

AIEEE – 2004 (MATHEMATICS)

Important Instructions:

- i) The test is of $1\frac{1}{2}$ hours duration.
 - ii) The test consists of 75 questions.
 - iii) The maximum marks are 225.
 - iv) For each correct answer you will get 3 marks and for a wrong answer you will get -1 mark.
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1. Let $R = \{(1, 3), (4, 2), (2, 4), (2, 3), (3, 1)\}$ be a relation on the set $A = \{1, 2, 3, 4\}$. The relation R is
(1) a function (2) reflexive
(3) not symmetric (4) transitive
2. The range of the function $f(x) = {}^{7-x}P_{x-3}$ is
(1) $\{1, 2, 3\}$ (2) $\{1, 2, 3, 4, 5\}$
(3) $\{1, 2, 3, 4\}$ (4) $\{1, 2, 3, 4, 5, 6\}$
3. Let z, w be complex numbers such that $\bar{z} + i\bar{w} = 0$ and $\arg zw = \pi$. Then $\arg z$ equals
(1) $\frac{\pi}{4}$ (2) $\frac{5\pi}{4}$
(3) $\frac{3\pi}{4}$ (4) $\frac{\pi}{2}$
4. If $z = x - iy$ and $z^{\frac{1}{3}} = p + iq$, then $\frac{\left(\frac{x}{p} + \frac{y}{q}\right)}{(p^2 + q^2)}$ is equal to
(1) 1 (2) -2
(3) 2 (4) -1
5. If $|z^2 - 1| = |z|^2 + 1$, then z lies on
(1) the real axis (2) an ellipse
(3) a circle (4) the imaginary axis.
6. Let $A = \begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$. The only correct statement about the matrix A is
(1) A is a zero matrix (2) $A^2 = I$
(3) A^{-1} does not exist (4) $A = (-1)I$, where I is a unit matrix

7. Let $A = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{pmatrix}$ (10) $B = \begin{pmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{pmatrix}$. If B is the inverse of matrix A, then α is

- (1) -2 (2) 5
(3) 2 (4) -1

8. If $a_1, a_2, a_3, \dots, a_n, \dots$ are in G.P., then the value of the determinant

$$\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}, \text{ is}$$

- (1) 0 (2) -2
(3) 2 (4) 1

9. Let two numbers have arithmetic mean 9 and geometric mean 4. Then these numbers are the roots of the quadratic equation

- (1) $x^2 + 18x + 16 = 0$ (2) $x^2 - 18x - 16 = 0$
(3) $x^2 + 18x - 16 = 0$ (4) $x^2 - 18x + 16 = 0$

10. If $(1 - p)$ is a root of quadratic equation $x^2 + px + (1 - p) = 0$, then its roots are

- (1) 0, 1 (2) -1, 2
(3) 0, -1 (4) -1, 1

11. Let $S(K) = 1 + 3 + 5 + \dots + (2K - 1) = 3 + K^2$. Then which of the following is true?

- (1) $S(1)$ is correct
(2) Principle of mathematical induction can be used to prove the formula
(3) $S(K) \not\Rightarrow S(K + 1)$
(4) $S(K) \Rightarrow S(K + 1)$

12. How many ways are there to arrange the letters in the word GARDEN with the vowels in alphabetical order?

- (1) 120 (2) 480
(3) 360 (4) 240

13. The number of ways of distributing 8 identical balls in 3 distinct boxes so that none of the boxes is empty is

- (1) 5 (2) 8C_3
(3) 3^8 (4) 21

14. If one root of the equation $x^2 + px + 12 = 0$ is 4, while the equation $x^2 + px + q = 0$ has equal roots, then the value of 'q' is

- (1) $\frac{49}{4}$ (2) 4
(3) 3 (4) 12

15. The coefficient of the middle term in the binomial expansion in powers of x of $(1 + \alpha x)^4$ and of $(1 - \alpha x)^6$ is the same if α equals

- (1) $-\frac{5}{3}$ (2) $\frac{3}{5}$
 (3) $\frac{-3}{10}$ (4) $\frac{10}{3}$

16. The coefficient of x^n in expansion of $(1 + x)(1 - x)^n$ is

- (1) $(n - 1)$ (2) $(-1)^n (1 - n)$
 (3) $(-1)^{n-1} (n - 1)^2$ (4) $(-1)^{n-1} n$

17. If $S_n = \sum_{r=0}^n \frac{1}{{}^n C_r}$ and $t_n = \sum_{r=0}^n \frac{r}{{}^n C_r}$, then $\frac{t_n}{S_n}$ is equal to

- (1) $\frac{1}{2}n$ (2) $\frac{1}{2}n - 1$
 (3) $n - 1$ (4) $\frac{2n - 1}{2}$

18. Let T_r be the r th term of an A.P. whose first term is a and common difference is d . If for some positive integers $m, n, m \neq n, T_m = \frac{1}{n}$ and $T_n = \frac{1}{m}$, then $a - d$ equals

- (1) 0 (2) 1
 (3) $\frac{1}{mn}$ (4) $\frac{1}{m} + \frac{1}{n}$

19. The sum of the first n terms of the series $1^2 + 2 \cdot 2^2 + 3^2 + 2 \cdot 4^2 + 5^2 + 2 \cdot 6^2 + \dots$ is $\frac{n(n+1)^2}{2}$ when n is even. When n is odd the sum is

- (1) $\frac{3n(n+1)}{2}$ (2) $\frac{n^2(n+1)}{2}$
 (3) $\frac{n(n+1)^2}{4}$ (4) $\left[\frac{n(n+1)}{2}\right]^2$

20. The sum of series $\frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \dots$ is

- (1) $\frac{(e^2 - 1)}{2}$ (2) $\frac{(e - 1)^2}{2e}$
 (3) $\frac{(e^2 - 1)}{2e}$ (4) $\frac{(e^2 - 2)}{e}$

21. Let α, β be such that $\pi < \alpha - \beta < 3\pi$. If $\sin\alpha + \sin\beta = -\frac{21}{65}$ and $\cos\alpha + \cos\beta = -\frac{27}{65}$, then the value of $\cos\frac{\alpha-\beta}{2}$ is

- (1) $-\frac{3}{\sqrt{130}}$ (2) $\frac{3}{\sqrt{130}}$
 (3) $\frac{6}{65}$ (4) $-\frac{6}{65}$

22. If $u = \sqrt{a^2 \cos^2 \theta + b^2 \sin^2 \theta} + \sqrt{a^2 \sin^2 \theta + b^2 \cos^2 \theta}$, then the difference between the maximum and minimum values of u^2 is given by

- (1) $2(a^2 + b^2)$ (2) $2\sqrt{a^2 + b^2}$
 (3) $(a + b)^2$ (4) $(a - b)^2$

23. The sides of a triangle are $\sin\alpha, \cos\alpha$ and $\sqrt{1 + \sin\alpha \cos\alpha}$ for some $0 < \alpha < \frac{\pi}{2}$. Then the greatest angle of the triangle is

