\therefore \quad T = SY\alpha\Delta T$



From FBD of one part of the wheel,

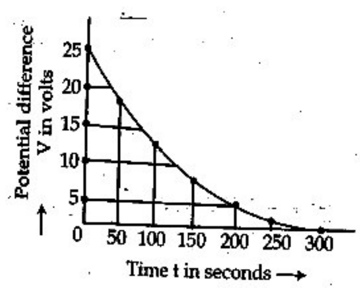
$F = 2T$

Where, F is the force that one part of the wheel applies on the other part.

$\therefore\ F = 2SY\alpha\Delta T$

32. The figure shows an experimental plot for discharging of a capacitor in an R-C circuit. The time constant τ of this circuit lies between:

(1) 150 sec and 200 sec (2) 0 and 50 sec
(3) 50 sec and 100 sec (4) 100 sec and 150 sec



32. 4

Sol. For discharging of an RC circuit,

$V = V_0 e^{-t/\tau}$

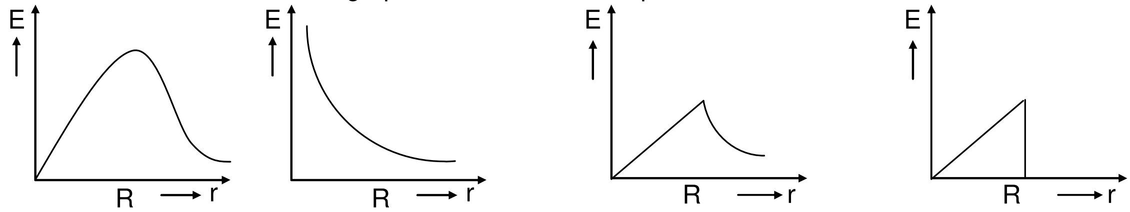
So, when $V = \frac{V_0}{2}$

$\frac{V_0}{2} = V_0 e^{-t/\tau}$

$\ln\frac{1}{2} = -\frac{t}{\tau} \Rightarrow \tau = \frac{t}{\ln 2}$

From graph when $V = \frac{V_0}{2}$, t = 100 s $\quad \therefore\ \tau = \frac{100}{\ln 2} = 144.3$ sec

33. In a uniformly charged sphere of total charge Q and radius R, the electric field E is plotted as a function of distance from the centre. The graph which would correspond to the above will be



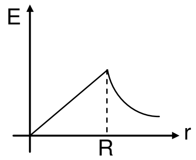
(1) (2) (3) (4)

33. 3

Sol. $\vec{E}_{inside} = \left(\frac{1}{4\pi\varepsilon_0}\frac{Q}{R^3}\right)\vec{r}$

$\vec{E}_{outside}\left(\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^3}\right)\vec{r}$

$\therefore$



34. An electromagnetic wave in vacuum has the electric and magnetic fields $\vec{E}$ and $\vec{B}$, which are always perpendicular to each other. The direction of polarization is given by $\vec{X}$ and that of wave propagation by $\vec{k}$. Then :

(1) $\vec{X} \parallel \vec{B}$ and $\vec{k} \parallel \vec{B}\times\vec{E}$ (2) $\vec{X} \parallel \vec{E}$ and $\vec{k} \parallel \vec{E}\times\vec{B}$ (3) $\vec{X} \parallel \vec{B}$ and $\vec{k} \parallel \vec{E}\times\vec{B}$ (4) $\vec{X} \parallel \vec{E}$ and $\vec{k} \parallel \vec{B}\times\vec{E}$

34. 3

Sol. Direction of polarization is parallel to magnetic field,

$\therefore \quad \vec{X} \parallel \vec{B}$

and direction of wave propagation is parallel to $\vec{E}\times\vec{B}$

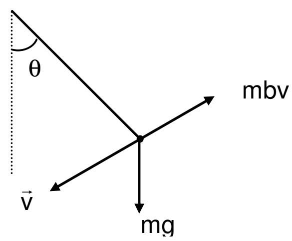
$\therefore \quad \vec{K} \parallel \vec{E}\times\vec{B}$

35. If a simple pendulum has significant amplitude (up to a factor of 1/e of original) only in the period between t = Os to t = τs, then τ may be called the average life of the pendulum. When the spherical bob of the pendulum suffers a retardation (due to viscous drag) proportional to its velocity, with 'b' as the constant of proportionality, the average life time of the pendulum is (assuming damping is small) in seconds:

(1) $\frac{0.693}{b}$ (2) b (3) $\frac{1}{b}$ (4) $\frac{2}{b}$

35. 4

Sol. As retardation = bv



$\therefore \quad$ retarding force = mbv

$\therefore$ net restoring torque when angular displacement is θ is given by

$= -mg\,\ell\sin\theta + mbv\,\ell$

$\therefore \quad I\alpha = -mg\,\ell\sin\theta + mbv\,\ell$

where, $I = m\ell^2$

$\therefore \quad \frac{d^2\theta}{dt^2} = \alpha = -\frac{g}{\ell}\sin\theta + \frac{bv}{\ell}$

for small damping, the solution of the above differential equation will be

$\therefore \quad \theta = \theta_0 e^{-\frac{bt}{2}} \sin(wt + \phi)$

$\therefore \quad$ angular amplitude will be = $\theta . e^{\frac{-bt}{2}}$

According to question, in τ time (average life–time),

angular amplitude drops to $\frac{1}{e}$ value of its original value (θ)

$\therefore \quad \frac{\theta_0}{e} = \theta_0 e^{-\frac{6\tau}{2}}$

$\frac{6\tau}{2} = 1$

$\therefore \quad \tau = \frac{2}{b}$

36. Hydrogen atom is excited from ground state to another state with principal quantum number equal to 4. Then the number of spectral lines in the emission spectra will be

(1) 2 (2) 3 (3) 5 (4) 6

36. 4

Sol. Number of spectral lines from a state n to ground state is

$= \frac{n(n-1)}{2} = 6$.

