AIEEE - 2003

1. A particle of mass M and charge Q moving with velocity $\vec{v}$ describes a circular path of radius R when subjected to a uniform transverse magnetic field of induction B. The work done by the field when the particle completes one full circle is

(A) $\left(\frac{Mv^2}{R}\right)2\pi R$ (B) zero

(C) $BQ2\pi R$ (D) $BQv2\pi R$

1. B.

Since the particle completes one full circle, therefore displacement of particle = 0

Work done = force $\times$ displacement = 0

2. A particle of charge -16×10^{-18} coulomb moving with velocity 10 ms^{-1} along the x-axis enters a region where a magnetic field of induction B is along the y –axis, and an electric field of induction B is along the y-axis, and an electric field of magnitude 10^4 V/m is along the negative z-axis. If the charged particle continues moving along the x-axis, the magnitude of B is

(A) 10^3 Wb/m^2 (B) 10^5 Wb/m^2

(C) 10^{16} Wb/m^2 (D) 10^{-3} Wb/m^2

2. A.

$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$ …(1)

The solution of this problem can be obtained by resolving the motion along the three coordinate axes namely

$$a_x = \frac{F_x}{m} = \frac{q}{m}(E_x + v_y B_z - v_z B_y)$$

$$a_y = \frac{F_y}{m} = \frac{q}{m}(E_y + v_z B_x - v_x B_z)$$

$$a_z = \frac{F_z}{m} = \frac{q}{m}(E_z + v_x B_y - v_y B_z)$$

For the given problem,

$E_x = E_y = 0,\ v_y = v_z = 0$ and $B_x = B_z = 0$

Substituting in equation (2), we get

$a_x = a_z = 0$ and $a_y = E_y - v_x B_z$

If the particle passes through the region undeflected a_y is also zero, then

$E_y = v_x B_z$

$$\Rightarrow\ B_z = \frac{E_y}{v_z} = \frac{10^4}{10} = 10^3\, Wb/m^2$$

3. A thin rectangular magnet suspended freely has a period of oscillation equal to T. Now it is broken into two equal halves (each having half of the original length) and one piece is made to oscillate freely in the same field. If its period of oscillation is T', the ratio T′/T is

(A) $\frac{1}{2\sqrt{2}}$ (B) 1/2

(C) 2 (D) 1/4

3. B.

When the magnet is divided into 2 equal parts, the magnetic dipole movement

M′ = pole strength $\times$ length $= \frac{M}{2}$ and moment of inertia

$$I' = \frac{1}{12} \times \text{mass} \times (\text{length})^2$$

$$= \frac{1}{12} \times \frac{m}{2}\left(\frac{\ell}{2}\right)^2$$

$$\Rightarrow \quad I' = \frac{I}{8}$$

$$\text{Time period} = 2\pi\sqrt{\frac{I'}{M'B}} = 2\pi\sqrt{\frac{I/8}{\frac{M}{2}B}}$$

$$T' = \frac{T}{2}$$

$$\Rightarrow \quad \frac{T'}{T} = \frac{1}{2}$$

4. A magnetic needle lying parallel to a magnetic field requires W units of work to turn it through 60°. The torque needed to maintain the needle in this position will be
(A) $\sqrt{3}$ W (B) W
(C) $(\sqrt{3}/2)$ W (D) 2W

4. A.

$W = -MB(\cos\theta_2 - \cos\theta_1)$

Initially magnetic needle is parallel to a magnet field, therefore

$\theta_1 = 0,$

$\theta_2 = 60°$

$\therefore \quad W = -MB(\cos 60° - \cos 0°)$

$= MB$

$e = MB \sin 60° = ZW \times \sqrt{3}/2 = \sqrt{3}W$

5. The magnetic lines of force inside a bar magnet
(A) are from north-pole to south-pole of the magnet
(B) do not exist
(C) depend upon the area of cross-section of the bar magnet
(D) are from south-pole to north-pole of the magnet.