- (1) 60° (2) 90°
 (3) 120° (4) 150°

24. A person standing on the bank of a river observes that the angle of elevation of the top of a tree on the opposite bank of the river is 60° and when he retires 40 meter away from the tree the angle of elevation becomes 30° . The breadth of the river is

- (1) 20 m (2) 30 m
 (3) 40 m (4) 60 m

25. If $f: \mathbb{R} \rightarrow \mathbb{S}$, defined by $f(x) = \sin x - \sqrt{3} \cos x + 1$, is onto, then the interval of \mathbb{S} is

- (1) $[0, 3]$ (2) $[-1, 1]$
 (3) $[0, 1]$ (4) $[-1, 3]$

26. The graph of the function $y = f(x)$ is symmetrical about the line $x = 2$, then

- (1) $f(x + 2) = f(x - 2)$ (2) $f(2 + x) = f(2 - x)$
 (3) $f(x) = f(-x)$ (4) $f(x) = -f(-x)$

27. The domain of the function $f(x) = \frac{\sin^{-1}(x-3)}{\sqrt{9-x^2}}$ is

- (1) $[2, 3]$ (2) $[2, 3]$
 (3) $[1, 2]$ (4) $[1, 2]$

28. If $\lim_{x \rightarrow \infty} \left(1 + \frac{a}{x} + \frac{b}{x^2}\right)^{2x} = e^2$, then the values of a and b , are

- (1) $a \in \underline{\underline{\mathbb{R}}}, b \in \underline{\underline{\mathbb{R}}}$ (2) $a = 1, b \in \underline{\underline{\mathbb{R}}}$
 (3) $a \in \underline{\underline{\mathbb{R}}}, b = 2$ (4) $a = 1$ and $b = 2$

29. Let $f(x) = \frac{1 - \tan x}{4x - \pi}$, $x \neq \frac{\pi}{4}$, $x \in \left[0, \frac{\pi}{2}\right]$. If $f(x)$ is continuous in $\left[0, \frac{\pi}{2}\right]$, then $f\left(\frac{\pi}{4}\right)$ is

- (1) 1 (2) $\frac{1}{2}$
 (3) $-\frac{1}{2}$ (4) -1

30. If $x = e^{y+e^{y+\dots\text{to } \infty}}$, $x > 0$, then $\frac{dy}{dx}$ is

- (1) $\frac{x}{1+x}$ (2) $\frac{1}{x}$
 (3) $\frac{1-x}{x}$ (4) $\frac{1+x}{x}$

31. A point on the parabola $y^2 = 18x$ at which the ordinate increases at twice the rate of the abscissa is

- (1) (2, 4) (2) (2, -4)
 (3) $\left(\frac{-9}{8}, \frac{9}{2}\right)$ (4) $\left(\frac{9}{8}, \frac{9}{2}\right)$

32. A function $y = f(x)$ has a second order derivative $f''(x) = 6(x - 1)$. If its graph passes through the point (2, 1) and at that point the tangent to the graph is $y = 3x - 5$, then the function is

- (1) $(x - 1)^2$ (2) $(x - 1)^3$
 (3) $(x + 1)^3$ (4) $(x + 1)^2$

33. The normal to the curve $x = a(1 + \cos\theta)$, $y = a\sin\theta$ at ' θ ' always passes through the fixed point

- (1) (a, 0) (2) (0, a)
 (3) (0, 0) (4) (a, a)

34. If $2a + 3b + 6c = 0$, then at least one root of the equation $ax^2 + bx + c = 0$ lies in the interval

- (1) (0, 1) (2) (1, 2)
 (3) (2, 3) (4) (1, 3)

35. $\lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{1}{n} e^{\frac{r}{n}}$ is

- (1) e (2) e - 1
 (3) 1 - e (4) e + 1

36. If $\int \frac{\sin x}{\sin(x - \alpha)} dx = Ax + B \log \sin(x - \alpha) + C$, then value of (A, B) is

- (1) $(\sin\alpha, \cos\alpha)$ (2) $(\cos\alpha, \sin\alpha)$
 (3) $(-\sin\alpha, \cos\alpha)$ (4) $(-\cos\alpha, \sin\alpha)$

37. $\int \frac{dx}{\cos x - \sin x}$ is equal to

$$(1) \frac{1}{\sqrt{2}} \log \left| \tan \left(\frac{x}{2} - \frac{\pi}{8} \right) \right| + C$$

$$(2) \frac{1}{\sqrt{2}} \log \left| \cot \left(\frac{x}{2} \right) \right| + C$$

$$(3) \frac{1}{\sqrt{2}} \log \left| \tan \left(\frac{x}{2} - \frac{3\pi}{8} \right) \right| + C$$

$$(4) \frac{1}{\sqrt{2}} \log \left| \tan \left(\frac{x}{2} + \frac{3\pi}{8} \right) \right| + C$$

38. The value of $\int_{-2}^3 |1-x^2| dx$ is

$$(1) \frac{28}{3}$$

$$(2) \frac{14}{3}$$

$$(3) \frac{7}{3}$$

$$(4) \frac{1}{3}$$

39. The value of $I = \int_0^{\pi/2} \frac{(\sin x + \cos x)^2}{\sqrt{1 + \sin 2x}} dx$ is

$$(1) 0$$

$$(2) 1$$

$$(3) 2$$

$$(4) 3$$

40. If $\int_0^{\pi} x f(\sin x) dx = A \int_0^{\pi/2} f(\sin x) dx$, then A is

$$(1) 0$$

$$(2) \pi$$

$$(3) \frac{\pi}{4}$$

$$(4) 2\pi$$

41. If $f(x) = \frac{e^x}{1+e^x}$, $I_1 = \int_{f(-a)}^{f(a)} xg\{x(1-x)\}dx$ and $I_2 = \int_{f(-a)}^{f(a)} g\{x(1-x)\}dx$ then the value of $\frac{I_2}{I_1}$ is