37. A coil is suspended in a uniform magnetic field, with the plane of the coil parallel to the magnetic lines of force. When a current is passed through the coil it starts oscillating; it is very difficult to stop. But if an aluminium plate is placed near to the coil, it stops. This is due to :

(1) development of air current when the plate is placed.

(2) induction of electrical charge on the plate

(3) shielding of magnetic lines of force as aluminium is a paramagnetic material.

(4) electromagnetic induction in the aluminium plate giving rise to electromagnetic damping.

37. 4

Sol. Oscillating coil produces time variable magnetic field. It cause eddy current in the aluminium plate which causes anti–torque on the coil, due to which is stops.

38. The mass of a spaceship is 1000 kg. It is to be launched from the earth's surface out into free space. The value of 'g' and 'R' (radius of earth) are 10 m/s^2 and 6400km respectively. The required energy for this work will be ;

(1) 6.4×10^{11} Joules (2) 6.4×10^{8} Joules (3) 6.4×10^{9} Joules (4) 6.4×10^{10} Joules

38. 4

Sol. To launch the spaceship out into free space, from energy conservation,

$\frac{-GMm}{R} + E = 0$

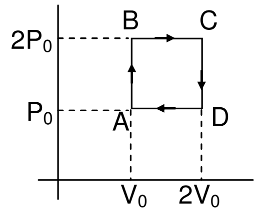
$E = \frac{GMm}{R} = \left(\frac{GM}{R^2}\right) mR = mgR$

$= 6.4 \times 10^{10}$ J

39. Helium gas goes through a cycle ABCDA (consisting of *two* isochoric and two isobaric lines) as shown in figure. Efficiency of this cycle is nearly: (Assume the gas to be close to ideal gas)

(1) 15.4% (2) 9.1%

(3) 10.5% (4) 12.5%



39. 1

Sol. Work done in complete cycle = Area under P–V graph

$= P_0V_0$

from A to B, heat given to the gas

$$= nC_v\Delta T = n\frac{3}{2}R\Delta T = \frac{3}{2}V_0\Delta P = \frac{3}{2}P_0V_0$$

from B to C, heat given to the system

$$= nC_p\Delta T = n\left(\frac{5}{2}R\right)\Delta T$$

$$= \frac{5}{2}(2P_0)\Delta V = 5P_0V_0$$

from C to D and D to A, heat is rejected.

efficiency, $\eta = \dfrac{\text{work done by gas}}{\text{heat given to the gas}} \times 100$

$$\eta = \frac{P_0V_0}{\frac{3}{2}P_0V_0 + 5P_0V_0} = 15.4\%$$

40. In Young's double slit experiment, one of the slit is wider than other, so that the amplitude of the light from one slit is double of that from other slit. If I_m be the maximum intensity, the resultant intensity I when they interfere at phase difference ϕ is given by

(1) $\frac{I_m}{9}(4+5\cos\phi)$ (2) $\frac{I_m}{3}\left(1+2\cos^2\frac{\phi}{2}\right)$ (3) $\frac{I_m}{5}\left(1+4\cos^2\frac{\phi}{2}\right)$ (4) $\frac{I_m}{9}\left(1+8\cos^2\frac{\phi}{2}\right)$

40. 4

Sol. Let $A_1 = A_0$, $A_2 = 2A_0$

If amplitude of resultant wave is A then

$$A^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\phi$$

For maximum intensity,

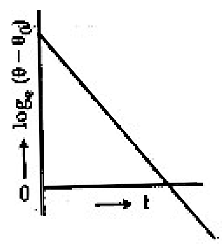
$$A_{max}^2 = A_1^2 + A_2^2 + 2A_1A_2$$

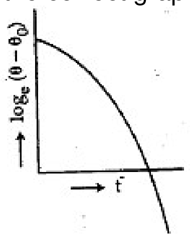
$$\therefore \quad \frac{A^2}{A_{max}^2} = \frac{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}{A_1^2 + A_2^2 + 2A_1A_2}$$

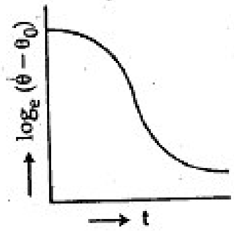
$$= \frac{A_0^2 + 4A_0^2 + 2(A_0)(2A_0)\cos\phi}{A_0^2 + 4A_0^2 + 2(A_0)(2A_0)}$$

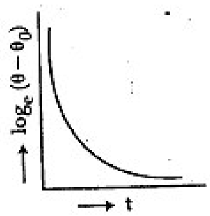
$$\frac{I}{I_m} = \frac{5+4\cos\phi}{9} = \frac{1+8\cos^2(\phi/2)}{9}$$

41. A liquid in a beaker has temperature $\theta(t)$ at time t and θ_0 is temperature of surroundings, then according to Newton's law of cooling the correct graph between $\log_e(\theta - \theta_0)$ and t is









(1) (2) (3) (4)

41. 1

Sol. According to Newtons law of cooling.

$$\frac{d\theta}{dt} \propto -(\theta - \theta_0)$$

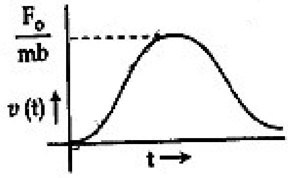
$\Rightarrow \quad \frac{d\theta}{dt} = -k(\theta - \theta_0)$

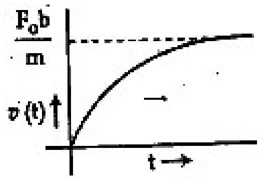
$\int \frac{d\theta}{\theta - \theta_0} = \int -k\,dt$

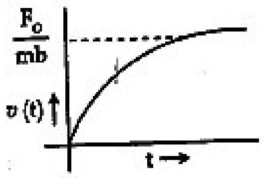
$\Rightarrow \quad \ln(\theta - \theta_0) = -kt + c$

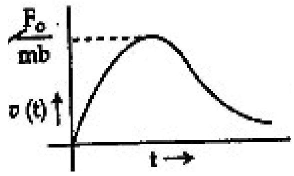
Hence the plot of $\ln(\theta - \theta_0)$ vs t should be a straight line with negative slope.

