SOLUTION TO AIEEE-2005

MATHEMATICS

1. If $A^2 - A + I = 0$, then the inverse of A is
(1) $A + I$ (2) A
(3) $A - I$ (4) $I - A$

1. **(4)**
Given $A^2 - A + I = 0$
$A^{-1}A^2 - A^{-1}A + A^{-1} - I = A^{-1}\cdot 0$ (Multiplying A^{-1} on both sides)
$\Rightarrow A - I + A^{-1} = 0$ or $A^{-1} = I - A$.

2. If the cube roots of unity are 1, ω, ω^2 then the roots of the equation $(x - 1)^3 + 8 = 0$, are
(1) $-1, -1 + 2\omega, -1 - 2\omega^2$ (2) $-1, -1, -1$
(3) $-1, 1 - 2\omega, 1 - 2\omega^2$ (4) $-1, 1 + 2\omega, 1 + 2\omega^2$

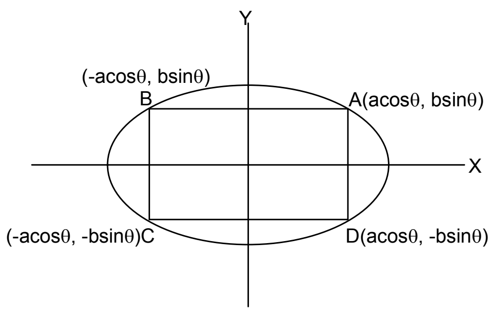
2. **(3)**
$(x - 1)^3 + 8 = 0 \Rightarrow (x - 1) = (-2)(1)^{1/3}$
$\Rightarrow x - 1 = -2$ or -2ω or $-2\omega^2$
or $n = -1$ or $1 - 2\omega$ or $1 - 2\omega^2$.

3. Let R = {(3, 3), (6, 6), (9, 9), (12, 12), (6, 12), (3, 9), (3, 12), (3, 6)} be a relation on the set A = {3, 6, 9, 12} be a relation on the set A = {3, 6, 9, 12}. The relation is
(1) reflexive and transitive only (2) reflexive only
(3) an equivalence relation (4) reflexive and symmetric only

3. **(1)**
Reflexive and transitive only.
e.g. (3, 3), (6, 6), (9, 9), (12, 12) [Reflexive]
(3, 6), (6, 12), (3, 12) [Transitive].

4. Area of the greatest rectangle that can be inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is
(1) 2ab (2) ab
(3) $\sqrt{ab}$ (4) $\frac{a}{b}$

4. **(1)**
Area of rectangle ABCD = $(2a\cos\theta)(2b\sin\theta) = 2ab\sin 2\theta$
$\Rightarrow$ Area of greatest rectangle is equal to 2ab
when $\sin 2\theta = 1$.



5. The differential equation representing the family of curves $y^2 = 2c\left(x + \sqrt{c}\right)$, where c > 0, is a parameter, is of order and degree as follows:
(1) order 1, degree 2 (2) order 1, degree 1

(3) order 1, degree 3 (4) order 2, degree 2

5. **(3)**

$y^2 = 2c(x + \sqrt{c})$ …(i)

$2yy' = 2c\cdot 1$ or $yy' = c$ …(ii)

$\Rightarrow y^2 = 2yy'(x + \sqrt{yy'})$ [on putting value of c from (ii) in (i)]

On simplifying, we get

$(y - 2xy')^2 = 4yy'^3$ …(iii)

Hence equation (iii) is of order 1 and degree 3.

6. $\lim_{n\to\infty}\left[\frac{1}{n^2}\sec^2\frac{1}{n^2} + \frac{2}{n^2}\sec^2\frac{4}{n^2} + ... + \frac{1}{n^2}\sec^2 1\right]$ equals

(1) $\frac{1}{2}\sec 1$ (2) $\frac{1}{2}\operatorname{cosec} 1$

(3) tan1 (4) $\frac{1}{2}\tan 1$

6. **(4)**

$\lim_{n\to\infty}\left[\frac{1}{n^2}\sec^2\frac{1}{n^2} + \frac{2}{n^2}\sec^2\frac{4}{n^2} + \frac{3}{n^2}\sec^2\frac{9}{n^2} + ... + \frac{1}{n}\sec^2 1\right]$ is equal to

$\lim_{n\to\infty}\frac{r}{n^2}\sec^2\frac{r^2}{n^2} = \lim_{n\to\infty}\frac{1}{n}\cdot\frac{r}{n}\sec^2\frac{r^2}{n^2}$

$\Rightarrow$ Given limit is equal to value of integral $\int_0^1 x\sec^2 x^2 dx$

or $\frac{1}{2}\int_0^1 2x\sec x^2 dx = \frac{1}{2}\int_0^1 \sec^2 t\,dt$ [put $x^2 = t$]

$= \frac{1}{2}(\tan t)_0^1 = \frac{1}{2}\tan 1$.

7. ABC is a triangle. Forces $\vec{P}, \vec{Q}, \vec{R}$ acting along IA, IB and IC respectively are in equilibrium, where I is the incentre of ΔABC. Then P : Q : R is

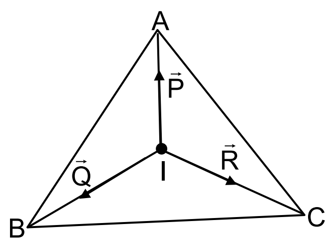
(1) sinA : sin B : sinC (2) $\sin\frac{A}{2} : \sin\frac{B}{2} : \sin\frac{C}{2}$

(3) $\cos\frac{A}{2} : \cos\frac{B}{2} : \cos\frac{C}{2}$ (4) cosA : cosB : cosC

7. **(3)**

Using Lami's Theorem

$\therefore P : Q : R = \cos\frac{A}{2} : \cos\frac{B}{2} : \cos\frac{C}{2}$.



8. If in a frequently distribution, the mean and median are 21 and 22 respectively, then its mode is approximately

(1) 22.0 (2) 20.5

(3) 25.5 (4) 24.0

8. **(4)**

Mode + 2Mean = 3 Median

$\Rightarrow$ Mode = $3 \times 22 - 2 \times 21$= 66 – 42= 24.

9. Let P be the point (1, 0) and Q a point on the locus $y^2 = 8x$. The locus of mid point of PQ is

(1) $y^2 - 4x + 2 = 0$ (2) $y^2 + 4x + 2 = 0$

(3) $x^2 + 4y + 2 = 0$ (4) $x^2 - 4y + 2 = 0$

9. (1)

P = (1, 0)

Q = (h, k) such that $k^2 = 8h$

Let (α, β) be the midpoint of PQ

$\alpha = \frac{h+1}{2}$, $\beta = \frac{k+0}{2}$

$2\alpha - 1 = h$ $2\beta = k$.

$(2\beta)^2 = 8(2\alpha - 1) \Rightarrow \beta^2 = 4\alpha - 2$

$\Rightarrow y^2 - 4x + 2 = 0$.