$$(1) 2$$

$$(2) -3$$

$$(3) -1$$

$$(4) 1$$

42. The area of the region bounded by the curves $y = |x - 2|$, $x = 1$, $x = 3$ and the x-axis is

$$(1) 1$$

$$(2) 2$$

$$(3) 3$$

$$(4) 4$$

43. The differential equation for the family of curves $x^2 + y^2 - 2ay = 0$, where a is an arbitrary constant is

$$(1) 2(x^2 - y^2)y' = xy$$

$$(2) 2(x^2 + y^2)y' = xy$$

$$(3) (x^2 - y^2)y' = 2xy$$

$$(4) (x^2 + y^2)y' = 2xy$$

44. The solution of the differential equation $y dx + (x + x^2y) dy = 0$ is

$$(1) -\frac{1}{xy} = C$$

$$(2) -\frac{1}{xy} + \log y = C$$

$$(3) \frac{1}{xy} + \log y = C$$

$$(4) \log y = Cx$$

45. Let A (2, -3) and B(-2, 1) be vertices of a triangle ABC. If the centroid of this triangle moves on the line $2x + 3y = 1$, then the locus of the vertex C is the line
 (1) $2x + 3y = 9$ (2) $2x - 3y = 7$
 (3) $3x + 2y = 5$ (4) $3x - 2y = 3$
46. The equation of the straight line passing through the point (4, 3) and making intercepts on the co-ordinate axes whose sum is -1 is
 (1) $\frac{x}{2} + \frac{y}{3} = -1$ and $\frac{x}{-2} + \frac{y}{1} = -1$ (2) $\frac{x}{2} - \frac{y}{3} = -1$ and $\frac{x}{-2} + \frac{y}{1} = -1$
 (3) $\frac{x}{2} + \frac{y}{3} = 1$ and $\frac{x}{2} + \frac{y}{1} = 1$ (4) $\frac{x}{2} - \frac{y}{3} = 1$ and $\frac{x}{-2} + \frac{y}{1} = 1$
47. If the sum of the slopes of the lines given by $x^2 - 2cxy - 7y^2 = 0$ is four times their product, then c has the value
 (1) 1 (2) -1
 (3) 2 (4) -2
48. If one of the lines given by $6x^2 - xy + 4cy^2 = 0$ is $3x + 4y = 0$, then c equals
 (1) 1 (2) -1
 (3) 3 (4) -3
49. If a circle passes through the point (a, b) and cuts the circle $x^2 + y^2 = 4$ orthogonally, then the locus of its centre is
 (1) $2ax + 2by + (a^2 + b^2 + 4) = 0$ (2) $2ax + 2by - (a^2 + b^2 + 4) = 0$
 (3) $2ax - 2by + (a^2 + b^2 + 4) = 0$ (4) $2ax - 2by - (a^2 + b^2 + 4) = 0$
50. A variable circle passes through the fixed point A (p, q) and touches x-axis. The locus of the other end of the diameter through A is
 (1) $(x - p)^2 = 4qy$ (2) $(x - q)^2 = 4py$
 (3) $(y - p)^2 = 4qx$ (4) $(y - q)^2 = 4px$
51. If the lines $2x + 3y + 1 = 0$ and $3x - y - 4 = 0$ lie along diameters of a circle of circumference 10π , then the equation of the circle is
 (1) $x^2 + y^2 - 2x + 2y - 23 = 0$ (2) $x^2 + y^2 - 2x - 2y - 23 = 0$
 (3) $x^2 + y^2 + 2x + 2y - 23 = 0$ (4) $x^2 + y^2 + 2x - 2y - 23 = 0$
52. The intercept on the line $y = x$ by the circle $x^2 + y^2 - 2x = 0$ is AB. Equation of the circle on AB as a diameter is
 (1) $x^2 + y^2 - x - y = 0$ (2) $x^2 + y^2 - x + y = 0$
 (3) $x^2 + y^2 + x + y = 0$ (4) $x^2 + y^2 + x - y = 0$
53. If $a \neq 0$ and the line $2bx + 3cy + 4d = 0$ passes through the points of intersection of the parabolas $y^2 = 4ax$ and $x^2 = 4ay$, then
 (1) $d^2 + (2b + 3c)^2 = 0$ (2) $d^2 + (3b + 2c)^2 = 0$
 (3) $d^2 + (2b - 3c)^2 = 0$ (4) $d^2 + (3b - 2c)^2 = 0$

54. The eccentricity of an ellipse, with its centre at the origin, is $\frac{1}{2}$. If one of the directrices is $x = 4$, then the equation of the ellipse is
- (1) $3x^2 + 4y^2 = 1$ (2) $3x^2 + 4y^2 = 12$
 (3) $4x^2 + 3y^2 = 12$ (4) $4x^2 + 3y^2 = 1$
55. A line makes the same angle θ , with each of the x and z axis. If the angle β , which it makes with y -axis, is such that $\sin^2 \beta = 3 \sin^2 \theta$, then $\cos^2 \theta$ equals
- (1) $\frac{2}{3}$ (2) $\frac{1}{5}$
 (3) $\frac{3}{5}$ (4) $\frac{2}{5}$
56. Distance between two parallel planes $2x + y + 2z = 8$ and $4x + 2y + 4z + 5 = 0$ is
- (1) $\frac{3}{2}$ (2) $\frac{5}{2}$
 (3) $\frac{7}{2}$ (4) $\frac{9}{2}$
57. A line with direction cosines proportional to 2, 1, 2 meets each of the lines $x = y + a = z$ and $x + a = 2y = 2z$. The co-ordinates of each of the point of intersection are given by
- (1) $(3a, 3a, 3a), (a, a, a)$ (2) $(3a, 2a, 3a), (a, a, a)$
 (3) $(3a, 2a, 3a), (a, a, 2a)$ (4) $(2a, 3a, 3a), (2a, a, a)$
58. If the straight lines $x = 1 + s, y = -3 - \lambda s, z = 1 + \lambda s$ and $x = \frac{t}{2}, y = 1 + t, z = 2 - t$ with parameters s and t respectively, are co-planar then λ equals
- (1) -2 (2) -1
 (3) $-\frac{1}{2}$ (4) 0
59. The intersection of the spheres $x^2 + y^2 + z^2 + 7x - 2y - z = 13$ and $x^2 + y^2 + z^2 - 3x + 3y + 4z = 8$ is the same as the intersection of one of the sphere and the plane
- (1) $x - y - z = 1$ (2) $x - 2y - z = 1$
 (3) $x - y - 2z = 1$ (4) $2x - y - z = 1$
60. Let \vec{a}, \vec{b} and \vec{c} be three non-zero vectors such that no two of these are collinear. If the vector $\vec{a} + 2\vec{b}$ is collinear with \vec{c} and $\vec{b} + 3\vec{c}$ is collinear with \vec{a} (λ being some non-zero scalar) then $\vec{a} + 2\vec{b} + 6\vec{c}$ equals
- (1) $\lambda\vec{a}$ (2) $\lambda\vec{b}$
 (3) $\lambda\vec{c}$ (4) 0
61. A particle is acted upon by constant forces $4\hat{i} + \hat{j} - 3\hat{k}$ and $3\hat{i} + \hat{j} - \hat{k}$ which displace it from a point $\hat{i} + 2\hat{j} + 3\hat{k}$ to the point $5\hat{i} + 4\hat{j} + \hat{k}$. The work done in standard units by the forces is given by
- (1) 40 (2) 30
 (3) 25 (4) 15

62. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors and λ is a real number, then the vectors $\vec{a} + 2\vec{b} + 3\vec{c}, \lambda\vec{b} + 4\vec{c}$ and $(2\lambda - 1)\vec{c}$ are non-coplanar for
- (1) all values of λ (2) all except one value of λ
 (3) all except two values of λ (4) no value of λ
63. Let u, v, w be such that $|\vec{u}| = 1, |\vec{v}| = 2, |\vec{w}| = 3$. If the projection v along u is equal to that of w along u and v, w are perpendicular to each other then $|\vec{u} \cdot \vec{v} + \vec{w}|$ equals
- (1) 2 (2) $\sqrt{7}$
 (3) $\sqrt{14}$ (4) 14
64. Let \vec{a}, \vec{b} and \vec{c} be non-zero vectors such that $(\vec{a} \times \vec{b}) \times \vec{c} = \frac{1}{3} |\vec{b}| |\vec{c}| \vec{a}$. If θ is the acute angle between the vectors b and c , then $\sin \theta$ equals
- (1) $\frac{1}{3}$ (2) $\frac{\sqrt{2}}{3}$
 (3) $\frac{2}{3}$ (4) $\frac{2\sqrt{2}}{3}$
65. Consider the following statements:
 (a) Mode can be computed from histogram
 (b) Median is not independent of change of scale
 (c) Variance is independent of change of origin and scale.
 Which of these is/are correct?
- (1) only (a) (2) only (b)
 (3) only (a) and (b) (4) (a), (b) and (c)
66. In a series of $2n$ observations, half of them equal a and remaining half equal $-a$. If the standard deviation of the observations is 2, then $|a|$ equals
- (1) $\frac{1}{n}$ (2) $\sqrt{2}$
 (3) 2 (4) $\frac{\sqrt{2}}{n}$
67. The probability that A speaks truth is $\frac{4}{5}$, while this probability for B is $\frac{3}{4}$. The probability that they contradict each other when asked to speak on a fact is
- (1) $\frac{3}{20}$ (2) $\frac{1}{5}$
 (3) $\frac{7}{20}$ (4) $\frac{4}{5}$
68. A random variable X has the probability distribution:
- | | | | | | | | | |
|-------|------|------|------|------|------|------|------|------|
| X: | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| p(X): | 0.15 | 0.23 | 0.12 | 0.10 | 0.20 | 0.08 | 0.07 | 0.05 |
- For the events $E = \{X \text{ is a prime number}\}$ and $F = \{X < 4\}$, the probability $P(E \cup F)$ is
- (1) 0.87 (2) 0.77
 (3) 0.35 (4) 0.50

69. The mean and the variance of a binomial distribution are 4 and 2 respectively. Then the probability of 2 successes is
- (1) $\frac{37}{256}$ (2) $\frac{219}{256}$
 (3) $\frac{128}{256}$ (4) $\frac{28}{256}$
70. With two forces acting at a point, the maximum effect is obtained when their resultant is 4N. If they act at right angles, then their resultant is 3N. Then the forces are
- (1) $(2 + \sqrt{2})\text{N}$ and $(2 - \sqrt{2})\text{N}$ (2) $(2 + \sqrt{3})\text{N}$ and $(2 - \sqrt{3})\text{N}$
 (3) $\left(2 + \frac{1}{2}\sqrt{2}\right)\text{N}$ and $\left(2 - \frac{1}{2}\sqrt{2}\right)\text{N}$ (4) $\left(2 + \frac{1}{2}\sqrt{3}\right)\text{N}$ and $\left(2 - \frac{1}{2}\sqrt{3}\right)\text{N}$
71. In a right angle $\triangle ABC$, $\angle A = 90^\circ$ and sides a, b, c are respectively, 5 cm, 4 cm and 3 cm. If a force \vec{F} has moments 0, 9 and 16 in N cm. units respectively about vertices A, B and C, then magnitude of \vec{F} is
- (1) 3 (2) 4
 (3) 5 (4) 9
72. Three forces P, \vec{Q} and R acting along IA, IB and IC, where I is the incentre of a $\triangle ABC$, are in equilibrium. Then P : \vec{Q} : R is
- (1) $\cos \frac{A}{2} : \cos \frac{B}{2} : \cos \frac{C}{2}$ (2) $\sin \frac{A}{2} : \sin \frac{B}{2} : \sin \frac{C}{2}$
 (3) $\sec \frac{A}{2} : \sec \frac{B}{2} : \sec \frac{C}{2}$ (4) $\operatorname{cosec} \frac{A}{2} : \operatorname{cosec} \frac{B}{2} : \operatorname{cosec} \frac{C}{2}$
73. A particle moves towards east from a point A to a point B at the rate of 4 km/h and then towards north from B to C at the rate of 5 km/h. If AB = 12 km and BC = 5 km, then its average speed for its journey from A to C and resultant average velocity direct from A to C are respectively
- (1) $\frac{17}{4}$ km/h and $\frac{13}{4}$ km/h (2) $\frac{13}{4}$ km/h and $\frac{17}{4}$ km/h
 (3) $\frac{17}{9}$ km/h and $\frac{13}{9}$ km/h (4) $\frac{13}{9}$ km/h and $\frac{17}{9}$ km/h
74. A velocity $\frac{1}{4}$ m/s is resolved into two components along OA and OB making angles 30° and 45° respectively with the given velocity. Then the component along OB is
- (1) $\frac{1}{8}$ m/s (2) $\frac{1}{4}(\sqrt{3} - 1)$ m/s
 (3) $\frac{1}{4}$ m/s (4) $\frac{1}{8}(\sqrt{6} - \sqrt{2})$ m/s
75. If t_1 and t_2 are the times of flight of two particles having the same initial velocity u and range R on the horizontal, then $t_1^2 + t_2^2$ is equal to
- (1) $\frac{u^2}{g}$ (2) $\frac{4u^2}{g^2}$
 (3) $\frac{u^2}{2g}$ (4) 1

AIEEE – 2004 (MATHEMATICS)

ANSWERS

1. 3	16. 2	31. 4	46. 4	61. 1
2. 1	17. 1	32. 2	47. 3	62. 3
3. 3	18. 1	33. 1	48. 4	63. 3
4. 2	19. 2	34. 1	49. 2	64. 4
5. 4	20. 2	35. 2	50. 1	65. 3
6. 2	21. 1	36. 2	51. 1	66. 3
7. 2	22. 4	37. 4	52. 1	67. 3
8. 1	23. 3	38. 1	53. 1	68. 2
9. 4	24. 1	39. 3	54. 2	69. 4
10. 3	25. 4	40. 2	55. 3	70. 3
11. 4	26. 2	41. 1	56. 3	71. 3
12. 3	27. 2	42. 1	57. 2	72. 1
13. 4	28. 2	43. 3	58. 1	73. 1
14. 1	29. 3	44. 2	59. 4	74. 4
15. 3	30. 3	45. 1	60. 4	75. 2

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SOLUTIONS

1. $(2, 3) \in R$ but $(3, 2) \notin R$.
Hence R is not symmetric.

2. $f(x) = {}^{7-x}P_{x-3}$
 $7-x \geq 0 \Rightarrow x \leq 7$
 $x-3 \geq 0 \Rightarrow x \geq 3$,
 and $7-x \geq x-3 \Rightarrow x \leq 5$
 $\Rightarrow 3 \leq x \leq 5 \Rightarrow x = 3, 4, 5 \Rightarrow \text{Range is } \{1, 2, 3\}$.