5. D.

The magnetic lines of force inside a bar magnet are from south pole to north pole of the magnet.

6. Curie temperature is the temperature above which
(A) a ferromagnetic material becomes paramagnetic
(B) a paramagnetic material becomes diamagnetic
(C) a ferromagnetic material becomes diamagnetic
(D) a paramagnetic material becomes ferromagnetic.

6. A.

Curie temperature is the temperature above which a ferromagnetic material becomes paramagnetic.

7. A spring balance is attached to the ceiling of a lift. A man hangs his bag on the spring and the spring reads 49 N, when the lift is stationary. If the lift moves downward with an acceleration of 5 m/s^2, the reading of the spring balance will be
(A) 24 N (B) 74 N
(C) 15 N (D) 49 N

7. A.
Reading of spring balance = m(g – a) = 5 × 4.8 = 24 N

8. The length of a wire of a potentiometer is 100 cm, and the e.m.f. of its stand and cell is E volt. It is employed to measure the e.m.f. of a battery whose internal resistance is 0.5 Ω. If the balance point is obtained at ℓ =30 cm from the positive end, the e.m.f. of the battery is
(A) $\frac{30\ E}{100.5}$
(B) $\frac{30\ E}{100-0.5}$
(C) $\frac{30(E-0.5i)}{100}$, where I is the current in the potentiometer wire.
(D) $\frac{30\ E}{100}$

8. A.
$V=\frac{E\ell}{L}=\frac{E\times 30}{100}=\frac{30E}{100}$.

9. A strip of copper and another germanium are cooled from room temperature to 80 K. The resistance of
(A) each of these decreases
(B) copper strip increases and that of germanium decreases
(C) copper strip decreases and that of germanium increases
(D) each of these increases.

9. C.
The temperature coefficient of resistance of copper is positive and that of germanium is negative, therefore when copper and germanium are cooled, resistance of copper strip decreases and that of germanium increases.

10. Consider telecommunication through optical fibres. Which of the following statements is not true?
(A) Optical fibres can be of graded refractive index.
(B) Optical fibres are subject to electromagnetic interference from outside.
(C) Optical fibres have extremely low transmission loss.
(D) Optical fibres may have homogeneous core with a suitable cladding

10. B.
Optical fibres are subject to electromagnetic interference from outside.

11. The thermo e.m.f. of a thermo-couple is 25 μV/°C at room temperature. A galvanometer of 40 ohm resistance, capable of detecting current as low as 10^{-5} A, is connected with the thermocouple. The smallest temperature difference that can be detected by this system is
(A) 16°C (B) 12°C
(C) 8°C (D) 20°C

11. A.

$E = 25\,\theta \times 10^{-6}$ V

$IR = 10^{-5} \times 40 = 4 \times 10^{-4}$ V

$$\theta = \frac{4\times10^{-4}}{25\times10^{-6}} = 16°C$$

12. The negative Zn pole of a Daniell cell, sending a constant current through a circuit, decreases in mass by 0.13 g in 30 minutes. If the electrochemical equivalent of Zn and Cu are 32.5 and 31.5 respectively, the increase in the mass of the positive Cu pole in this time is

(A) 0.180 g (B) 0.141 g

(C) 0.126 g (D) 0.242 g

12. C.

$$\frac{m_{Zn}}{m_{Cu}} = \frac{Z_{zx}}{Z_{Cx}}$$

I and t are same for both Cu and Zn electrodes

$$\frac{0.13}{m_{Cu}} = \frac{31.5}{32.5}$$

$$m_{Cu} = \frac{0.13\times32.5}{32.5} = 0.126 \text{ g.}$$

13. Dimensions of $\frac{1}{\mu_0\varepsilon_0}$, where symbols have their usual meaning, are

(A) $[L^{-1}T]$ (B) $[L^{-2}T^2]$

(C) $[L^2T^{-2}]$ (D) $[LT^{-1}]$

13. C.

14. A circular disc X of radius R is made from an iron pole of thickness t, and another disc Y of radius 4R is made from an iron plate of thickness t/4. then the relation between the moment of inertia I_X and I_Y is

(A) $I_Y = 32I_X$ (B) $I_Y = 16I_X$

(C) $I_Y = 32\ I_X$ (D) $I_Y = 64\ I_X$

14. D.

If t is the thickness and R is the radius of the disc, then mass = $\pi R^2 t\rho$

ρ = density of the material of the disc.