10. If C is the mid point of AB and P is any point outside AB, then

(1) $\overrightarrow{PA} + \overrightarrow{PB} = 2\overrightarrow{PC}$ (2) $\overrightarrow{PA} + \overrightarrow{PB} = \overrightarrow{PC}$

(3) $\overrightarrow{PA} + \overrightarrow{PB} + 2\overrightarrow{PC} = 0$ (4) $\overrightarrow{PA} + \overrightarrow{PB} + \overrightarrow{PC} = 0$

10. (1)

$\overrightarrow{PA} + \overrightarrow{AC} + \overrightarrow{CP} = 0$

$\overrightarrow{PB} + \overrightarrow{BC} + \overrightarrow{CP} = 0$

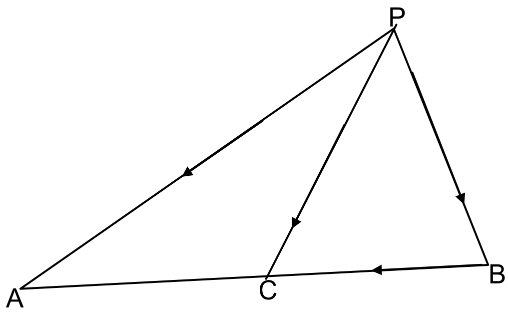
Adding, we get

$\overrightarrow{PA} + \overrightarrow{PB} + \overrightarrow{AC} + \overrightarrow{BC} + 2\overrightarrow{CP} = 0$

Since $\overrightarrow{AC} = -\overrightarrow{BC}$

& $\overrightarrow{CP} = -\overrightarrow{PC}$

$\Rightarrow \overrightarrow{PA} + \overrightarrow{PB} - 2\overrightarrow{PC} = 0$.



11. If the coefficients of rth, (r+ 1)th and (r + 2)th terms in the binomial expansion of $(1 + y)^m$ are in A.P., then m and r satisfy the equation

(1) $m^2 - m(4r - 1) + 4r^2 - 2 = 0$ (2) $m^2 - m(4r+1) + 4r^2 + 2 = 0$

(3) $m^2 - m(4r + 1) + 4r^2 - 2 = 0$ (4) $m^2 - m(4r - 1) + 4r^2 + 2 = 0$

11. (3)

Given ${}^mC_{r-1}$, ${}^mC_r$, ${}^mC_{r+1}$ are in A.P.

$2\,{}^mC_r = {}^mC_{r-1} + {}^mC_{r+1}$

$\Rightarrow 2 = \frac{{}^mC_{r-1}}{{}^mC_r} + \frac{{}^mC_{r+1}}{{}^mC_r}$

$= \frac{r}{m-r+1} + \frac{m-r}{r+1}$

$\Rightarrow m^2 - m(4r + 1) + 4r^2 - 2 = 0$.

12. In a triangle PQR, $\angle R = \frac{\pi}{2}$. If $\tan\left(\frac{P}{2}\right)$ and $\tan\left(\frac{Q}{2}\right)$ are the roots of $ax^2 + bx + c = 0$, $a \neq 0$ then

(1) a = b + c (2) c = a + b

(3) b = c (4) b = a + c

12. (2)

$\tan\left(\frac{P}{2}\right)$, $\tan\left(\frac{Q}{2}\right)$ are the roots of $ax^2 + bx + c = 0$

$\tan\left(\frac{P}{2}\right) + \tan\left(\frac{Q}{2}\right) = -\frac{b}{a}$

$$\tan\left(\frac{P}{2}\right)\tan\left(\frac{Q}{2}\right) = \frac{c}{a}$$

$$\frac{\tan\left(\frac{P}{2}\right) + \tan\left(\frac{Q}{2}\right)}{1 - \tan\left(\frac{P}{2}\right)\tan\left(\frac{Q}{2}\right)} = \tan\left(\frac{P}{2} + \frac{Q}{2}\right) = 1$$

$$\Rightarrow \frac{-\frac{b}{a}}{1 - \frac{c}{a}} = 1 \Rightarrow -\frac{b}{a} = \frac{a}{a} - \frac{c}{a} \Rightarrow -b = a - c$$

c = a + b.

13. The system of equations
$\alpha x + y + z = \alpha - 1$,
$x + \alpha y + z = \alpha - 1$,
$x + y + \alpha z = \alpha - 1$
has no solution, if α is

(1) -2 (2) either – 2 or 1
(3) not -2 (4) 1

13. (1)

$\alpha x + y + z = \alpha - 1$
$x + \alpha y + z = \alpha - 1$
$x + y + z\alpha = \alpha - 1$

$$\Delta = \begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & \alpha \end{vmatrix}$$

$= \alpha(\alpha^2 - 1) - 1(\alpha - 1) + 1(1 - \alpha)$
$= \alpha(\alpha - 1)(\alpha + 1) - 1(\alpha - 1) - 1(\alpha - 1)$
$\Rightarrow (\alpha - 1)[\alpha^2 + \alpha - 1 - 1] = 0$
$\Rightarrow (\alpha - 1)[\alpha^2 + \alpha - 2] = 0$
$[\alpha^2 + 2\alpha - \alpha - 2] = 0$
$(\alpha - 1)[\alpha(\alpha + 2) - 1(\alpha + 2)] = 0$
$(\alpha - 1) = 0, \alpha + 2 = 0 \Rightarrow \alpha = -2, 1;$ but $\alpha \neq 1$.

14. The value of α for which the sum of the squares of the roots of the equation $x^2 - (a - 2)x - a - 1 = 0$ assume the least value is

(1) 1 (2) 0
(3) 3 (4) 2

14. (1)

$x^2 - (a - 2)x - a - 1 = 0$
$\Rightarrow \alpha + \beta = a - 2$
$\alpha\beta = -(a + 1)$
$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= a^2 - 2a + 6 = (a - 1)^2 + 5$
$\Rightarrow a = 1$.

15. If roots of the equation $x^2 - bx + c = 0$ be two consecutive integers, then $b^2 - 4c$ equals

(1) – 2 (2) 3
(3) 2 (4) 1

15. **(4)**

Let $\alpha, \alpha + 1$ be roots

$\alpha + \alpha + 1 = b$

$\alpha(\alpha + 1) = c$

$\therefore b^2 - 4c = (2\alpha + 1)^2 - 4\alpha(\alpha + 1) = 1.$

16. If the letters of word SACHIN are arranged in all possible ways and these words are written out as in dictionary, then the word SACHIN appears at serial number

(1) 601 (2) 600

(3) 603 (4) 602

16. **(1)**

Alphabetical order is

A, C, H, I, N, S

No. of words starting with A – 5!

No. of words starting with C – 5!

No. of words starting with H – 5!

No. of words starting with I – 5!

No. of words starting with N – 5!

SACHIN – 1

601.

17. The value of $^{50}C_4 + \sum_{r=1}^{6} {}^{56-r}C_3$ is

(1) $^{55}C_4$ (2) $^{55}C_3$

(3) $^{56}C_3$ (4) $^{56}C_4$

17. **(4)**

$^{50}C_4 + \sum_{r=1}^{6} {}^{56-r}C_3$

$\Rightarrow {}^{50}C_4 + \left[{}^{55}C_3 + {}^{54}C_3 + {}^{53}C_3 + {}^{52}C_3 + {}^{51}C_3 + {}^{50}C_3\right]$

$= \left({}^{50}C_4 + {}^{50}C_3\right) + {}^{51}C_3 + {}^{52}C_3 + {}^{53}C_3 + {}^{54}C_3 + {}^{55}C_3$

$\Rightarrow \left({}^{51}C_4 + {}^{51}C_3\right) + {}^{52}C_3 + {}^{53}C_3 + {}^{54}C_3 + {}^{55}C_3$

$\Rightarrow {}^{55}C_4 + {}^{55}C_3 = {}^{56}C_4.$

18. If A = $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and I = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then which one of the following holds for all $n \geq 1$, by the principle of mathematical indunction

(1) $A^n = nA - (n - 1)I$ (2) $A^n = 2^{n-1}A - (n - 1)I$

(3) $A^n = nA + (n - 1)I$ (4) $A^n = 2^{n-1}A + (n - 1)I$

18. **(1)**

By the principle of mathematical induction (1) is true.