3. Here $\omega = \frac{z}{i} \Rightarrow \arg\left(z \cdot \frac{z}{i}\right) = \pi \Rightarrow 2 \arg(z) - \arg(i) = \pi \Rightarrow \arg(z) = \frac{3\pi}{4}$.

4. $z = (p+iq)^3 = p(p^2-3q^2) - iq(q^2-3p^2)$
 $\Rightarrow \frac{x}{p} = p^2 - 3q^2$ & $\frac{y}{q} = q^2 - 3p^2 \Rightarrow \frac{\frac{x}{p} + \frac{y}{q}}{(p^2+q^2)} = -2$.

5. $|z^2 - 1|^2 = (|z|^2 + 1)^2 \Rightarrow (z^2 - 1)(\bar{z}^2 - 1) = |z|^4 + 2|z|^2 + 1$
 $\Rightarrow z^2 + \bar{z}^2 + 2z\bar{z} = 0 \Rightarrow z + \bar{z} = 0$
 $\Rightarrow \text{R}(z) = 0 \Rightarrow z$ lies on the imaginary axis.

6. $A \cdot A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$.

7. $AB = I \Rightarrow A(10B) = 10I$
 $\Rightarrow \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 10 & 0 & 5-\alpha \\ 0 & 10 & \alpha-5 \\ 0 & 0 & 5+\alpha \end{bmatrix} = 10 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ if $\alpha = 5$.

8. $\begin{vmatrix} \log_a n & \log_a n+1 & \log_a n+2 \\ \log_a n+3 & \log_a n+4 & \log_a n+5 \\ \log_a n+6 & \log_a n+7 & \log_a n+8 \end{vmatrix}$
 $C_3 \rightarrow C_3 - C_2, C_2 \rightarrow C_3 - C_1$
 $= \begin{vmatrix} \log_a n & \log r & \log r \\ \log_a n+3 & \log r & \log r \\ \log_a n+6 & \log r & \log r \end{vmatrix} = 0$ (where r is a common ratio).

9. Let numbers be $a, b \Rightarrow a+b = 18, \sqrt{ab} = 4 \Rightarrow ab = 16$, a and b are roots of the equation
 $\Rightarrow x^2 - 18x + 16 = 0$.

10. **(3)**
 $(1-p)^2 + p(1-p) + (1-p) = 0$ (since $(1-p)$ is a root of the equation $x^2 + px + (1-p) = 0$)
 $\Rightarrow (1-p)(1-p+p+1) = 0$
 $\Rightarrow 2(1-p) = 0 \Rightarrow (1-p) = 0 \Rightarrow p = 1$
sum of root is $\alpha + \beta = -p$ and product $\alpha\beta = 1-p = 0$ (where $\beta = 1-p = 0$)
 $\Rightarrow \alpha + 0 = -1 \Rightarrow \alpha = -1 \Rightarrow$ Roots are $0, -1$

11. $S(k) = 1 + 3 + 5 + \dots + (2k-1) = 3 + k^2$
 $S(k+1) = 1 + 3 + 5 + \dots + (2k-1) + (2k+1)$
 $= (3 + k^2) + 2k + 1 = k^2 + 2k + 4$ [from $S(k) = 3 + k^2$]
 $= 3 + (k^2 + 2k + 1) = 3 + (k+1)^2 = S(k+1)$.
Although $S(k)$ in itself is not true but it considered true will always imply towards $S(k+1)$.

12. Since in half the arrangement A will be before E and other half E will be before A.
Hence total number of ways = $\frac{6!}{2} = 360$.

13. Number of balls = 8
number of boxes = 3
Hence number of ways = ${}^7C_2 = 21$.

14. Since 4 is one of the root of $x^2 + px + 12 = 0 \Rightarrow 16 + 4p + 12 = 0 \Rightarrow p = -7$
and equation $x^2 + px + q = 0$ has equal roots
 $\Rightarrow D = 49 - 4q = 0 \Rightarrow q = \frac{49}{4}$.

15. Coefficient of Middle term in $(1 + \alpha x)^4 = t_3 = {}^4C_2 \cdot \alpha^2$
Coefficient of Middle term in $(1 - \alpha x)^6 = t_4 = {}^6C_3 (-\alpha)^3$
 ${}^4C_2 \alpha^2 = -{}^6C_3 \cdot \alpha^3 \Rightarrow -6 = 20\alpha \Rightarrow \alpha = \frac{-3}{10}$

16. Coefficient of x^n in $(1+x)(1-x)^n = (1+x)({}^nC_0 - {}^nC_1x + \dots + (-1)^{n-1} {}^nC_{n-1}x^{n-1} + (-1)^n {}^nC_n x^n)$
 $= (-1)^n {}^nC_n + (-1)^{n-1} {}^nC_{n-1} = (-1)^n (1-n)$.

17. $t = \sum_{r=0}^n \frac{r}{{}^nC_r} = \sum_{r=0}^n \frac{n-r}{{}^nC_{n-r}} = \sum_{r=0}^n \frac{n-r}{{}^nC_r}$ ($\because {}^nC_r = {}^nC_{n-r}$)
 $2t_n = \sum_{r=0}^n \frac{r+n-r}{{}^nC_r} = \sum_{r=0}^n \frac{n}{{}^nC_r} \Rightarrow t_n = \frac{n}{2} \sum_{r=0}^n \frac{1}{{}^nC_r} = \frac{n}{2} S_n \Rightarrow \frac{t_n}{S_n} = \frac{n}{2}$

18. $T_m = \frac{1}{n} = a + (m-1)d$ (1)

and $T_n = \frac{1}{m} = a + (n-1)d$ (2)

from (1) and (2) we get $a = \frac{1}{mn}$, $d = \frac{1}{mn}$

Hence $a - d = 0$

19. If n is odd then $(n - 1)$ is even \Rightarrow sum of odd terms $= \frac{(n-1)n^2}{2} + n^2 = \frac{n^2(n+1)}{2}$.

20. $\frac{e^\alpha + e^{-\alpha}}{2} = 1 + \frac{\alpha^2}{2!} + \frac{\alpha^4}{4!} + \frac{\alpha^6}{6!} + \dots$

$\frac{e^\alpha + e^{-\alpha}}{2} - 1 = \frac{\alpha^2}{2!} + \frac{\alpha^4}{4!} + \frac{\alpha^6}{6!} + \dots$

put $\alpha = 1$, we get

$\frac{(e-1)^2}{2e} = \frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \dots$

21. $\sin \alpha + \sin \beta = -\frac{21}{65}$ and $\cos \alpha + \cos \beta = -\frac{27}{65}$.

Squaring and adding, we get

$2 + 2 \cos(\alpha - \beta) = \frac{1170}{(65)^2}$

$\Rightarrow \cos^2\left(\frac{\alpha - \beta}{2}\right) = \frac{9}{130} \Rightarrow \cos\left(\frac{\alpha - \beta}{2}\right) = \frac{-3}{\sqrt{130}} \quad \left(\because \frac{\pi}{2} < \frac{\alpha - \beta}{2} < \frac{3\pi}{2}\right)$.