Moment of inertia of disc X,

$$I_x = \frac{1}{2}\pi R^4 t\rho \quad \ldots(i)$$

Moment of inertia of disc Y,

$$I_y = 32\ \pi R^4 t\rho \quad \ldots(ii)$$

From equation (i) and (ii)

$$I_y = 64\ I_x$$

15. The time period of a satellite of earth is 5 hours. If the separation between the earth and the satellite is increased to 4 times the previous value, the new time period will become

(A) 10 hours (B) 80 hours

(C) 40 hours (D) 20 hours

15. C.

Time period of a satellite $T = \frac{2\pi}{R_e}\sqrt{\frac{r^3}{g}}$

r = distance between satellite and the earth.

$T \propto r^{3/2}$

$\Rightarrow \frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^{3/2}$

$T_2 = 8T_1 = 8 \times 5 = 40$ hours

16. A particle performing uniform circular motion has angular momentum L. If its angular frequency is doubled and its kinetic energy halved, then the new angular momentum is
(A) L/4 (B) 2L
(C) 4L (D) L/2

16. A.

Angular momentum of a particle performing uniform circular motion

$L = I\omega$

Kinetic energy, $K = \frac{1}{2}I\omega^2$

Therefore, $L = \frac{2K}{\omega^2}\omega = \frac{2K}{\omega}$

$\frac{L_1}{L_2} = \frac{K_1\omega_2}{K_2\omega_1}$

$\frac{L_1}{L_2} = 2 \times 2 = 4$

$L_2 = \frac{L}{4}$.

17. Which of the following radiations has the least wavelength?
(A) γ-rays (B) β-rays
(C) α-rays (D) X-rays

17. D.

18. When U^{238} nucleus originally at rest, decays by emitting an alpha particle having a speed u, the recoil speed of the residual nucleus is
(A) $\frac{4u}{238}$ (B) $-\frac{4u}{234}$
(C) $\frac{4u}{234}$ (D) $-\frac{4u}{238}$

18. B.

According to principle of conservation of linear momentum the momentum of the system remains the same before and after the decay.

Atomic mass of uranium = 238 and after emitting an alpha particle.

$= 238 - 4 = 234$

$\therefore \quad 238 \times 0 = 4u + 234\ v$

$\therefore \quad v = -\frac{4u}{234}$.

19. Two spherical bodies of mass M and 5M and radii R and 2R respectively are released in free space with initial separation between their centres equal to 12R. If they attract each other

due to gravitational force only, then the distance covered by the smaller body just before collision is

(A) 2.5R (B) 4.5R
(C) 7.5R (D) 1.5R

19. C.

The two spheres collide when the smaller sphere covered the distance of 7.5 R.

20. The difference in the variation of resistance with temperature in a metal and a semiconductor arises essentially due to the difference in the

(A) crystal structure
(B) variation of the number of charge carries with temperature
(C) type of bonding
(D) variation for scattering mechanism with temperature.

20. B.

Variation of the number charge carriers with temperature.

21. A car moving with a speed of 50 km/hr, can be stopped by brakes after at least 6 m. If the same car is moving at a speed of 100 km/hr, the minimum stopping distance is

(A) 12 m (B) 18 m
(C) 24 m (D) 6 m

21. C.

22. A boy playing on the roof of a 10 m high building throws a ball with a speed of 10 m/s at an angle of 30° with the horizontal. How far from the throwing point will the ball be at the height of 10 m from the ground?