19. If the coefficient of x^7 in $\left[ax^2 + \left(\frac{1}{bx}\right)\right]^{11}$ equals the coefficient of x^{-7} in $\left[ax^2 - \left(\frac{1}{bx}\right)\right]^{11}$, then a and b satisfy the relation

(1) $a - b = 1$ (2) $a + b = 1$

(3) $\frac{a}{b} = 1$ (4) $ab = 1$

19. **(4)**

T_{r+1} in the expansion $\left[ax^2 + \frac{1}{bx}\right]^{11} = {}^{11}C_r \left(ax^2\right)^{11-r} \left(\frac{1}{bx}\right)^r$

$= {}^{11}C_r\ (a)^{11-r}\ (b)^{-r}\ (x)^{22-2r-r}$

$\Rightarrow 22 - 3r = 7 \quad \Rightarrow r = 5$

$\therefore$ coefficient of $x^7 = {}^{11}C_5(a)^6\ (b)^{-5}$ ……(1)

Again T_{r+1} in the expansion $\left[ax - \frac{1}{bx^2}\right]^{11} = {}^{11}C_r \left(ax\right)^{11-r} \left(-\frac{1}{bx^2}\right)^r$

$= {}^{11}C_r\ a^{11-r}\ (-1)^r \times (b)^{-r}\ (x)^{-2r}\ (x)^{11-r}$

Now $11 - 3r = -7 \quad \Rightarrow 3r = 18 \quad \Rightarrow \quad r = 6$

$\therefore$ coefficient of $x^{-7} = {}^{11}C_6\ a^5 \times 1 \times (b)^{-6}$

$\Rightarrow\ {}^{11}C_5 \left(a\right)^6 \left(b\right)^{-5} = {}^{11}C_6 a^5 \times \left(b\right)^{-6}$

$\Rightarrow ab = 1.$

20. Let $f : (-1, 1) \to B$, be a function defined by $f(x) = \tan^{-1}\frac{2x}{1-x^2}$, then f is both one-one and onto when B is the interval

(1) $\left(0, \frac{\pi}{2}\right)$ (2) $\left[0, \frac{\pi}{2}\right)$

(3) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (4) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

20. (4)

Given $f(x) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$ for $x \in (-1, 1)$

clearly range of $f(x) = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\therefore$ co-domain of function = B = $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

21. If z_1 and z_2 are two non-zero complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$ then $\arg z_1 - \arg z_2$ is equal to

(1) $\frac{\pi}{2}$ (2) $-\pi$

(3) 0 (4) $-\frac{\pi}{2}$

21. (3)

$|z_1 + z_2| = |z_1| + |z_2| \quad \Rightarrow z_1$ and z_2 are collinear and are to the same side of origin; hence $\arg z_1 - \arg z_2 = 0$.

22. If $\omega = \frac{z}{z - \frac{1}{3}i}$ and $|\omega| = 1$, then z lies on

(1) an ellipse (2) a circle

(3) a straight line (4) a parabola.

22. (3)

As given $w = \dfrac{z}{z - \frac{1}{3}i}$ $\Rightarrow |w| = \dfrac{|z|}{|z - \frac{1}{3}i|} = 1$ $\Rightarrow$ distance of z from origin and point $\left(0, \dfrac{1}{3}\right)$ is same hence z lies on bisector of the line joining points (0, 0) and (0, 1/3).
Hence z lies on a straight line.

23. If $a^2 + b^2 + c^2 = -2$ and $f(x) = \begin{vmatrix} 1+a^2x & (1+b^2)x & (1+c^2)x \\ (1+a^2)x & 1+b^2x & (1+c^2)x \\ (1+a^2)x & (1+b^2)x & 1+c^2x \end{vmatrix}$ then f(x) is a polynomial of degree

(1) 1 (2) 0
(3) 3 (4) 2

23. **(4)**

$$f(x) = \begin{vmatrix} 1+(a^2+b^2+c^2+2)x & (1+b^2)x & (1+c^2)x \\ 1+(a^2+b^2+c^2+2)x & 1+b^2x & (1+c^2)x \\ 1+(a^2+b^2+c^2+2)x & (1+b^2)x & 1+c^2x \end{vmatrix}$$, Applying $C_1 \to C_1 + C_2 + C_3$

$$= \begin{vmatrix} 1 & (1+b^2)x & (1+c^2)x \\ 1 & 1+b^2x & (1+c^2)x \\ 1 & (1+b^2)x & 1+c^2x \end{vmatrix} \quad \because\ a^2+b^2+c^2+2=0$$

$$f(x) = \begin{vmatrix} 0 & x-1 & 0 \\ 0 & 1-x & x-1 \\ 1 & (1+b^2)x & 1+c^2x \end{vmatrix}$$; Applying $R_1 \to R_1 - R_2$, $R_2 \to R_2 - R_3$

$f(x) = (x-1)^2$
Hence degree = 2.

24. The normal to the curve $x = a(\cos\theta + \theta\sin\theta)$, $y = a(\sin\theta - \theta\cos\theta)$ at any point 'θ' is such that

(1) it passes through the origin
(2) it makes angle $\dfrac{\pi}{2} + \theta$ with the x-axis
(3) it passes through $\left(a\dfrac{\pi}{2}, -a\right)$
(4) it is at a constant distance from the origin

24. **(4)**

Clearly $\dfrac{dy}{dx} = \tan\theta$ $\Rightarrow$ slope of normal = $-\cot\theta$
Equation of normal at 'θ' is
$y - a(\sin\theta - \theta\cos\theta) = -\cot\theta(x - a(\cos\theta + \theta\sin\theta)$
$\Rightarrow y\sin\theta - a\sin^2\theta + a\theta\cos\theta\sin\theta = -x\cos\theta + a\cos^2\theta + a\theta\sin\theta\cos\theta$
$\Rightarrow x\cos\theta + y\sin\theta = a$
Clearly this is an equation of straight line which is at a constant distance 'a' from origin.

25. A function is matched below against an interval where it is supposed to be increasing. Which of the following pairs is incorrectly matched?

Interval	Function
(1) $(-\infty, \infty)$	$x^3 - 3x^2 + 3x + 3$
(2) $[2, \infty)$	$2x^3 - 3x^2 - 12x + 6$
(3) $\left(-\infty, \frac{1}{3}\right]$	$3x^2 - 2x + 1$
(4) $(-\infty, -4]$	$x^3 + 6x^2 + 6$

25. (3)

Clearly function $f(x) = 3x^2 - 2x + 1$ is increasing when

$f'(x) = 6x - 2 \geq 0 \Rightarrow x \in [1/3, \infty)$

Hence (3) is incorrect.

26. Let α and β be the distinct roots of $ax^2 + bx + c = 0$, then $\lim_{x\to\alpha} \frac{1-\cos\left(ax^2+bx+c\right)}{\left(x-\alpha\right)^2}$ is equal to

(1) $\frac{a^2}{2}\left(\alpha-\beta\right)^2$ (2) 0

(3) $-\frac{a^2}{2}\left(\alpha-\beta\right)^2$ (4) $\frac{1}{2}\left(\alpha-\beta\right)^2$

26. (1)

Given limit = $\lim_{x\to\alpha} \frac{1-\cos a\left(x-\alpha\right)\left(x-\beta\right)}{\left(x-\alpha\right)^2} = \lim_{x\to\alpha} \frac{2\sin^2\left(a\frac{\left(x-\alpha\right)\left(x-\beta\right)}{2}\right)}{\left(x-\alpha\right)^2}$

$$= \lim_{x\to\alpha} \frac{2}{\left(x-\alpha\right)^2} \times \frac{\sin^2\left(a\frac{\left(x-\alpha\right)\left(x-\beta\right)}{2}\right)}{\frac{a^2\left(x-\alpha\right)^2\left(x-\beta\right)^2}{4}} \times \frac{a^2\left(x-\alpha\right)^2\left(x-\beta\right)^2}{4}$$

$= \frac{a^2\left(\alpha-\beta\right)^2}{2}$.