22. $u = \sqrt{a^2 \cos^2 \theta + b^2 \sin^2 \theta} + \sqrt{a^2 \sin^2 \theta + b^2 \cos^2 \theta}$
 $= \sqrt{\frac{a^2 + b^2}{2} + \frac{a^2 - b^2}{2} \cos 2\theta} + \sqrt{\frac{a^2 + b^2}{2} + \frac{b^2 - a^2}{2} \cos 2\theta}$

$\Rightarrow u^2 = a^2 + b^2 + 2\sqrt{\left(\frac{a^2 + b^2}{2}\right)^2 - \left(\frac{a^2 - b^2}{2}\right)^2} \cos^2 2\theta$

min value of $u^2 = a^2 + b^2 + 2ab$

max value of $u^2 = 2(a^2 + b^2)$

$\Rightarrow u_{\max}^2 - u_{\min}^2 = (a - b)^2$.

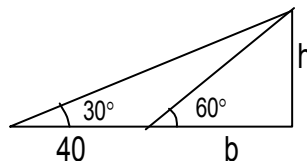
23. Greatest side is $\sqrt{1 + \sin \alpha \cos \alpha}$, by applying cos rule we get greatest angle = 120° .

24. $\tan 30^\circ = \frac{h}{40 + b}$

$\Rightarrow \sqrt{3}h = 40 + b$ (1)

$\tan 60^\circ = h/b \Rightarrow h = \sqrt{3}b$ (2)

$\Rightarrow b = 20 \text{ m}$



25. $-2 \leq \sin x - \sqrt{3} \cos x \leq 2 \Rightarrow -1 \leq \sin x - \sqrt{3} \cos x + 1 \leq 3$

\Rightarrow range of $f(x)$ is $[-1, 3]$.

Hence S is $[-1, 3]$.

26. If $y = f(x)$ is symmetric about the line $x = 2$ then $f(2 + x) = f(2 - x)$.

27. $9 - x^2 > 0$ and $-1 \leq x - 3 \leq 1 \Rightarrow x \in [2, 3)$

$$28. \lim_{x \rightarrow \infty} \left(1 + \frac{a}{x} + \frac{b}{x^2}\right)^{2x} = \lim_{x \rightarrow \infty} \left(1 + \frac{a}{x} + \frac{b}{x^2}\right)^{\left(\frac{1}{\frac{a+b}{x} + \frac{b}{x^2}}\right) \times 2x \times \left(\frac{a+b}{x} + \frac{b}{x^2}\right)} = e^{2a} \Rightarrow a = 1, b \in \mathbb{R}$$

$$29. f(x) = \frac{1 - \tan x}{4x - \pi} \Rightarrow \lim_{x \rightarrow \frac{\pi}{4}} \frac{1 - \tan x}{4x - \pi} = -\frac{1}{2}$$

$$30. x = e^{y+e^{y+e^{y+\dots}} \infty} \Rightarrow x = e^{y+x} \\ \Rightarrow \ln x - x = y \Rightarrow \frac{dy}{dx} = \frac{1}{x} - 1 = \frac{1-x}{x}.$$

31. Any point be $\left(\frac{9}{2}t^2, 9t\right)$; differentiating $y^2 = 18x$

$$\Rightarrow \frac{dy}{dx} = \frac{9}{y} = \frac{1}{t} = 2 \text{ (given)} \Rightarrow t = \frac{1}{2}.$$

$$\Rightarrow \text{Point is } \left(\frac{9}{8}, \frac{9}{2}\right)$$

$$32. f''(x) = 6(x-1) \Rightarrow f'(x) = 3(x-1)^2 + c \\ \text{and } f'(2) = 3 \Rightarrow c = 0 \\ \Rightarrow f(x) = (x-1)^3 + k \text{ and } f(2) = 1 \Rightarrow k = 0 \\ \Rightarrow f(x) = (x-1)^3.$$

33. Eliminating θ , we get $(x-a)^2 + y^2 = a^2$.
Hence normal always pass through $(a, 0)$.

$$34. \text{Let } f'(x) = ax^2 + bx + c \Rightarrow f(x) = \frac{ax^3}{3} + \frac{bx^2}{2} + cx + d \\ \Rightarrow f(x) = \frac{1}{6}(2ax^3 + 3bx^2 + 6cx + 6d), \text{ Now } f(1) = f(0) = d, \text{ then according to Rolle's theorem} \\ \Rightarrow f'(x) = ax^2 + bx + c = 0 \text{ has at least one root in } (0, 1)$$

$$35. \lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{1}{n} e^{\frac{r}{n}} = \int_0^1 e^x dx = (e-1)$$

$$36. \text{Put } x - \alpha = t \\ \Rightarrow \int \frac{\sin(\alpha+t)}{\sin t} dt = \sin \alpha \int \cot t dt + \cos \alpha \int dt \\ = \cos \alpha (x - \alpha) + \sin \alpha \ln |\sin t| + c \\ A = \cos \alpha, B = \sin \alpha$$

$$37. \int \frac{dx}{\cos x - \sin x} = \frac{1}{\sqrt{2}} \int \frac{1}{\cos\left(x + \frac{\pi}{4}\right)} dx = \frac{1}{\sqrt{2}} \int \sec\left(x + \frac{\pi}{4}\right) dx = \frac{1}{\sqrt{2}} \log \left| \tan\left(\frac{x}{2} + \frac{3\pi}{8}\right) \right| + C$$

$$38. \int_{-2}^{-1} (x^2 - 1) dx + \int_{-1}^1 (1 - x^2) dx + \int_1^3 (x^2 - 1) dx = \frac{x^3}{3} - x \Big|_{-2}^{-1} + x - \frac{x^3}{3} \Big|_{-1}^1 + \frac{x^3}{3} - x \Big|_1^3 = \frac{28}{3}.$$

$$39. \int_0^{\frac{\pi}{2}} \frac{(\sin x + \cos x)^2}{\sqrt{(\sin x + \cos x)^2}} dx = \int_0^{\frac{\pi}{2}} (\sin x + \cos x) dx = \left| -\cos x + \sin x \right|_0^{\frac{\pi}{2}} = 2.$$

$$40. \text{ Let } I = \int_0^{\pi} x f(\sin x) dx = \int_0^{\pi} (\pi - x) f(\sin x) dx = \pi \int_0^{\pi} f(\sin x) dx - I \quad (\text{since } f(2a - x) = f(x))$$