42. A particle of mass m is at rest at the origin at time t = 0. It is subjected to a force F (t) = F_0e^{-bt} in the *x* direction. Its speed *v(t)* is depicted by which of the following curves?

(1)


(2)


(3)


(4)


42. 3

Sol. $F = F_0e^{-bt}$

$\Rightarrow \quad a = \frac{F}{m} = \frac{F_0}{m}e^{-bt}$

$\Rightarrow \quad \frac{dv}{dt} = \frac{F_0}{m}e^{-bt}$

$\int dv = \int_0^t \frac{F}{m}e^{-bt}dt$

$\Rightarrow \quad v = \frac{F}{m}\left[\frac{-1}{b}\right]\left[e^{-bt}\right]_0^t$

$\Rightarrow \quad v = \frac{F}{mb}\left[e^{-bt}\right]$

v = 0 at t = 0

and $\quad v \to \frac{F}{mb}$ as $t \to \infty$

So, velocity increases continuously and attains a maximum value of $v = \frac{F}{mb}$ as $t \to \infty$.

43. Two electric bulbs marked 25W – 220V and 100W – 220V are connected in series to a 440Vsupply. Which of the bulbs will fuse?

(1) both (2) 100 W (3) 25 W (4) neither

43. 3

Sol. Resistances of both the bulbs are

$R_1 = \frac{V^2}{P_1} = \frac{220^2}{25}$

$R_2 = \frac{V^2}{P_2} = \frac{220^2}{100}$

Hence $R_1 > R_2$

When connected in series, the voltages divide in them in the ratio of their resistances. The voltage of 440 V devides in such a way that voltage across 25 w bulb will be more than 220 V.
Hence 25 w bulb will fuse.

44. Resistance of a given wire is obtained by measuring the current flowing in it and the voltage difference applied across it. If the percentage errors in the measurement of the current and the voltage difference are 3% each, then error in the value of resistance of the wire is
(1) 6% (2) zero (3) 1% (4) 3%

44. 1

Sol. $R = \frac{V}{i}$

$\Rightarrow \quad \left|\frac{\Delta R}{R}\right| = \left|\frac{\Delta V}{V}\right| + \left|\frac{\Delta i}{i}\right|$

$\frac{\Delta V}{V} \times 100 = 3$

$\Rightarrow \quad \frac{\Delta V}{V} = 0.03$

Similarly, $\frac{\Delta i}{i} = 0.03$

Hence $\frac{\Delta R}{R} = 0.06$

So percentage error is $\frac{\Delta R}{R} \times 100 = 6\%$

45. A boy can throw a stone up to a maximum height of 10 m. The maximum horizontal distance that the boy can throw the same stone up to will be
(1) $20\sqrt{2}$ m (2) 10 m (3) $10\sqrt{2}$ m (4) 20 m

45. 4

Sol. maximum vertical height = $\frac{u^2}{2g} = 10\,m$

Horizontal range of a projectile = $\frac{u^2 \sin 2\theta}{g}$

Range is maximum when $\theta = 45^0$

Maximum horizontal range = $\frac{u^2}{g}$

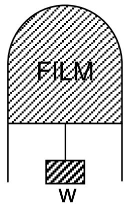
Hence maximum horizontal distance = 20 m.

46. This question has statement 1 and statement 2. Of the four choices given after the statements, choose the one that best describes the two statements
Statement 1 : Davisson – germer experiment established the wave nature of electrons.
Statement 2 : If electrons have wave nature, they can interfere and show diffraction.
(1) Statement 1 is false, Statement 2 is true
(2) Statement 1 is true, Statement 2 is false
(3) Statement 1 is true, Statement 2 is the correct explanation for statement 1
(4) Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation for statement 1.

46. 3

Sol. Davisson – Germer experiment showed that electron beams can undergo diffraction when passed through atomic crystals. This shows the wave nature of electrons as waves can exhibit interference and diffraction.

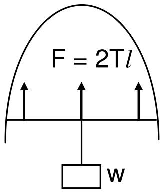
47. A thin liquid film formed between a U-shaped wire and a light slider supports a weight of 1.5×10^{-2}N (see figure). The length of the slider is 30 cm and its weight negligible. The surface tension of the liquid film is



(1) 0.0125 Nm^{-1} (2) 0.1 Nm^{-1}
(3) 0.05 Nm^{-1} (4) 0.025 Nm^{-1}

47. 4

Sol. The force of surface tension acting on the slider balances the force due to the weight.

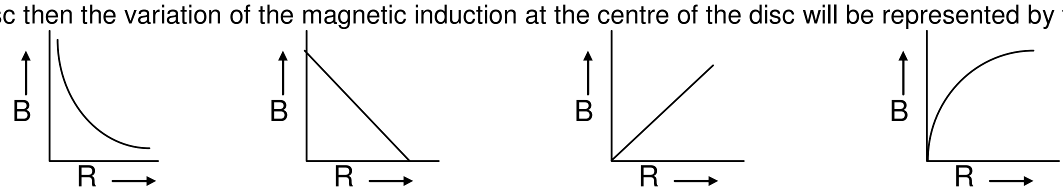


$\Rightarrow$ $F = 2T\ell = w$

$\Rightarrow$ $2T(0.3) = 1.5 \times 10^{-2}$

$\Rightarrow$ $T = 2.5 \times 10^{-2}$ N/m

48. A charge Q is uniformly distributed over the surface of non conducting disc of radius R. The disc rotates about an axis perpendicular to its plane and passing through its centre with an angular velocity ω. As a result of this rotation a magnetic field of induction B is obtained at the centre of the disc. If we keep both the amount of charge placed on the disc and its angular velocity to be constant and vary the radius of the disc then the variation of the magnetic induction at the centre of the disc will be represented by the figure



(1) (2) (3) (4)

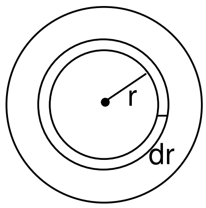
48. 1

Sol. Consider ring like element of disc of radius r and thickness dr.