[g = 10 m/s^2, sin 30° = ½, cos 30° = $\sqrt{3}/2$]

(A) 5.20 m (B) 4.33 m
(C) 2.60 m (d) 8.66 m.

22. D.

The ball will be at the height of 10 m from the ground when it cover its maximum horizontal range.

Maximum horizontal range $R = \dfrac{u^2 \sin 2\theta}{g}$

$$R = \frac{(10)^2 \times 2 \times \frac{\sqrt{3}}{2} \times \frac{1}{2}}{10} = 8.66 \text{ m.}$$

23. An ammeter reads upto 1 ampere. Its internal resistance is 0.81 ohm. To increase the range to 10 A the value of the required shunt is

(A) 0.03 Ω (B) 0.3 Ω
(C) 0.9 Ω (D) 0.09 Ω

23. D.

$$S = \frac{I_g G}{I - I_g} = \frac{1 \times 0.81}{10 - 1} = 0.09\ \Omega$$

24. The physical quantities not having same dimensions are

(A) torque and work (B) momentum and Planck's constant

(C) stress and Young's modulus (D) speed and $(\mu_0\varepsilon_0)^{-1/2}$

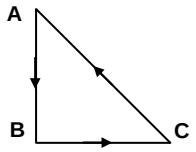
24. B.

Dimensions of momentum = kg m/sec = $[MLT^{-2}]$

Dimensions of Planck's constant = joule sec = $[ML^2T^{-1}]$

$\therefore$ Dimensions of momentum $\neq$ dimensions of Planck's constant.

25. Three forces start acting simultaneously on a particle moving with velocity $\vec{v}$. These forces are represented in magnitude and direction by the three sides of a triangle ABC(as shown). The particle will now move with velocity



(A) less than $\vec{v}$

(B) greater than $\vec{v}$

(C) $|v|$ in the direction of the largest force BC

(D) $\vec{v}$, remaining unchanged.

25. D.

According to triangle law of vector addition if three vectors addition if three vectors are represented by three sides of a triangle taken in same order, then their resultant is zero. Therefore resultant of the forces acting on the particle is zero, so the particles velocity remains unchanged.

26. If the electric flux entering and leaving an enclosed surface respectively is ϕ_1 and ϕ_2, the electric charge inside the surface will be

(A) $(\phi_2 - \phi_1)\varepsilon_0$ (B) $\dfrac{(\phi_2 + \phi_1)}{\varepsilon_0}$

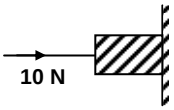
(C) $\dfrac{(\phi_2 - \phi_1)}{\varepsilon_0}$ (D) $(\phi_2 + \phi_1)\varepsilon_0$

26. A.

According to Gauss's theorem, charge in flux = $\dfrac{\text{charge enclosed by the surface}}{\varepsilon_0}$

$\therefore\ q = (\phi_2 - \phi_1)\varepsilon_0$.

27. A horizontal force of 10 N is necessary to just hold a block stationary against a wall. The coefficient of friction between the block and the wall is 0.2. The weight of the block is



(A) 20 N (B) 50 N

(C) 100 N (D) 2 N

27. D.

Weight of the block = $\mu R = 0.2 \times 10 = 2N$.

28. A marble block of mass 2 kg lying on ice when given a velocity of 6 m/s is stopped by friction in 10 s. then the coefficient of friction is

(A) 002 (B) 0.03

(C) 0.04 (D) 0.01

28. C.

Retardation $= \dfrac{u}{t} = \dfrac{6}{10} = 0.6$ m/sec^2

Frictional force = μ mg = ma

$\therefore\ \mu = \dfrac{a}{g} = \dfrac{0.6}{10} = 0.06.$

29. Consider the following two statements.

(1) Linear momentum of a system of particles is zero.
(2) Kinetic energy of system of particles is zero.
(A) A does not imply B and B does not imply A.
(B) A implies B but B does not imply A
(C) A does not imply B but b implies A'
(D) A implies B and B implies A.