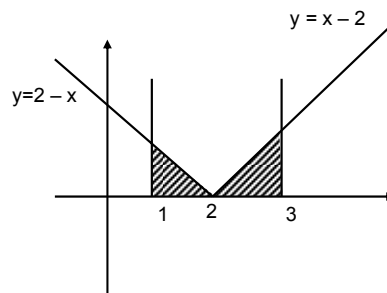
$$\Rightarrow I = \pi \int_0^{\pi/2} f(\sin x) dx \Rightarrow A = \pi.$$

$$41. f(-a) + f(a) = 1$$

$$I_1 = \int_{f(-a)}^{f(a)} x g\{x(1-x)\} dx = \int_{f(-a)}^{f(a)} (1-x) g\{x(1-x)\} dx \quad \left(\because \int_a^b f(x) dx = \int_a^b f(a+b-x) dx \right)$$

$$2I_1 = \int_{f(-a)}^{f(a)} g\{x(1-x)\} dx = I_2 \Rightarrow I_2 / I_1 = 2.$$

$$42. \text{ Area} = \int_1^2 (2-x) dx + \int_2^3 (x-2) dx = 1.$$



$$43. 2x + 2yy' - 2ay' = 0$$

$$a = \frac{x + yy'}{y'} \quad (\text{eliminating } a)$$

$$\Rightarrow (x^2 - y^2)y' = 2xy.$$

$$45. y dx + x dy + x^2 y dy = 0.$$

$$\frac{d(xy)}{x^2 y^2} + \frac{1}{y} dy = 0 \Rightarrow -\frac{1}{xy} + \log y = C.$$

$$45. \text{ If } C \text{ be } (h, k) \text{ then centroid is } (h/3, (k-2)/3) \text{ it lies on } 2x + 3y = 1.$$

$$\Rightarrow \text{locus is } 2x + 3y = 9.$$

46. $\frac{x}{a} + \frac{y}{b} = 1$ where $a + b = -1$ and $\frac{4}{a} + \frac{3}{b} = 1$

$\Rightarrow a = 2, b = -3$ or $a = -2, b = 1$.

Hence $\frac{x}{2} - \frac{y}{3} = 1$ and $\frac{x}{-2} + \frac{y}{1} = 1$.

47. $m_1 + m_2 = -\frac{2c}{7}$ and $m_1 m_2 = -\frac{1}{7}$

$m_1 + m_2 = 4m_1 m_2$ (given)

$\Rightarrow c = 2$.

48. $m_1 + m_2 = \frac{1}{4c}, m_1 m_2 = \frac{6}{4c}$ and $m_1 = -\frac{3}{4}$.

Hence $c = -3$.

49. Let the circle be $x^2 + y^2 + 2gx + 2fy + c = 0 \Rightarrow c = 4$ and it passes through (a, b)
 $\Rightarrow a^2 + b^2 + 2ga + 2fb + 4 = 0$.

Hence locus of the centre is $2ax + 2by - (a^2 + b^2 + 4) = 0$.

50. Let the other end of diameter is (h, k) then equation of circle is

$(x - h)(x - p) + (y - k)(y - q) = 0$

Put $y = 0$, since x-axis touches the circle

$\Rightarrow x^2 - (h + p)x + (hp + kq) = 0 \Rightarrow (h + p)^2 = 4(hp + kq)$ (D = 0)

$\Rightarrow (x - p)^2 = 4qy$.

51. Intersection of given lines is the centre of the circle i.e. $(1, -1)$

Circumference = $10\pi \Rightarrow$ radius $r = 5$

\Rightarrow equation of circle is $x^2 + y^2 - 2x + 2y - 23 = 0$.

52. Points of intersection of line $y = x$ with $x^2 + y^2 - 2x = 0$ are $(0, 0)$ and $(1, 1)$
hence equation of circle having end points of diameter $(0, 0)$ and $(1, 1)$ is

$x^2 + y^2 - x - y = 0$.

53. Points of intersection of given parabolas are $(0, 0)$ and $(4a, 4a)$

\Rightarrow equation of line passing through these points is $y = x$

On comparing this line with the given line $2bx + 3cy + 4d = 0$, we get

$d = 0$ and $2b + 3c = 0 \Rightarrow (2b + 3c)^2 + d^2 = 0$.

54. Equation of directrix is $x = a/e = 4 \Rightarrow a = 2$

$b^2 = a^2(1 - e^2) \Rightarrow b^2 = 3$

Hence equation of ellipse is $3x^2 + 4y^2 = 12$.

55. $l = \cos \theta, m = \cos \theta, n = \cos \beta$

$\cos^2 \theta + \cos^2 \theta + \cos^2 \beta = 1 \Rightarrow 2 \cos^2 \theta = \sin^2 \beta = 3 \sin^2 \theta$ (given)

$\cos^2 \theta = 3/5$.

56. Given planes are

$2x + y + 2z - 8 = 0, 4x + 2y + 4z + 5 = 0 \Rightarrow 2x + y + 2z + 5/2 = 0$

Distance between planes = $\frac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}} = \frac{|-8 - 5/2|}{\sqrt{2^2 + 1^2 + 2^2}} = \frac{7}{2}$.

57. Any point on the line $\frac{x}{1} = \frac{y+a}{1} = \frac{z}{1} = t_1$ (say) is $(t_1, t_1 - a, t_1)$ and any point on the line

$$\frac{x+a}{2} = \frac{y}{1} = \frac{z}{1} = t_2 \text{ (say) is } (2t_2 - a, t_2, t_2).$$

Now direction cosine of the lines intersecting the above lines is proportional to $(2t_2 - a - t_1, t_2 - t_1 + a, t_2 - t_1)$.

$$\text{Hence } 2t_2 - a - t_1 = 2k, \quad t_2 - t_1 + a = k \text{ and } t_2 - t_1 = 2k$$

On solving these, we get $t_1 = 3a, t_2 = a$.

Hence points are $(3a, 2a, 3a)$ and (a, a, a) .

58. Given lines $\frac{x-1}{1} = \frac{y+3}{-\lambda} = \frac{z-1}{\lambda} = s$ and $\frac{x}{1/2} = \frac{y-1}{1} = \frac{z-2}{-1} = t$ are coplanar then plan passing through these lines has normal perpendicular to these lines

$$\Rightarrow a - b\lambda + c\lambda = 0 \quad \text{and} \quad \frac{a}{2} + b - c = 0 \text{ (where } a, b, c \text{ are direction ratios of the normal to the plan)}$$

On solving, we get $\lambda = -2$.

59. Required plane is $S_1 - S_2 = 0$
 where $S_1 = x^2 + y^2 + z^2 + 7x - 2y - z - 13 = 0$ and
 $S_2 = x^2 + y^2 + z^2 - 3x + 3y + 4z - 8 = 0$
 $\Rightarrow 2x - y - z = 1$.

$$60. (\vec{a} + 2\vec{b}) = t_1\vec{c} \quad \dots(1)$$

$$\text{and } \vec{b} + 3\vec{c} = t_2\vec{a} \quad \dots(2)$$

$$(1) - 2 \times (2) \Rightarrow \vec{a}(1 + 2t_2) + \vec{c}(-t_1 - 6) = 0 \Rightarrow 1 + 2t_2 = 0 \Rightarrow t_2 = -1/2 \text{ \& } t_1 = -6.$$

Since \vec{a} and \vec{c} are non-collinear.

Putting the value of t_1 and t_2 in (1) and (2), we get $\vec{a} + 2\vec{b} + 6\vec{c} = \vec{0}$.