27. Suppose f(x) is differentiable x = 1 and $\lim_{h\to 0}\frac{1}{h}f\left(1+h\right) = 5$, then $f'(1)$ equals

(1) 3 (2) 4

(3) 5 (4) 6

27. (3)

$f'\left(1\right) = \lim_{h\to 0}\frac{f\left(1+h\right)-f\left(1\right)}{h}$; As function is differentiable so it is continuous as it is given that $\lim_{h\to 0}\frac{f\left(1+h\right)}{h} = 5$ and hence f(1) = 0

Hence $f'(1) = \lim_{h\to 0}\frac{f\left(1+h\right)}{h} = 5$

Hence (3) is the correct answer.

28. Let f be differentiable for all x. If f(1) = - 2 and $f'(x) \geq 2$ for $x \in [1, 6]$, then

(1) $f(6) \geq 8$ (2) $f(6) < 8$

(3) $f(6) < 5$ (4) $f(6) = 5$

28. (1)

As f(1) = - 2 & $f'(x) \ge 2 \ \forall \ x \in [1, 6]$

Applying Lagrange's mean value theorem

$$\frac{f(6)-f(1)}{5} = f'(c) \ge 2$$

$\Rightarrow f(6) \ge 10 + f(1)$

$\Rightarrow f(6) \ge 10 - 2$

$\Rightarrow f(6) \ge 8$.

29. If f is a real-valued differentiable function satisfying $|f(x) - f(y)| \le (x - y)^2$, x, y $\in$ R and f(0) = 0, then f(1) equals

(1) -1 (2) 0

(3) 2 (4) 1

29. (2)

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$|f'(x)| = \lim_{h\to 0}\left|\frac{f(x+h)-f(x)}{h}\right| \le \lim_{h\to 0}\left|\frac{(h)^2}{h}\right|$$

$\Rightarrow |f'(x)| \le 0 \quad \Rightarrow f'(x) = 0 \quad \Rightarrow f(x) =$ constant

As f(0) = 0 $\quad\Rightarrow$ f(1) = 0.

30. If x is so small that x^3 and higher powers of x may be neglected, then $\dfrac{(1+x)^{3/2}-\left(1+\frac{1}{2}x\right)^3}{(1-x)^{1/2}}$ may be approximated as

(1) $1-\frac{3}{8}x^2$ (2) $3x+\frac{3}{8}x^2$

(3) $-\frac{3}{8}x^2$ (4) $\frac{x}{2}-\frac{3}{8}x^2$

30. (3)

$$(1-x)^{1/2}\left[1+\frac{3}{2}x+\frac{3}{2}\left(\frac{3}{2}-1\right)x^2-1-3\left(\frac{1}{2}x\right)-3(2)\left(\frac{1}{2}x\right)^2\right]$$

$$= (1-x)^{1/2}\left[-\frac{3}{8}x^2\right] = -\frac{3}{8}x^2.$$

31. If $x = \sum_{n=0}^{\infty}a^n,\ y=\sum_{n=0}^{\infty}b^n,\ z=\sum_{n=0}^{\infty}c^n$ where a, b, c are in A.P. and |a| < 1, |b|<1, |c|< 1, then x, y, z are in

(1) G.P. (2) A.P.

(3) Arithmetic – Geometric Progression (4) H.P.

31. (4)

$x = \sum_{n=0}^{\infty}a^n = \frac{1}{1-a}$ $\qquad$ $a = 1-\frac{1}{x}$

$y = \sum_{n=0}^{\infty}b^n = \frac{1}{1-b}$ $\qquad$ $b = 1-\frac{1}{y}$

$z = \sum_{n=0}^{\infty} c^n = \frac{1}{1-c}$ $\qquad$ $c = 1 - \frac{1}{z}$

a, b, c are in A.P.

2b = a + c

$2\left(1-\frac{1}{y}\right) = 1 - \frac{1}{x} + 1 - \frac{1}{y}$

$\frac{2}{y} = \frac{1}{x} + \frac{1}{z}$

$\Rightarrow$ x, y, z are in H.P.

32. In a triangle ABC, let $\angle C = \frac{\pi}{2}$. If r is the inradius and R is the circumradius of the the triangle ABC, then 2 (r + R) equals

(1) b + c $\qquad$ (2) a + b

(3) a + b + c $\qquad$ (4) c + a

32. (2)

$2r + 2R = c + \frac{2ab}{(a+b+c)} = \frac{(a+b)^2 + c(a+b)}{(a+b+c)} = a + b$ $\qquad$ (since $c^2 = a^2 + b^2$).

33. If $\cos^{-1} x - \cos^{-1} \frac{y}{2} = \alpha$, then $4x^2 - 4xy \cos \alpha + y^2$ is equal to

(1) $2 \sin 2\alpha$ $\qquad$ (2) 4

(3) $4 \sin^2 \alpha$ $\qquad$ (4) $-4 \sin^2 \alpha$

33. (3)

$\cos^{-1}x - \cos^{-1}\frac{y}{2} = \alpha$

$\cos^{-1}\left(\frac{xy}{2} + \sqrt{(1-x^2)\left(1-\frac{y^2}{4}\right)}\right) = \alpha$

$\cos^{-1}\left(\frac{xy + \sqrt{4 - y^2 - 4x^2 + x^2y^2}}{2}\right) = \alpha$

$\Rightarrow 4 - y^2 - 4x^2 + x^2y^2 = 4\cos^2\alpha + x^2y^2 - 4xy\cos\alpha$

$\Rightarrow 4x^2 + y^2 - 4xy\cos\alpha = 4\sin^2\alpha$.

34. If in a triangle ABC, the altitudes from the vertices A, B, C on opposite sides are in H.P., then sin A, sin B, sin C are in

(1) G.P. $\qquad$ (2) A.P.

(3) Arithmetic – Geometric Progression $\qquad$ (4) H.P.

34. (2)

$\Delta = \frac{1}{2}p_1a = \frac{1}{2}p_2b = \frac{1}{2}p_3b$

p_1, p_2, p_3 are in H.P.

$\Rightarrow \frac{2\Delta}{a}, \frac{2\Delta}{b}, \frac{2\Delta}{c}$ are in H.P.

$\Rightarrow \frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in H.P

$\Rightarrow$ a, b, c are in A.P.

$\Rightarrow$ sinA, sinB, sinC are in A.P.

35. If $I_1 = \int_0^1 2^{x^2} dx$, $I_2 = \int_0^1 2^{x^3} dx$, $I_3 = \int_1^2 2^{x^2} dx$ and $I_4 = \int_1^2 2^{x^3} dx$ then

(1) $I_2 > I_1$ (2) $I_1 > I_2$

(3) $I_3 = I_4$ (4) $I_3 > I_4$

35. (2)

$I_1 = \int_0^1 2^{x^2} dx$, $I_2 = \int_0^1 2^{x^3} dx$, $I_3 = \int_0^1 2^{x^2} dx$, $I_4 = \int_0^1 2^{x^3} dx$

$\forall\ 0 < x < 1,\ x^2 > x^3$

$\Rightarrow \int_0^1 2^{x^2} dx > \int_0^1 2^{x^3} dx$

$\Rightarrow I_1 > I_2$.