If σ is charge per unit area, then charge on the element

$$dq = \sigma(2\pi r\, dr)$$

current 'i' associated with rotating charge dq is



$$i = \frac{(dq)w}{2\pi} = \sigma w\, r\, dr$$

Magnetic field dB at center due to element

$$dB = \frac{\mu_0 i}{2r} = \frac{\mu_0 \sigma \omega dr}{2}$$

$$B_{net} = \int dB = \frac{\mu_0 \sigma \omega}{2} \int_0^R dr = \frac{\mu_0 \sigma \omega R}{2}$$

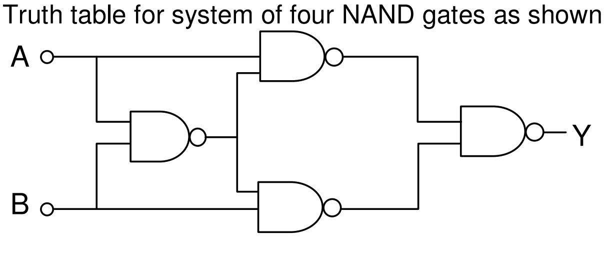
$\Rightarrow$ $B_{net} = \dfrac{\mu_0 Q \omega}{2\pi R}$ $\left[\because Q = \sigma \pi R^2\right]$

So if Q and w are unchanged then

$$B_{net} \propto \frac{1}{R}$$

Hence variation of B_{net} with R should be a rectangular hyperbola as represented in (1).

49. Truth table for system of four NAND gates as shown in figure is



A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

(1)

A	B	Y
0	0	0
0	1	0
1	0	1
1	1	1

(2)

A	B	Y
0	0	1
0	1	1
1	0	0
1	1	0

(3)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

(4)

49. 1

Sol.

A	**B**	**y**	**y_1**	**y_2**	**y**
0	0	1	1	1	0
0	1	1	1	0	1
1	0	1	0	1	1
1	1	0	1	1	0

50. A radar has a power of 1 Kw and is operating at a frequency of 10 GHz. It is located on a mountain top of height 500 m. The maximum distance upto which it can detect object located on the surface of the earth (Radius of earth = 6.4×10^6 m) is

(1) 80 km (2) 16 km (3) 40 km (4) 64 km

50. 1

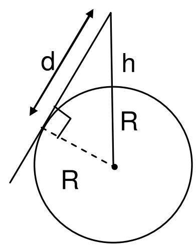
Sol. Maximum distance on earth where object can be detected is d, then

$$(h+R)^2 = d^2 + R^2$$

$\Rightarrow$ $d^2 = h^2 + 2Rh$

since $h << R$, $\Rightarrow$ $d^2 = 2hR$

$\Rightarrow$ $d = \sqrt{2(500)(6.4\times10^6)} = 80$ km



51. Assume that a neutron breaks into a proton and an electron. The energy released during this process is (Mass of neutron = 1.6725×10^{-27} kg; mass of proton = 1.6725×10^{-27} kg; mass of electron = 9×10^{-31} kg)

(1) 0.73 MeV (2) 7.10 MeV (3) 6.30 MeV (4) 5.4 MeV

51. 1

Sol. $\Delta m = (m_p + m_e) - m_n$

$= 9 \times 10^{-31}$ kg.

Energy released = $(9 \times 10^{-31}$ kg$)c^2$ joules

$$= \frac{9\times10^{-31}\times(3\times10^8)^2}{1.6\times10^{-13}}\text{MeV}$$

= 0.73 MeV.

52. A Carnot engine, whose efficiency is 40%, takes in heat from a source maintained at a temperature of 500 K It is desired to have an engine of efficiency 60%. Then, the intake temperature for the same exhaust (sink) temperature must be

(1) efficiency of Carnot engine cannot be made larger than 50%

(2) 1200 K (3) 750 K (4) 600 K

52. 3

Sol. $\frac{40}{100} = \frac{500 - T_S}{500}$, $T_S = 300$ K

$\frac{600}{100} = \frac{T - 300}{T} \Rightarrow T = 750$ K

53. This question has statement 1 and statement 2. Of the four choices given after the statements, choose the one that best describes the two statements.

If two springs S_1 and S_2 of force constants k_1 and k_2, respectively, are stretched by the same force, it is found that more work is done on spring S_1 than on spring S_2.

Statement 1 : If stretched by the same amount, work done on S_1, will be more than that on S_2

Statement 2 : $k_1 < k_2$

(1) Statement 1 is false, Statement 2 is true
(2) Statement 1 is true, Statement 2 is false
(3) Statement 1 is true, Statement 2 is the correct explanation for statement 1
(4) Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation for statement 1.