29. C.

30. Two coils are placed close to each other. The mutual inductance of the pair of coils depends upon
(A) the rates at which current are changing in the two coils
(B) relative position and orientation of the two coils
(C) the materials of the wires of the coils
(D) the currents in the two coils

30. C.
The mutual inductance of the pair of coils depends on geometry of two coils, distance between two coils, distance between two coils, relative placement of two coils etc.

31. A block of mass M is pulled along a horizontal friction surface by a rope of mass m. If a force P is applied at the free end of the rope, the force exerted by the rope on the block is
(A) $\frac{Pm}{M+m}$ (B) $\frac{Pm}{M-m}$
(C) P (D) $\frac{Pm}{M+m}$

31. D.
Force on block = mass $\times$ acceleration = $\frac{PM}{M+m}$

32. A light spring balance hangs from the hook of the other light spring balance and a block of mass M kg hangs from the former one. Then the true statement about scale reading is
(A) both the scales read M kg each
(B) the scale of the lower one reads M kg and of upper one zero
(C) the reading of the two scales can be anything but sum of the reading will be M kg
(D) both the scales read M/2 kg.

32. A.
Both the scales read M kg each.

33. A wire suspended vertically from one of its ends stretched by attaching weight of 200 N to the lower end. The weight stretches the wire by 1 mm. Then the elastic energy stored in the wire is
(A) 0.2 J (B) 10 J
(C) 20 J (D) 0.1 J

33. D.
The elastic potential energy stored in the wire,

$$U = \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume}$$

$$= \frac{1}{2} \times \frac{F}{A} \times \frac{\Delta \ell}{\ell} \times A\ell = \frac{1}{2} F \Delta \ell = \frac{1}{2} \times 200 \times 10^{-3} = 0.1 \text{ J}$$

34. The escape velocity for a body projected vertically upwards from the surface of earth is 11 km/s. If the body is projected at an angle of 45° with the vertical, the escape velocity will be
(A) $11\sqrt{2}$ km/s (B) 22 km/s
(C) 11 km/s (D) $11/\sqrt{2}$ m/s

34. C.
The escape velocity of a body is independent of the angle of projection.

35. A mass M is suspended from a spring of negligible mass. The spring is pulled a little and then released so that the mass executes SHM of time period T. If the mass is increased by m, the time period becomes 5T/3. then the ratio of m/M is
(A) 3/5 (B) 25/9
(C) 16/9 (D) 5/3

35. C.

$$\frac{T}{T'} = \sqrt{\frac{M}{M+m}}$$

$$\Rightarrow \quad \frac{9}{25} = \frac{M}{M+m}$$

$\Rightarrow 9M + 9m = 25\ M$

$$\therefore \quad \frac{m}{M} = \frac{16}{9}$$

36. "Heat cannot by itself flow from a body at lower temperature to a body at higher temperature" is a statement of consequence of
(A) second law of thermodynamics (B) conservation of momentum
(C) conservation of mass (D) first law of thermodynamics.

36. A.
Second law of thermodynamics.

37. Two particles A and B of equal masses are suspended from two massless springs of spring constants k_1 and k_2 respectively. If the maximum velocities, during oscillations, are equal, the ratio of amplitudes of A and B is
(A) $\sqrt{\frac{k_1}{k_2}}$ (B) $\frac{k_2}{k_1}$
(C) $\sqrt{\frac{k_2}{k_1}}$ (D) $\frac{k_1}{k_2}$

37. C.

$$\frac{a_1}{a_2} = \sqrt{\frac{k_2}{k_1}}.$$

38. The length of a simple pendulum executing simple harmonic motion is increased by 21%. The percentage increase in the time period of the pendulum of increased length is
(A) 11% (B) 21%
(C) 42% (D) 10%

38. D.

Time period of simple pendulum is given by.