36. The area enclosed between the curve $y = \log_e (x + e)$ and the coordinate axes is

(1) 1 (2) 2

(3) 3 (4) 4

36. (1)

Required area (OAB) = $\int_{1-e}^{0} \ln(x+e) dx$

$= \left[x\ln(x+e) - \int \frac{1}{x+e} x\,dx \right]_0^1 = 1.$

37. The parabolas $y^2 = 4x$ and $x^2 = 4y$ divide the square region bounded by the lines $x = 4$, $y = 4$ and the coordinate axes. If S_1, S_2, S_3 are respectively the areas of these parts numbered from top to bottom; then $S_1 : S_2 : S_3$ is

(1) 1 : 2 : 1 (2) 1 : 2 : 3

(3) 2 : 1 : 2 (4) 1 : 1 : 1

37. (4)

$y^2 = 4x$ and $x^2 = 4y$ are symmetric about line $y = x$

$\Rightarrow$ area bounded between $y^2 = 4x$ and $y = x$ is $\int_0^4 \left(2\sqrt{x} - x\right) dx = \frac{8}{3}$

$\Rightarrow A_{s_2} = \frac{16}{3}$ and $A_{s_1} = A_{s_3} = \frac{16}{3}$

$\Rightarrow A_{s_1} : A_{s_2} : A_{s_3} :: 1 : 1 : 1.$

38. If $x\frac{dy}{dx} = y (\log y - \log x + 1)$, then the solution of the equation is

(1) $y \log\left(\frac{x}{y}\right) = cx$ (2) $x \log\left(\frac{y}{x}\right) = cy$

(3) $\log\left(\frac{y}{x}\right) = cx$ (4) $\log\left(\frac{x}{y}\right) = cy$

38. (3)

$\frac{x\,dy}{dx} = y (\log y - \log x + 1)$

$\frac{dy}{dx} = \frac{y}{x}\left(\log\left(\frac{y}{x}\right) + 1\right)$

Put $y = vx$

$$\frac{dy}{dx} = v + \frac{x\,dv}{dx}$$

$$\Rightarrow\ v + \frac{xdv}{dx} = v(\log v + 1)$$

$$\frac{xdv}{dx} = v\log v$$

$$\Rightarrow\ \frac{dv}{v\log v} = \frac{dx}{x}$$

put log v = z

$$\frac{1}{v}dv = dz$$

$$\Rightarrow\ \frac{dz}{z} = \frac{dx}{x}$$

ln z = ln x + ln c

z = cx

log v = cx

$$\log\left(\frac{y}{x}\right) = cx.$$

39. The line parallel to the x–axis and passing through the intersection of the lines ax + 2by + 3b = 0 and bx – 2ay – 3a = 0, where (a, b) $\neq$ (0, 0) is

(1) below the x–axis at a distance of $\frac{3}{2}$ from it

(2) below the x–axis at a distance of $\frac{2}{3}$ from it

(3) above the x–axis at a distance of $\frac{3}{2}$ from it

(4) above the x–axis at a distance of $\frac{2}{3}$ from it

39. (1)

ax + 2by + 3b + λ(bx – 2ay – 3a) = 0

$\Rightarrow$ (a + bλ)x + (2b – 2aλ)y + 3b - 3λa = 0

a + bλ = 0 $\Rightarrow$ λ = -a/b

$\Rightarrow$ ax + 2by + 3b - $\frac{a}{b}$ (bx – 2ay – 3a) = 0

$$\Rightarrow \text{ax + 2by + 3b – ax + } \frac{2a^2}{b}y + \frac{3a^2}{b} = 0$$

$$y\left(2b + \frac{2a^2}{b}\right) + 3b + \frac{3a^2}{b} = 0$$

$$y\left(\frac{2b^2 + 2a^2}{b}\right) = -\left(\frac{3b^2 + 3a^2}{b}\right)$$

$$y = \frac{-3(a^2 + b^2)}{2(b^2 + a^2)} = \frac{-3}{2}$$

$y = -\frac{3}{2}$ so it is 3/2 units below x-axis.

40. A spherical iron ball 10 cm in radius is coated with a layer of ice of uniform thickness than melts at a rate of 50 cm^3/min. When the thickness of ice is 5 cm, then the rate at which the thickness of ice decreases, is

(1) $\frac{1}{36\pi}$ cm/min (2) $\frac{1}{18\pi}$ cm/min

(3) $\frac{1}{54\pi}$ cm/min (4) $\frac{5}{6\pi}$ cm/min

40. (2)

$$\frac{dv}{dt} = 50$$

$$4\pi r^2 \frac{dr}{dt} = 50$$

$$\Rightarrow \frac{dr}{dt} = \frac{50}{4\pi(15)^2} \quad \text{where } r = 15$$

$$= \frac{1}{16\pi}.$$

41. $\int \left\{ \frac{(\log x - 1)}{(1 + (\log x)^2} \right\}^2 dx$ is equal to

(1) $\frac{\log x}{(\log x)^2 + 1} + C$ (2) $\frac{x}{x^2 + 1} + C$

(3) $\frac{xe^x}{1 + x^2} + C$ (4) $\frac{x}{(\log x)^2 + 1} + C$

41. (4)

$$\int \frac{(\log x - 1)^2}{\left(1 + (\log x)^2\right)^2} dx$$

$$= \int \left[\frac{1}{\left(1 + (\log x)^2\right)} - \frac{2 \log x}{\left(1 + (\log x)^2\right)^2} \right] dx$$

$$= \int \left[\frac{e^t}{1 + t^2} - \frac{2t\ e^t}{\left(1 + t^2\right)^2} \right] dt \quad \text{put } \log x = t \Rightarrow dx = e^t\, dt$$

$$\int e^t \left[\frac{1}{1 + t^2} - \frac{2t}{\left(1 + t^2\right)^2} \right] dt$$

$$= \frac{e^t}{1 + t^2} + c = \frac{x}{1 + (\log x)^2} + c$$

42. Let f : R → R be a differentiable function having f (2) = 6, f′ (2) = $\left(\frac{1}{48}\right)$. Then $\lim_{x \to 2} \int_6^{f(x)} \frac{4t^3}{x - 2} dt$ equals

(1) 24 (2) 36

(3) 12 (4) 18

42. **(4)**

$$\lim_{x\to 2}\int_0^{f(x)}\frac{4t^3}{x-2}dt$$

Applying L Hospital rule

$\lim_{x\to 2}\left[4f(x)^2\ f'(x)\right]$ = 4f(2)^3 f′(2)

= $4 \times 6^3 \times \frac{1}{48}$ = 18.

43. Let f (x) be a non–negative continuous function such that the area bounded by the curve y = f (x), x–axis and the ordinates x = $\frac{\pi}{4}$ and x = β > $\frac{\pi}{4}$ is $\left(\beta\sin\beta+\frac{\pi}{4}\cos\beta+\sqrt{2}\beta\right)$. Then $f\left(\frac{\pi}{2}\right)$ is

(1) $\left(\frac{\pi}{4}+\sqrt{2}-1\right)$ (2) $\left(\frac{\pi}{4}-\sqrt{2}+1\right)$

(3) $\left(1-\frac{\pi}{4}-\sqrt{2}\right)$ (4) $\left(1-\frac{\pi}{4}+\sqrt{2}\right)$

43. **(4)**

Given that $\int_{\pi/4}^{\beta} f(x)dx = \beta\sin\beta+\frac{\pi}{4}\cos\beta+\sqrt{2}\,\beta$

Differentiating w. r. t β

f(β) = β cosβ + sinβ - $\frac{\pi}{4}$ sinβ + $\sqrt{2}$

$f\left(\frac{\pi}{2}\right)=\left(1-\frac{\pi}{4}\right)\sin\frac{\pi}{2}+\sqrt{2}=1-\frac{\pi}{4}+\sqrt{2}$.

44. The locus of a point P (α, β) moving under the condition that the line y = αx + β is a tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is

(1) an ellipse (2) a circle

(3) a parabola (4) a hyperbola

44. **(4)**

Tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is

y = mx $\pm\sqrt{a^2m^2-b^2}$

Given that y = αx + β is the tangent of hyperbola

$\Rightarrow$ m = α and $a^2m^2 - b^2 = \beta^2$

$\therefore$ $a^2\alpha^2 - b^2 = \beta^2$

Locus is $a^2x^2 - y^2 = b^2$ which is hyperbola.