53. 1

Sol. $F = K_1S_1 = K_2 S_2$
$W_1 = FS_1$, $W_2 = FS_2$
$K_1S_1^2 > K_2S_2^2$
$S_1 > S_2$
$K_1 < K_2$
$W \propto K$
$W_1 < W_2$

54. Two cars of masses m_1 and m_2 are moving in circles of radii r_1 and r_2, respectively. Their speeds are such that they make complete circles in the same time t. The ratio of their centripetal acceleration is

(1) $m_1r_1 : m_2r_2$ (2) $m_1 : m_2$ (3) $r_1 : r_2$ (4) 1 : 1

54. 3

Sol. $a \propto r$

55. A cylindrical tube, open at both ends, has a fundamental frequency, f, in air. The tube is dipped vertically in water so that half of it is in water. The fundamental frequency of the air-column is now

(1) f (2) $\frac{f}{2}$ (3) $\frac{3f}{4}$ (4) 2f

55. 1

Sol. $f_0 = \frac{v}{2\ell}$

$f_C = \frac{v}{2\ell}$

56. An object 2.4 m in front of a lens forms a sharp image on a film 12 cm behind the lens. A glass plate 1cm thick, of refractive index 1.50 is interposed between lens and film with its plane faces parallel to film. At what distance (from lens) should object be shifted to be in sharp focus on film?

(1) 7.2 m (2) 2.4 m (3) 3.2 m (4) 5.6 m

56. 4

Sol. Case I: u = –240cm, v = 12, by Lens formula

$$\frac{1}{f} = \frac{7}{80}$$

Case II: $v = 12 - \frac{1}{3} = \frac{35}{3}$ (normal shift = $1 - \frac{2}{3} = \frac{1}{3}$)

$$f = \frac{7}{80}$$

$$u = 5.6$$

57. A diatomic molecule is made of two masses m_1 and m_2 which are separated by a distance r. If we calculate its rotational energy by applying Bohr's rule of angular momentum quantization, its energy will be given by (n is an integer)

(1) $\frac{(m_1 + m_2)^2 n^2 h^2}{2m_1^2 m_2^2 r^2}$ (2) $\frac{n^2 h^2}{2(m_1 + m_2) r^2}$ (3) $\frac{2n^2 h^2}{(m_1 + m_2) r^2}$ (4) $\frac{(m_1 + m_2) n^2 h^2}{2m_1 m_2 r^2}$

57. 4

Sol. $r_1 = \frac{m_2 r}{m_1 + m_2}$; $r_2 = \frac{m_1 r}{m_1 + m_2}$

$(I_1 + I_2)\omega = \frac{nh}{2\pi} = n\hbar$

$K.E = \frac{1}{2}(I_1 + I_2)\,\omega^2 = \frac{n^2\hbar^2(m_1 + m_2)}{2m_1 m_2 r^2}$

58. A spectrometer gives the following reading when used to measure the angle of a prism.
Main scale reading: 58.5 degree
Vernier scale reading : 09 divisions
Given that 1 division on main scale corresponds to 0.5 degree. Total divisions on the vernier scale is 30 and match with 29 divisions of the main scale. The angle of the prism from the above data
(1) 58.59° (2) 58.77° (3) 58.65° (4) 59°

58. 3

Sol. $L.C = \frac{1}{60}$

Total Reading = $585 + \frac{9}{60} = 58.65$

59. This question has statement 1 and statement 2. Of the four choices given after the statements, choose the one that best describes the two statements.
An insulating solid sphere of radius R has a uniformly positive charge density ρ. As a result of this uniform charge distribution there is a finite value of electric potential at the centre of the sphere, at the surface of the sphere and also at a point out side the sphere. The electric potential at infinity is zero.
Statement 1 : When a charge q is taken from the centre to the surface of the sphere, its potential energy changes by $\frac{qp}{3\varepsilon_0}$

Statement 2 : The electric field at a distance r(r < R) from the centre of the sphere is $\frac{\rho r}{3\varepsilon_0}$

(1) Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation for statement 1.
(2) Statement 1 is true, Statement 2 is false
(3) Statement 1 is false, Statement 2 is true
(4) Statement 1 is true, Statement 2 is the correct explanation for statement 1

59. 3

Sol. $\oint \vec{E}\cdot\vec{d}A = \frac{1}{\varepsilon_0}\left(\rho \times \frac{4}{3}\pi r^3\right)$

$E = \frac{\rho r}{3\varepsilon_0}$

Statement 2 is correct

$\Delta PE = (V_{sur} - V_{cent})q = -\frac{q}{6\varepsilon_0}\rho R^2$

Statement 1 is incorrect

60. Proton, Deuteron and alpha particle of the same kinetic energy are moving in circular trajectories in a constant magnetic field. The radii of proton, deuteron and alpha particle are respectively r_p, r_d and r_α. Which one of the following relations is correct?
(1) $r_\alpha = r_p = r_d$ (2) $r_\alpha = r_p < r_d$ (3) $r_\alpha > r_d > r_p$ (4) $r_\alpha = r_d > r_p$

60. 2

Sol. $r = \frac{\sqrt{2mK}}{Bq}$

$r \propto \frac{\sqrt{m}}{q}$

$r_\alpha = r_p < r_d$

PART C: CHEMISTRY

61. Which among the following will be named as dibromidobis(ethylene diamine)chromium(III) bromide ?

(1) $\left[Cr(en)_3\right]Br_3$ (2) $\left[Cr(en)_2Br_2\right]Br$ (3) $\left[Cr(en)Br_4\right]^-$ (4) $\left[Cr(en)Br_2\right]Br$

61. 2

Sol. $\left[Cr(en)_2Br_2\right]Br$ – dibromido bis (ethylene diamine)chromium(III) bromide

62. Which method of purification is represented by the following equation :

$Ti(s)+2I_2(g)\xrightarrow{523K}TiI_4(g)\xrightarrow{1700K}Ti(s)+2I_2(g)$

(1) zone refining (2) cupellation (3) Poling (4) Van Arkel

62. 4

Sol. Van Arkel method

$Ti(s)+2I_2(g)\xrightarrow{523K}TiI_4(g)$

$TiI_4(g)\xrightarrow{1700\text{ K}}Ti(s)+2I_2(g)$

63. Lithium forms body centred cubic structure. The length of the side of its unit cell is 351 pm. Atomic radius of the lithium will be :

(1) 75 pm (2) 300 pm (3) 240 pm (4) 152 pm

63. 4

Sol. For BCC, $\sqrt{3}a = 4r$

$$r=\frac{\sqrt{3}\times 351}{4}=152pm$$

64. The molecule having smallest bond angle is :

(1) NCl_3 (2) $AsCl_3$ (3) $SbCl_3$ (4) PCl_3

64. 3

Sol. As the size of central atom increases lone pair bond pair repulsions increases so, bond angle decreases

65. Which of the following compounds can be detected by Molisch's test ?

(1) Nitro compounds (2) Sugars (3) Amines (4) Primary alcohols

65. 2

Sol. Molisch's Test : when a drop or two of alcoholic solution of α–naphthol is added to sugar solution and then conc. H_2SO_4 is added along the sides of test tube, formation of violet ring takes place at the junction of two liquids.