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

New length $\ell' = \ell + \frac{21}{100}\ell = \frac{121}{199}\ell$

$$\therefore \quad \frac{T'}{T} = \sqrt{\frac{\ell'}{\ell}} = \sqrt{\frac{21}{100}}$$

$$\frac{T'}{T} = \frac{11}{10}$$

$$\Rightarrow \quad T' = T + \frac{1}{10}T$$

$T \neq 10\%$ of T.

39. The displacement y of wave travelling in the x-direction is given by

$y = 10^{-4}\sin\left(600t - 2x + \frac{\pi}{3}\right)$ metres,

where x is expressed in metres and t in seconds. The speed of the wave-motion, in ms^{-1} is

(A) 300 (B) 600

(C) 1200 (D) 200

39. A.

Velocity of wave = $n\lambda = \frac{600}{2\pi} \times \frac{2\pi}{2}$ = 300 m/sec.

40. When the current changes from +2 A to –2 A in 0.05 second, an e.m.f. of 8 V is induced in a coil. The coefficient of self-induction of the coil is

(A) 0.2 H (B) 0.4 H

(C) 0.8 H (D) 0.1 H

40. D.

If e is the induced e.m.f. in the coil, then $e = -L\frac{di}{dt}$

Therefore, $L = -\frac{e}{di/dt}$

Substituting values, we get $L = \frac{-8 \times 0.05}{-4} = 0.1\,H$

41. In an oscillating LC circuit the maximum charge on the capacitor is Q. The charge on the capacitor when the energy is stored equally between the electric and magnetic field is

(A) Q/2 (B) Q/√3

(C) Q/√2 (D) Q

41. C.

energy stored in capacitor = $E = \frac{1}{2}\frac{Q^2}{C}$

$$\Rightarrow \quad \frac{1}{2} \times \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{q^2}{C}$$

$$\Rightarrow \quad q = \frac{Q}{\sqrt{2}}.$$

42. The core of any transformer is laminated so as to

(A) reduce the energy loss due to eddy currents
(B) make it light weight
(C) make it robust and strong
(D) increase the secondary voltage.

42. A.

43. Let $\vec{F}$ be the force acting on a particle having position vector $\vec{r}$ and $\vec{T}$ be the torque of this force about the origin. Then

(A) $\vec{r}\cdot\vec{T}=0$ and $\vec{F}\cdot\vec{T}\neq 0$ (B) $\vec{r}\cdot\vec{T}\neq 0$ and $\vec{F}\cdot\vec{T}=0$

(C) $\vec{r}\cdot\vec{T}\neq 0$ and $\vec{F}\cdot\vec{T}\neq 0$ (D) $\vec{r}\cdot\vec{T}=0$ and $\vec{F}\cdot\vec{T}=0$

43. D.

Torque = Force $\times$ Position vector

$$\vec{T}=\vec{F}\times\vec{r}$$

$$\vec{r}\cdot\vec{T}=\vec{r}\cdot(\vec{F}\times\vec{r})=0$$

$$\vec{F}\cdot\vec{T}=\vec{F}\cdot(\vec{F}\times\vec{r})=0$$

44. A radioactive sample at any instant has its disintegration rate 5000 disintegrations per minute. After 5 minutes, the rate is 1250 disintegrations per minute.
Then, the decay constant (per minute) is

(A) 0.4 In 2 (B) 0.2 In 2

(C) 0.1 In 2 (D) 0.8 In 2.