45. If the angle θ between the line $\frac{x+1}{1}=\frac{y-1}{2}=\frac{z-2}{2}$ and the plane 2x − y + $\sqrt{\lambda}$ z + 4 = 0 is such that sin θ = $\frac{1}{3}$ the value of λ is

(1) $\frac{5}{3}$ (2) $\frac{-3}{5}$

(3) $\frac{3}{4}$ (4) $\frac{-4}{3}$

45. **(1)**

Angle between line and normal to plane is

$\cos\left(\frac{\pi}{2}-\theta\right)=\frac{2-2+2\sqrt{\lambda}}{3\times\sqrt{5+\lambda}}$ where θ is angle between line & plane

$\Rightarrow \sin\theta = \frac{2\sqrt{\lambda}}{3\sqrt{5+\lambda}}=\frac{1}{3}$

$\Rightarrow \lambda = \frac{5}{3}$.

46. The angle between the lines 2x = 3y = – z and 6x = – y = – 4z is

(1) 0^0 (2) 90^0

(3) 45^0 (4) 30^0

46. **(2)**

Angle between the lines 2x = 3y = - z & 6x = -y = -4z is 90°

Since $a_1a_2 + b_1b_2 + c_1c_2 = 0$.

47. If the plane 2ax – 3ay + 4az + 6 = 0 passes through the midpoint of the line joining the centres of the spheres

$x^2 + y^2 + z^2 + 6x - 8y - 2z = 13$ and

$x^2 + y^2 + z^2 - 10x + 4y - 2z = 8$, then a equals

(1) – 1 (2) 1

(3) – 2 (4) 2

47. **(3)**

Plane

2ax – 3ay + 4az + 6 = 0 passes through the mid point of the centre of spheres

$x^2 + y^2 + z^2 + 6x - 8y - 2z = 13$ and $x^2 + y^2 + z^2 - 10x + 4y - 2z = 8$ respectively

centre of spheres are (-3, 4, 1) & (5, - 2, 1)

Mid point of centre is (1, 1, 1)

Satisfying this in the equation of plane, we get

2a – 3a + 4a + 6 = 0 $\Rightarrow$ a = -2.

48. The distance between the line $\vec{r}=2\hat{i}-2\hat{j}+3\hat{k}+\lambda(\hat{i}-\hat{j}+4\hat{k})$ and the plane $\vec{r}\cdot(\hat{i}+5\hat{j}+\hat{k})=5$ is

(1) $\frac{10}{9}$ (2) $\frac{10}{3\sqrt{3}}$

(3) $\frac{3}{10}$ (4) $\frac{10}{3}$

48. **(2)**

Distance between the line

$\vec{r}=2\hat{i}-2\hat{j}+3\hat{k}+\lambda\left(\hat{i}-\hat{j}+4\hat{k}\right)$ and the plane $\vec{r}\cdot\left(\hat{i}+5\hat{j}+\hat{k}\right)$ = 5 is

equation of plane is x + 5y + z = 5

$\therefore$ Distance of line from this plane

= perpendicular distance of point (2, -2, 3) from the plane

i.e. $\left|\frac{2-10+3-5}{\sqrt{1+5^2+1}}\right|=\frac{10}{3\sqrt{3}}$.

49. For any vector $\vec{a}$, the value of $(\vec{a}\times\hat{i})^2+(\vec{a}\times\hat{j})^2+(\vec{a}\times\hat{k})^2$ is equal to

(1) $3\vec{a}^2$ (2) $\vec{a}^2$

(3) $2\vec{a}^2$ (4) $4\vec{a}^2$

49. (3)

Let $\vec{a}=x\hat{i}+y\hat{j}+z\hat{k}$

$\vec{a}\times\hat{i}=z\hat{j}-y\hat{k}$

$\Rightarrow\left(\vec{a}\times\hat{i}\right)^2=y^2+z^2$

similarly $\left(\vec{a}\times\hat{j}\right)^2=x^2+z^2$

and $\left(\vec{a}\times\hat{k}\right)^2=x^2+y^2 \Rightarrow \left(\vec{a}\times\hat{i}\right)^2=y^2+z^2$

similarly $\left(\vec{a}\times\hat{j}\right)^2=x^2+z^2$

and $\left(\vec{a}\times\hat{k}\right)^2=x^2+y^2$

$\Rightarrow\left(\vec{a}\times\hat{i}\right)^2+\left(\vec{a}\times\hat{j}\right)^2+\left(\vec{a}\times\hat{k}\right)^2=2\left(x^2+y^2+z^2\right) = 2\vec{a}^2$.

50. If non-zero numbers a, b, c are in H.P., then the straight line $\frac{x}{a}+\frac{y}{b}+\frac{1}{c}=0$ always passes through a fixed point. That point is

(1) (-1, 2) (2) (-1, -2)

(3) (1, -2) (4) $\left(1, -\frac{1}{2}\right)$

50. (3)

a, b, c are in H.P.

$\Rightarrow \frac{2}{b}-\frac{1}{a}-\frac{1}{c}=0$

$\frac{x}{a}+\frac{y}{b}+\frac{1}{c}=0$

$\Rightarrow \frac{x}{-1}=\frac{y}{2}=\frac{1}{-1}$ $\therefore$ x = 1, y = -2

51. If a vertex of a triangle is (1, 1) and the mid-points of two sides through this vertex are (-1, 2) and (3, 2), then the centroid of the triangle is

(1) $\left(-1, \frac{7}{3}\right)$ (2) $\left(\frac{-1}{3}, \frac{7}{3}\right)$

(3) $\left(1, \frac{7}{3}\right)$ (4) $\left(\frac{1}{3}, \frac{7}{3}\right)$

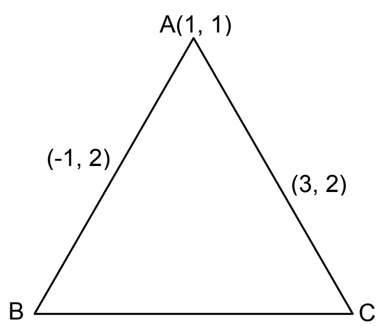
51. (3)

Vertex of triangle is (1, 1) and midpoint of sides through this vertex is (-1, 2) and (3, 2)

$\Rightarrow$ vertex B and C come out to be (-3, 3) and (5, 3)

$\therefore$ centroid is $\frac{1-3+5}{3}, \frac{1+3+3}{3}$

$\Rightarrow$ (1, 7/3)

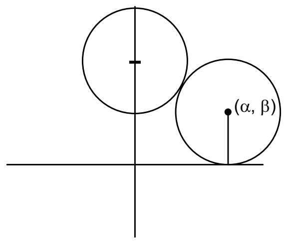


52. If the circles $x^2 + y^2 + 2ax + cy + a = 0$ and $x^2 + y^2 - 3ax + dy - 1 = 0$ intersect in two distinct points P and Q then the line $5x + by - a = 0$ passes through P and Q for
(1) exactly one value of a
(2) no value of a
(3) infinitely many values of a
(4) exactly two values of a

52. (2)
$S_1 = x^2 + y^2 + 2ax + cy + a = 0$
$S_2 = x^2 + y^2 - 3ax + dy - 1 = 0$
Equation of radical axis of S_1 and S_2
$S_1 - S_2 = 0$
$\Rightarrow 5ax + (c - d)y + a + 1 = 0$
Given that $5x + by - a = 0$ passes through P and Q
$\Rightarrow \frac{a}{1} = \frac{c-d}{b} = \frac{a+1}{-a}$
$\Rightarrow a + 1 = -a^2$
$a^2 + a + 1 = 0$
No real value of a.