66. The incorrect expression among the following is :

(1) $\dfrac{\Delta G_{system}}{\Delta S_{total}}=-T$ (2) In isothermal process $w_{reversible}=-nRT\ln\dfrac{V_f}{V_i}$

(3) $\ln K=\dfrac{\Delta H^0-T\Delta S^0}{RT}$ (4) $K=e^{-\Delta G^0/RT}$

66. 3

Sol. $\Delta G°=-RT\ln K$ and $\Delta G^0=\Delta H^0-T\Delta S^0$

67. The density of a solution prepared by dissolving 120 g of urea (mol. Mass = 60 u) in 1000g of water is 1.15 g/mL. The molarity of this solution is :

(1) 0.50 M (2) 1.78 M (3) 1.02 M (4) 2.05 M

67. 4

Sol. Total weight of solution = 1000 + 120 = 1120 g

Molarity = $\frac{120}{60} \times \frac{1000}{1120/1.15} = 2.05M$

68. The species which can best serve as an initiator for the cationic polymerization is :

(1) $LiAlH_4$ (2) HNO_3 (3) $AlCl_3$ (4) BuLi

68. 3

Sol. lewis acids can initiate the cationic polymerization.

69. Which of the following on thermal decomposition yields a basic as well as an acidic oxide ?

(1) $NaNO_3$ (2) $KClO_3$ (3) $CaCO_3$ (4) NH_4NO_3

69. 3

Sol. $CaCO_3 \rightarrow \underset{\text{Basic}}{CaO} + \underset{\text{Acidic}}{CO_2}$

70. The standard reduction potentials for Zn^{2+}/Zn, Ni^{2+}/Ni, and Fe^{2+}/Fe are –0.76, –0.23 and –0.44 V respectively. The reaction $X + Y^{2+} \rightarrow X^{2+} + Y$ will be spontaneous when :

(1) X = Ni, Y = Fe (2) X = Ni, Y = Zn (3) X = Fe, Y = Zn (4) X = Zn, Y = Ni

70. 4

Sol. $Zn + Fe^{+2} \rightarrow Zn^{+2} + Fe$

$Fe + Ni^{+2} \rightarrow Fe^{2+} + Ni$

$Zn + Ni^{2+} \rightarrow Zn^{+2} + Ni$

All these are spontaneous

71. According to Freundlich adsorption isotherm, which of the following is correct ?

(1) $\frac{x}{m} \propto P^0$ (2) $\frac{x}{m} \propto p^1$ (3) $\frac{x}{m} \propto p^{1/n}$

(4) All the above are correct for different ranges of pressure

71. 4

Sol. $\frac{x}{m} \propto P^0$ is true at extremely high pressures

$\frac{x}{m} \propto p^1$; $\frac{x}{m} \propto p^{1/n}$ are true at low and moderate pressures

72. The equilibrium constant (K_C) for the reaction $N_2(g) + O_2(g) \rightarrow 2NO(g)$ at temperature T is 4 x 10^{-4}. The value of K_C for the reaction, $NO(g) \rightarrow ½ N_2(g) + ½ O_2(g)$ at the same temperature is :

(1) 0.02 (2) 2.5 x 10^2 (3) 4 x 10^{-4} (4) 50.0

72. 4

Sol. $N_2 + O_2 \rightleftharpoons 2NO \qquad K_C = 4 \times 10^{-4}$

$NO \rightleftharpoons \frac{1}{2}N_2 + \frac{1}{2}O_2 \qquad K_C^1 = \sqrt{\frac{1}{K_C}}$

$K_C^1 = \frac{1}{\sqrt{4 \times 10^{-4}}} = 50$

73. The compressibility factor for a real gas at high pressure is :

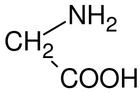
(1) 1 + RT/pb (2) 1 (3) 1 + pb/RT (4) 1–pb/RT

73. 3

Sol. At high pressure $Z = 1 + \frac{Pb}{RT}$

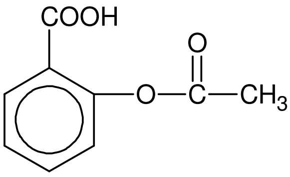
74. Which one of the following statements is correct ?
(1) All amino acids except lysine are optically active
(2) All amino acids are optically active
(3) All amino acids except glycine are optically active
(4) All amino acids except glutamic acid are optically active

74. 3

Sol. Glycine $\longrightarrow$ 

75. Aspirin is known as :
(1) Acetyl salicylic acid (2) Phenyl salicylate
(3) Acetyl salicylate (4) Methyl salicylic acid

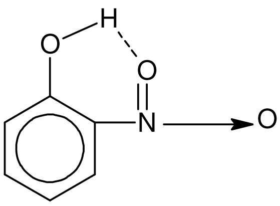
75. 1

Sol. Aspirin 

Acetyl salicylic acid

76. Ortho–Nitrophenol is less soluble in water than p– and m– Nitrophenols because :
(1) o–Nitrophenol is more volatile in steam than those of m – and p–isomers
(2) o–Nitrophenol shows Intramolecular H–bonding
(3) o–Nitrophenol shows Intermolecular H–bonding
(4) Melting point of o–Nitrophenol is lower than those of m–and p–isomers.