61. Work done by the forces \vec{F}_1 and \vec{F}_2 is $(\vec{F}_1 + \vec{F}_2) \cdot \vec{d}$, where \vec{d} is displacement

$$\text{According to question } \vec{F}_1 + \vec{F}_2 = (4\hat{i} + \hat{j} - 3\hat{k}) + (3\hat{i} + \hat{j} - \hat{k}) = 7\hat{i} + 2\hat{j} - 4\hat{k}$$

and $\vec{d} = (5\hat{i} + 4\hat{j} + \hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k}) = 4\hat{i} + 2\hat{j} - 2\hat{k}$. Hence $(\vec{F}_1 + \vec{F}_2) \cdot \vec{d}$ is 40.

$$63. \text{ Condition for given three vectors to be coplanar is } \begin{vmatrix} 1 & 2 & 3 \\ 0 & \lambda & 4 \\ 0 & 0 & 2\lambda - 1 \end{vmatrix} = 0 \Rightarrow \lambda = 0, 1/2.$$

Hence given vectors will be non coplanar for all real values of λ except 0, 1/2.

63. Projection of \vec{v} along \vec{u} and \vec{w} along \vec{u} is $\frac{\vec{v} \cdot \vec{u}}{|\vec{u}|}$ and $\frac{\vec{w} \cdot \vec{u}}{|\vec{u}|}$ respectively

$$\text{According to question } \frac{\vec{v} \cdot \vec{u}}{|\vec{u}|} = \frac{\vec{w} \cdot \vec{u}}{|\vec{u}|} \Rightarrow \vec{v} \cdot \vec{u} = \vec{w} \cdot \vec{u} \text{ and } \vec{v} \cdot \vec{w} = 0$$

$$|\vec{u} - \vec{v} + \vec{w}|^2 = |\vec{u}|^2 + |\vec{v}|^2 + |\vec{w}|^2 - 2\vec{u} \cdot \vec{v} + 2\vec{u} \cdot \vec{w} - 2\vec{v} \cdot \vec{w} = 14 \Rightarrow |\vec{u} - \vec{v} + \vec{w}| = \sqrt{14}.$$

64. $(\vec{a} \times \vec{b}) \times \vec{c} = \frac{1}{3} |\vec{b}| |\vec{c}| \vec{a} \Rightarrow (\vec{a} \cdot \vec{c}) \vec{b} - (\vec{b} \cdot \vec{c}) \vec{a} = \frac{1}{3} |\vec{b}| |\vec{c}| \vec{a}$
 $\Rightarrow (\vec{a} \cdot \vec{c}) \vec{b} = \left(\frac{1}{3} |\vec{b}| |\vec{c}| + (\vec{b} \cdot \vec{c}) \right) \vec{a} \Rightarrow \vec{a} \cdot \vec{c} = 0 \text{ and } \frac{1}{3} |\vec{b}| |\vec{c}| + (\vec{b} \cdot \vec{c}) = 0$
 $\Rightarrow |\vec{b}| |\vec{c}| \left(\frac{1}{3} + \cos \theta \right) = 0 \Rightarrow \cos \theta = -1/3 \Rightarrow \sin \theta = \frac{2\sqrt{2}}{3}.$

65. Mode can be computed from histogram and median is dependent on the scale. Hence statement (a) and (b) are correct.

66. $x_i = a$ for $i = 1, 2, \dots, n$ and $x_i = -a$ for $i = n+1, \dots, 2n$
S.D. = $\sqrt{\frac{1}{2n} \sum_{i=1}^{2n} (x_i - \bar{x})^2} \Rightarrow 2 = \sqrt{\frac{1}{2n} \sum_{i=1}^{2n} x_i^2} \quad \left(\text{Since } \sum_{i=1}^{2n} x_i = 0 \right) \Rightarrow 2 = \sqrt{\frac{1}{2n} \cdot 2na^2} \Rightarrow |a| = 2$

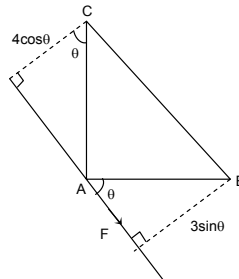
67. E_1 : event denoting that A speaks truth
 E_2 : event denoting that B speaks truth
Probability that both contradicts each other = $P(E_1 \cap \bar{E}_2) + P(\bar{E}_1 \cap E_2) = \frac{4}{5} \cdot \frac{1}{4} + \frac{1}{5} \cdot \frac{3}{4} = \frac{7}{20}$

68. $P(E \cup F) = P(E) + P(F) - P(E \cap F) = 0.62 + 0.50 - 0.35 = 0.77$

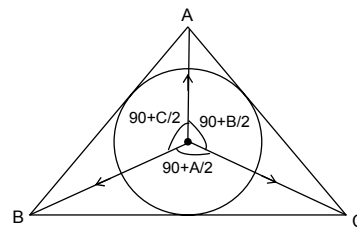
69. Given that $np = 4, npq = 2 \Rightarrow q = 1/2 \Rightarrow p = 1/2, n = 8 \Rightarrow p(x = 2) = {}^8C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^6 = \frac{28}{256}$

70. $P + Q = 4, P^2 + Q^2 = 9 \Rightarrow P = \left(2 + \frac{1}{2}\sqrt{2}\right)N$ and $Q = \left(2 - \frac{1}{2}\sqrt{2}\right)N.$

71. F . $3 \sin \theta = 9$
F . $4 \cos \theta = 16$
 $\Rightarrow F = 5.$



72. By Lami's theorem
 $\vec{P} : \vec{Q} : \vec{R} = \sin\left(90^\circ + \frac{A}{2}\right) : \sin\left(90^\circ + \frac{B}{2}\right) : \sin\left(90^\circ + \frac{C}{2}\right)$
 $\Rightarrow \cos \frac{A}{2} : \cos \frac{B}{2} : \cos \frac{C}{2}.$



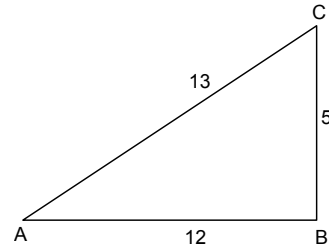
73. Time T_1 from A to B = $\frac{12}{4} = 3$ hrs.

T_2 from B to C = $\frac{5}{5} = 1$ hrs.

Total time = 4 hrs.

Average speed = $\frac{17}{4}$ km/hr.

Resultant average velocity = $\frac{13}{4}$ km/hr.



74. Component along OB = $\frac{\frac{1}{4} \sin 30^\circ}{\sin(45^\circ + 30^\circ)} = \frac{1}{8}(\sqrt{6} - \sqrt{2})$ m/s.

75. $t_1 = \frac{2u \sin \alpha}{g}$, $t_2 = \frac{2u \sin \beta}{g}$ where $\alpha + \beta = 90^\circ$

$\therefore t_1^2 + t_2^2 = \frac{4u^2}{g^2}$.