53. A circle touches the x-axis and also touches the circle with centre at (0, 3) and radius 2. The locus of the centre of the circle is
(1) an ellipse
(2) a circle
(3) a hyperbola
(4) a parabola

53. (4)
Equation of circle with centre (0, 3) and radius 2 is
$x^2 + (y - 3)^2 = 4$.
Let locus of the variable circle is (α, β)
$\because$ It touches x-axis.
$\therefore$ It equation $(x - \alpha)^2 + (y - \beta)^2 = \beta^2$
Circles touch externally
$\therefore \sqrt{\alpha^2 + (\beta - 3)^2} = 2 + \beta$
$\alpha^2 + (\beta - 3)^2 = \beta^2 + 4 + 4\beta$
$\alpha^2 = 10(\beta - 1/2)$
$\therefore$ Locus is $x^2 = 10(y - 1/2)$ which is parabola.



54. If a circle passes through the point (a, b) and cuts the circle $x^2 + y^2 = p^2$ orthogonally, then the equation of the locus of its centre is
(1) $x^2 + y^2 - 3ax - 4by + (a^2 + b^2 - p^2) = 0$
(2) $2ax + 2by - (a^2 - b^2 + p^2) = 0$
(3) $x^2 + y^2 - 2ax - 3by + (a^2 - b^2 - p^2) = 0$
(4) $2ax + 2by - (a^2 + b^2 + p^2) = 0$

54. (4)
Let the centre be (α, β)
$\because$ It cut the circle $x^2 + y^2 = p^2$ orthogonally
$2(-\alpha) \times 0 + 2(-\beta) \times 0 = c_1 - p^2$
$c_1 = p^2$
Let equation of circle is $x^2 + y^2 - 2\alpha x - 2\beta y + p^2 = 0$
It pass through (a, b) $\Rightarrow a^2 + b^2 - 2\alpha a - 2\beta b + p^2 = 0$
Locus $\therefore$ $2ax + 2by - (a^2 + b^2 + p^2) = 0$.

55. An ellipse has OB as semi minor axis, F and F′ its focii and the angle FBF′ is a right angle. Then the eccentricity of the ellipse is
(1) $\frac{1}{\sqrt{2}}$
(2) $\frac{1}{2}$
(3) $\frac{1}{4}$
(4) $\frac{1}{\sqrt{3}}$

55. **(1)**

$\because \angle FBF' = 90^o$

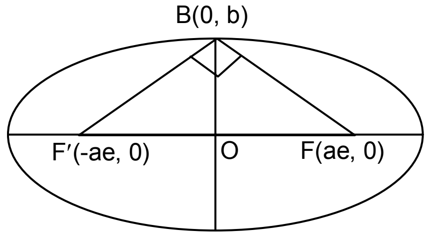
$\therefore \left(\sqrt{a^2e^2+b^2}\right)^2 + \left(\sqrt{a^2e^2+b^2}\right)^2 = (2ae)^2$

$\Rightarrow 2(a^2e^2 + b^2) = 4a^2e^2$

$\Rightarrow e^2 = b^2/a^2$

Also $e^2 = 1 - b^2/a^2 = 1 - e^2$

$\Rightarrow 2e^2 = 1,\ e = \frac{1}{\sqrt{2}}$.



56. Let a, b and c be distinct non-negative numbers. If the vectors $a\hat{i}+a\hat{j}+c\hat{k},\ \hat{i}+\hat{k}$ and $c\hat{i}+c\hat{j}+b\hat{k}$ lie in a plane, then c is

(1) the Geometric Mean of a and b
(2) the Arithmetic Mean of a and b
(3) equal to zero
(4) the Harmonic Mean of a and b

56. **(1)**

Vector $a\hat{i}+a\hat{j}+c\hat{k}$, $\hat{i}+\hat{k}$ and $c\hat{i}+c\hat{j}+b\hat{k}$ are coplanar

$$\begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix} = 0 \Rightarrow c^2 = ab$$

$\therefore$ a, b, c are in G.P.

57. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors and λ is a real number then $\left[\lambda(\vec{a}+\vec{b})\lambda^2\vec{b}\ \lambda\vec{c}\right] = \left[\vec{a}\ \vec{b}+\vec{c}\ \vec{b}\right]$ for

(1) exactly one value of λ
(2) no value of λ
(3) exactly three values of λ
(4) exactly two values of λ

57. **(2)**

$\left[\lambda(\vec{a}+\vec{b})\ \ \lambda^2\vec{b}\ \ \lambda\vec{c}\right] = \left[\vec{a}\ \ \vec{b}+\vec{c}\ \ \vec{b}\right]$

$$\begin{vmatrix} \lambda & \lambda & 0 \\ 0 & \lambda^2 & 0 \\ 0 & 0 & \lambda \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{vmatrix}$$

$\Rightarrow \lambda^4 = -1$

Hence no real value of λ.

58. Let $\vec{a} = \hat{i} - \hat{k},\ \vec{b} = x\hat{i} + \hat{j} + (1-x)\hat{k}$ and $\vec{c} = y\hat{i} + x\hat{j} + (1+x-y)\hat{k}$. Then $\left[\vec{a},\ \vec{b},\ \vec{c}\right]$ depends on

(1) only y
(2) only x
(3) both x and y
(4) neither x nor y

58. **(4)**

$\vec{a} = \hat{i} - \hat{k}$, $\vec{b} = x\hat{i} + \hat{j} + (1-x)\hat{k}$ and $\vec{c} = y\hat{i} + x\hat{j} + (1+x-y)\hat{k}$

$\left[\vec{a}\ \vec{b}\ \vec{c}\right] = \vec{a}\cdot\left(\vec{b}\times\vec{c}\right)$

$$\vec{b}\times\vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & 1 & 1-x \\ y & x & 1+x-y \end{vmatrix} = \hat{i}\,(1 + x - x - x^2) - \hat{j}\,(x + x^2 - xy - y + xy) + \hat{k}\,(x^2 - y)$$

$\vec{a}.\left(\vec{b}\times\vec{c}\right) = 1$

which does not depend on x and y.

59. Three houses are available in a locality. Three persons apply for the houses. Each applies for one house without consulting others. The probability that all the three apply for the same house is

(1) $\frac{2}{9}$ (2) $\frac{1}{9}$

(3) $\frac{8}{9}$ (4) $\frac{7}{9}$

59. (2)

For a particular house being selected

Probability = $\frac{1}{3}$

Prob(all the persons apply for the same house) = $\left(\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}\right)3 = \frac{1}{9}$.

60. A random variable X has Poisson distribution with mean 2. Then $P(X > 1.5)$ equals

(1) $\frac{2}{e^2}$ (2) 0

(3) $1-\frac{3}{e^2}$ (4) $\frac{3}{e^2}$

60. (3)

$P(x = k) = e^{-\lambda}\,\frac{\lambda^k}{k!}$

$P(x \geq 2) = 1 - P(x = 0) - P(x = 1)$

$= 1 - e^{-\lambda} - e^{-\lambda}\left(\frac{\lambda}{1!}\right)$

$= 1 - \frac{3}{e^2}$.

61. Let A and B be two events such that $P\left(\overline{A\cup B}\right) = \frac{1}{6}$, $P\left(A\cap B\right) = \frac{1}{4}$ and $P\left(\bar{A}\right) = \frac{1}{4}$, where $\bar{A}$ stands for complement of event A. Then events A and B are

(1) equally likely and mutually exclusive

(2) equally likely but not independent

(3) independent but not equally likely

(4) mutually exclusive and independent

61. (3)

$P\left(\overline{A\cup B}\right) = \frac{1}{6}$, $P(A \cap B) = \frac{1}{4}$ and $P\left(\bar{A}\right) = \frac{1}{4}$

$\Rightarrow P(A \cup B) = 5/6 \quad P(A) = 3/4$

Also $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(B) = 5/6 - 3/4 + 1/4 = 1/3$

$P(A)\,P(B) = 3/4 - 1/3 = 1/4 = P(A \cap B)$

Hence A and B are independent but not equally likely.