76. 2

Sol. 

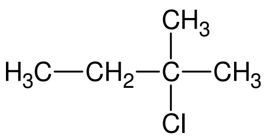
Intramolecular H–bonding decreases water solubility.

77. How many chiral compounds are possible on monochlorination of 2–methyl butane ?
(1) 8 (2) 2 (3) 4 (4) 6

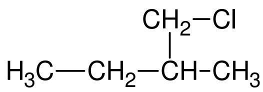
77. 2

Sol. $H_3C-CH_2-CH(CH_3)-CH_3$ on monochlorination gives

$\underset{\substack{(I)\\ \text{Achiral}}}{H_2C(Cl)-CH_2-CH(CH_3)-CH_3}$ $\underset{\substack{(II)\\ \text{Chiral}}}{H_3C-CH(Cl)-CH(CH_3)-CH_3}$



(III)
Achiral



(IV)
chiral

78. Very pure hydrogen (99.9%) can be made by which of the following processes ?
(1) Reaction of methane with steam
(2) Mixing natural hydrocarbons of high molecular weight
(3) Electrolysis of water
(4) Reaction of salt like hydrides with water

78. 3

Sol. Highly pure hydrogen is obtained by the electrolysis of water.

79. The electrons identified by quantum numbers n and l :
(a) n = 4, l = 1 (b) n = 4, l = 0 (c) n = 3, l = 2 (d) n = 3 , l = 1
Can be placed in order of increasing energy as :
(1) (c) < (d) < (b) < (a) (2) (d) < (b) < (c) < (a) (3) (b) < (d) < (a) < (c) (4) (a) < (c) < (b) < (d)

79. 2

Sol. (a) (n + l) = 4 + 1 = 5 (b) (n + l) = 4 + 0 = 4 (c) (n + 1) = 3 + 2 = 5 (d) (n + 1) = 3 + 1 = 4

80. For a first order reaction, (A) $\rightarrow$ products, the concentration of A changes from 0.1 M to 0.025 M in 40 minutes. The rate of reaction when the concentration of A is 0.01 M is :
(1) 1.73×10^{-5} M/ min (2) 3.47×10^{-4} M/min
(3) 3.47×10^{-5} M/min (4) 1.73×10^{-4} M/min

80. 2

Sol. $k = \frac{2.303}{40}\log\frac{0.1}{0.025}$

$k = \frac{0.693}{20}$

For a F.O.R., rate=k[A]; rate = $\frac{0.693}{20}\times 10^{-2} = 3.47\times 10^{-4} M/min.$

81. Iron exhibits + 2 and +3 oxidation states. Which of the following statements about iron is incorrect ?
(1) Ferrous oxide is more basic in nature than the ferric oxide.
(2) Ferrous compounds are relatively more ionic than the corresponding ferric compounds
(3) Ferrous compounds are less volatile than the corresponding ferric compounds.
(4) Ferrous compounds are more easily hydrolysed than the corresponding ferric compounds.

81. 4

Sol. FeO $\rightarrow$ More basic, more ionic, less volatile

82. The pH of a 0.1 molar solution of the acid HQ is 3. The value of the ionization constant, Ka of this acid is :
(1) 3×10^{-1} (2) 1×10^{-3} (3) 1×10^{-5} (4) 1×10^{-7}

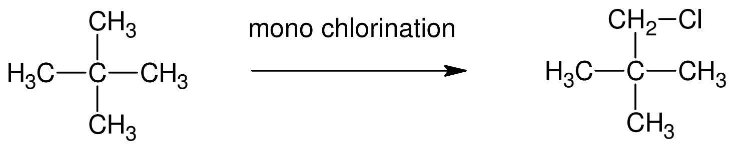
82. 3

Sol. $\left[H^+\right] = \sqrt{K_a.C} \Rightarrow 10^{-3} = \sqrt{K_a.10^{-1}}$

$\Rightarrow K_a = 10^{-5}$

83. Which branched chain isomer of the hydrocarbon with molecular mass 72u gives only one isomer of mono substituted alky halide ?
(1) Tertiary butyl chloride (2) Neopentane
(3) Isohexane (4) Neohexane

83. 2



Neopentane only one compound

Sol. Mol. wt = 72u

84. K_f for water is 1.86K kg mol^{-1}. If your automobile radiator holds 1.0 kg of water, how many grams of ethylene glycol ($C_2H_6O_2$) must you add to get the freezing point of the solution lowered to –2.8°C ?
(1) 72g (2) 93g (3) 39g (4) 27g
84. 2
Sol. $\Delta T_f = K_f.m$

$$2.8 = 1.86 \times \frac{wt}{62} \times \frac{1000}{1000}$$

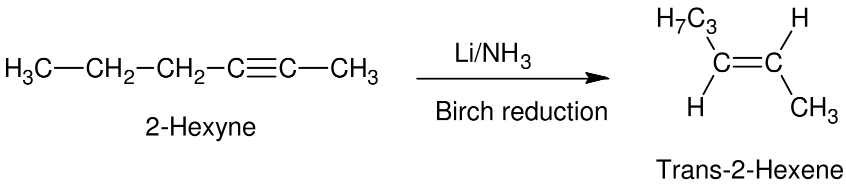
Wt = 93g

85. What is DDT among the following :
(1) Greenhouse gas (2) A fertilizer
(3) Biodegradable pollutant (4) Non–biodegradable pollutant
85. 4
Sol. DDT – non–biodegradable pollutant.

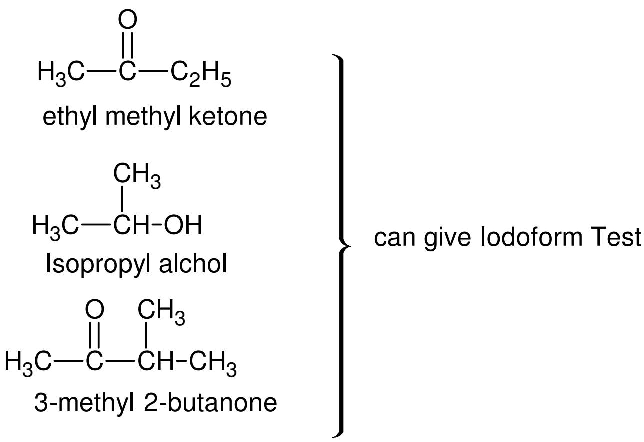
86. The increasing order of the ionic radii of the given isoelectronic species is :
(1) Cl^-, Ca^{2+}, K^+, S^{2-} (2) S^{2-}, Cl^-, Ca^{2+}, K^+ (3) Ca^{2+}, K^+, Cl^-, S^{2-} (4) K^+, S^{2-}, Ca^{2+}, Cl^-
86. 3
Sol. For isoelectronic species, as the z/e decreases, ionic radius increases

87. 2–Hexyne gives trans–2–Hexene on treatment with :
(1) Pt/H_2 (2) Li/NH_3 (3) $Pd/BaSO_4$ (4) $LiAlH_4$
87. 2

Sol.



88. Iodoform can be prepared from all except :
(1) Ethyl methyl ketone (2) Isopropyl alcohol
(3) 3–Methyl – 2– butanone (4) Isobutyl alcohol
88. 4
Sol. Iodoform is given by 1) methyl ketones $R\text{-}CO\text{-}CH_3$
2) alcohols of the type $R\text{-}CH(OH)CH_3$
where R can be hydrogen also



$$H_3C-\underset{}{\overset{CH_3}{\overset{|}{C}}}H-CH_2-OH \longrightarrow \text{can't give}$$

Isobutyl alcohol

89. In which of the following pairs the two species are not isostructural ?

(1) CO_3^{2-} and NO_3^- (2) PCl_4^+ and $SiCl_4$ (3) PF_5 and BrF_5 (4) AlF_6^{3-} and SF_6

89. 3

Sol. (1) CO_3^{2-} & $NO_3^- \rightarrow Sp^2$ hybridized, Trigonal planar

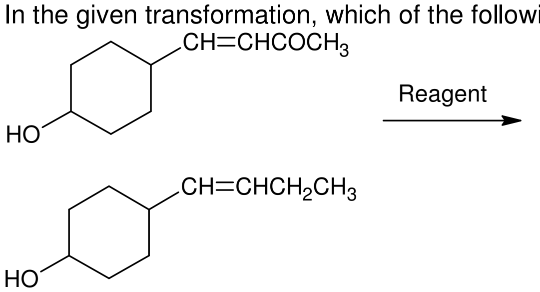
(2) PCl_4^+ & $SiCl_4 \rightarrow Sp^3$ hybridized, Tetrahedral

(3) $PF_5 \rightarrow Sp^3d$ hybridized, Trigonal bipyramidal

$BrF_5 \rightarrow Sp^3d^2$ hybridized, square pyramidal

(4) AlF_6^{3-} & $SF_6 \rightarrow Sp^3d^2$ hybridized, octahedral

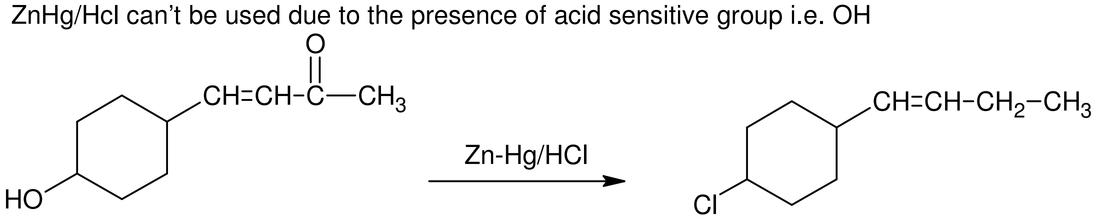
90. In the given transformation, which of the following is the most appropriate reagent ?



(1) $NH_2NH_2, \overset{(-)}{O}H$ (2) $Zn-Hg/HCl$ (3) $Na, Liq. NH_3$ (4) $NaBH_4$

90. 1

Sol. ZnHg/Hcl can't be used due to the presence of acid sensitive group i.e. OH



and Na/Liq. NH_3 and $NaBH_4$ convert – CO – into – CH(OH)–

KEY

PART A: MATHEMATICS

1.	2	2.	3	3.	3	4.	2
5.	1	6.	4	7.	2	8.	4
9.	1	10.	4	11.	1	12.	1
13.	3	14.	4	15.	1	16.	3
17.	3	18.	4	19.	1	20.	3
21.	2	22.	1	23.	3	24.	2 or 4
25.	1	26.	2	27.	4	28.	2
29.	3	30.	2				

PART B: PHYSICS

31.	4	32.	4	33.	3	34.	3
35.	4	36.	4	37.	4	38.	4
39.	1	40.	4	41.	1	42.	3
43.	3	44.	1	45.	4	46.	3
47.	4	48.	1	49.	1	50.	1
51.	1	52.	3	53.	1	54.	3
55.	1	56.	4	57.	4	58.	3
59.	3	60.	2				

PART C: CHEMISTRY

61.	2	62.	4	63.	4	64.	3
65.	2	66.	3	67.	4	68.	3
69.	3	70.	4	71.	4	72.	4
73.	3	74.	3	75.	1	76.	2
77.	2	78.	3	79.	2	80.	2
81.	4	82.	3	83.	2	84.	2
85.	4	86.	3	87.	2	88.	4
89.	3	90.	1				