62. A lizard, at an initial distance of 21 cm behind an insect, moves from rest with an acceleration of 2 cm/s^2 and pursues the insect which is crawling uniformly along a straight line at a speed of 20 cm/s. Then the lizard will catch the insect after

(1) 20 s (2) 1 s
(3) 21 s (4) 24 s

62. (3)

$\frac{1}{2}2t^2 = 21 + 20t$

$\Rightarrow t = 21.$

63. Two points A and B move from rest along a straight line with constant acceleration f and f′ respectively. If A takes m sec. more than B and describes 'n' units more than B in acquiring the same speed then

(1) $(f - f')m^2 = ff'n$ (2) $(f + f')m^2 = ff'n$
(3) $\frac{1}{2}(f + f')m = ff'n^2$ (4) $(f' - f)n = \frac{1}{2}ff'm^2$

63. (4)

$v^2 = 2f(d + n) = 2f'd$

$v = f'(t) = (m + t)f$

eliminate d and m we get

$(f' - f)n = \frac{1}{2}ff'm^2.$

64. A and B are two like parallel forces. A couple of moment H lies in the plane of A and B and is contained with them. The resultant of A and B after combining is displaced through a distance

(1) $\frac{2H}{A - B}$ (2) $\frac{H}{A + B}$
(3) $\frac{H}{2(A + B)}$ (4) $\frac{H}{A - B}$

64. (2)

$(A + B) = d = H$

$d = \left(\frac{H}{A + B}\right).$

65. The resultant R of two forces acting on a particle is at right angles to one of them and its magnitude is one third of the other force. The ratio of larger force to smaller one is

(1) 2 : 1 (2) $3 : \sqrt{2}$
(3) 3 : 2 (4) $3 : 2\sqrt{2}$

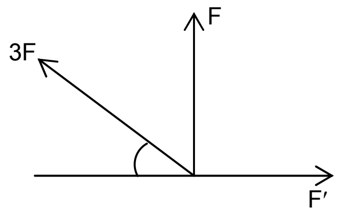
65. (4)

$F' = 3F \cos\theta$

$F = 3F \sin\theta$

$\Rightarrow F' = 2\sqrt{2}\, F$

$F : F' :: 3 : 2\sqrt{2}.$



66. The sum of the series $1+\frac{1}{4.2!}+\frac{1}{16.4!}+\frac{1}{64.6!}+.........$ ad inf. is

(1) $\frac{e-1}{\sqrt{e}}$ (2) $\frac{e+1}{\sqrt{e}}$

(3) $\frac{e-1}{2\sqrt{e}}$ (4) $\frac{e+1}{2\sqrt{e}}$

66. (4)

$$\frac{e^x+e^{-x}}{2}=1+\frac{x^2}{2!}+\frac{x^4}{4!}+\frac{x^6}{6!}+.......$$

putting x = 1/2 we get

$$\frac{e+1}{2\sqrt{e}}.$$

67. The value of $\int_{-\pi}^{\pi}\frac{\cos^2 x}{1+a^x}dx$, a > 0, is

(1) $a\pi$ (2) $\frac{\pi}{2}$

(3) $\frac{\pi}{a}$ (4) 2π

67. (2)

$$\int_{-\pi}^{\pi}\frac{\cos^2 x}{1+a^x}\,dx=\int_{0}^{\pi}\cos^2 x\,dx=\frac{\pi}{2}.$$

68. The plane x + 2y – z = 4 cuts the sphere $x^2 + y^2 + z^2 - x + z - 2 = 0$ in a circle of radius

(1) 3 (2) 1

(3) 2 (4) $\sqrt{2}$

68. (2)

Perpendicular distance of centre $\left(\frac{1}{2},\ 0,\ -\frac{1}{2}\right)$ from x + 2y – 2 = 4

$$\frac{\left|\frac{1}{2}+\frac{1}{2}-4\right|}{\sqrt{6}}=\sqrt{\frac{3}{2}}$$

radius = $\sqrt{\frac{5}{2}-\frac{3}{2}}=1.$

69. If the pair of lines $ax^2 + 2(a + b)xy + by^2 = 0$ lie along diameters of a circle and divide the circle into four sectors such that the area of one of the sectors is thrice the area of another sector then

(1) $3a^2 - 10ab + 3b^2 = 0$ (2) $3a^2 - 2ab + 3b^2 = 0$

(3) $3a^2 + 10ab + 3b^2 = 0$ (4) $3a^2 + 2ab + 3b^2 = 0$

69. (4)

$$\left|\frac{2\sqrt{(a+b)^2-ab}}{a+b}\right|=1$$

$\Rightarrow (a + b)^2 = 4(a^2 + b^2 + ab)$

$\Rightarrow 3a^2 + 3b^2 + 2ab = 0.$

70. Let $x_1, x_2, \ldots, x_n$ be n observations such that $\sum x_i^2 = 400$ and $\sum x_i = 80$. Then a possible value of n among the following is

(1) 15 (2) 18

(3) 9 (4) 12

70. (2)

$$\frac{\sum x_i^2}{n} \geq \left(\frac{\sum x_i}{n}\right)^2$$

$\Rightarrow n \geq 16.$

71. A particle is projected from a point O with velocity *u* at an angle of 60^o with the horizontal. When it is moving in a direction at right angles to its direction at O, its velocity then is given by

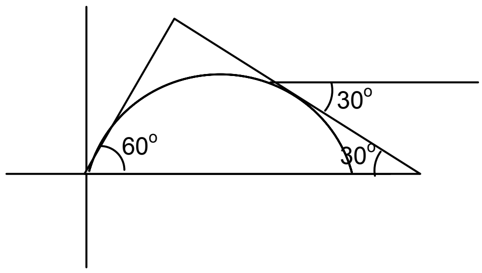
(1) $\frac{u}{3}$ (2) $\frac{u}{2}$

(3) $\frac{2u}{3}$ (4) $\frac{u}{\sqrt{3}}$

71. (4)

u cos 60^o = v cos 30^o

v = $\frac{4}{\sqrt{3}}$.



72. If both the roots of the quadratic equation $x^2 - 2kx + k^2 + k - 5 = 0$ are less than 5, then k lies in the interval

(1) (5, 6] (2) $(6, \infty)$

(3) $(-\infty, 4)$ (4) [4, 5]

72. (3)

$\frac{-b}{2a} < 5$

$f(5) > 0$

$\Rightarrow k \in (-\infty, 4)$.

73. If $a_1, a_2, a_3, \ldots, a_n, \ldots$ are in G.P., then the determinant

$$\Delta = \begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$$ is equal to

(1) 1 (2) 0

(3) 4 (4) 2

73. (2)

$C_1 - C_2, C_2 - C_3$

two rows becomes identical

Answer: 0.

74. A real valued function f(x) satisfies the functional equation f(x – y) = f(x) f(y) – f(a – x) f(a + y) where a is a given constant and f(0) = 1, f(2a – x) is equal to

(1) –f(x) (2) f(x)

(3) f(a) + f(a – x) (4) f(-x)

74. **(1)**

f(a – (x – a)) = f(a) f(x – a) – f(0) f(x)

= -f(x) $\left[\because\ x=0,\ y=0,\ f(0)=f^2(0)-f^2(a)\ \Rightarrow\ f^2(a)=0\ \Rightarrow\ f(a)=0\right]$.

75. If the equation

$a_nx^n + a_{n-1}x^{n-1} + ... + a_1x = 0$, $a_1 \neq 0$, $n \geq 2$, has a positive root x = α, then the equation $na_nx^{n-1} + (n-1)a_{n-1}x^{n-2} + ... + a_1 = 0$ has a positive root, which is

(1) greater than α
(2) smaller than α
(3) greater than or equal to α
(4) equal to α

75. **(2)**

f(0) = 0, f(α) = 0

$\Rightarrow$ $f'(k) = 0$ for some $k \in (0, \alpha)$.