44. A.

$$\lambda=\frac{2\text{In}2}{5}=0.4\text{ In }2.$$

45. A nucleus with Z = 92 emits the following in a sequence; α, α, β^-, β^-, α, α, α, α, β^-, β^-, α, β^+, β^+, α. The Z of the resulting nucleus is

(A) 76 (B) 78

(C) 82 (D) 74

45. B.

The Z of resultant nucleus = 92 – 16 + 4 – 2 = 78

46. Two identical photo cathodes receive light of frequencies f_1 and f_2. if the velocities of the photoelectrons (of mass m) coming out are respectively v_1 and v_2, then

(A) $v_1^2-v_2^2=\frac{2h}{m}(f_1-f_2)$ (B) $v_1+v_2=\left[\frac{2h}{m}(f_1+f_2)\right]^{1/2}$

(C) $v_1^2+v_2^2=\frac{2h}{m}(f_1+f_2)$ (D) $v_1-v_2=\left[\frac{2h}{m}(f_1-f_2)\right]^{1/2}$

46. A.

$$\frac{1}{2}m(v_1^2-v_2^2)=h(f_1-f_2)$$

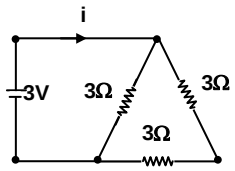
$$\Rightarrow\quad v_1^2-v_2^2=\frac{2h}{m}(f_1-f_2)\,.$$

47. Which of the following cannot be emitted by radioactive substance during their decay?

(A) protons (B) neutrinos
(C) helium nuclei (D) electrons

47. A.

48. A 3 volt battery with negligible internal resistance is connected in a circuit as shown in the figure. The current I, in the circuit will be



(A) 1 A (B) 1.5 A
(C) 2 A (D) 1/3 A

48. B.

The current through the circuit, $I = \frac{V}{R} = \frac{3}{2} = 1.5$ A

49. A sheet of aluminium foil of negligible thickness is introduced between the plates of a capacitor. The capacitance of the capacitor
(A) decreases (B) remains unchanged
(C) becomes infinite (D) increases.

49. B.
When a sheet of aluminium foil of negligible thickness is introduced between the plates of a capacitor, the capacitance of capacitor remains unchanged.

50. The displacement of a particle varies according to the relation x = 4(cos πt + sin πt). the amplitude of the particle is
(A) –4 (B) 4
(C) $4\sqrt{2}$ (D) 8

50. C.
The amplitude of given wave equation = $4\sqrt{2}$.

51. A thin spherical conduction shell of radius R has a charge q. another charge Q is placed at the centre of the shell. The electrostatic potential at a point P at a distance R/2 from the centre of the shell is
(A) $\frac{2Q}{4\pi\varepsilon_0 R}$ (B) $\frac{2Q}{4\pi\varepsilon_0 R} - \frac{2q}{4\pi\varepsilon_0 R}$
(C) $\frac{2Q}{4\pi\varepsilon_0 R} + \frac{q}{4\pi\varepsilon_0 R}$ (D) $\frac{(q+Q)}{4\pi\varepsilon_0}\frac{2}{R}$

51. C.
The total potential at P = $\frac{1}{4\pi\varepsilon_0}\cdot\frac{1}{R}(q+2Q)$

52. The work done in placing a charge of 8×10^{-18} coulomb on a condenser of capacity 100 micro-farad is
(A) 16×10^{-32} joule (B) 3.2×10^{-26} joule
(C) 4×10^{-10} joule (D) 32×10^{-32} joule

52. D.
Required work done is $w = \frac{1}{2}\frac{Q^2}{C^2}$
$= \frac{1}{2} \times \frac{(8\times10^{-18})^2}{10^{-4}} = 32\times10^{-32}$ J

53. The co-ordinates of a moving particle at any time t are given by $x = \alpha t^3$ and $y = \beta t^3$. The speed to the particle at time t is given by

(A) $3t\sqrt{\alpha^2+\beta^2}$ (B) $3t^2\sqrt{\alpha^2+\beta^2}$

(C) $t^2\sqrt{\alpha^2+\beta^2}$ (D) $\sqrt{\alpha^2+\beta^2}$

53. B.

Speed = $|\vec{v}| = \sqrt{(3\alpha t^2)^2 + (3\beta t^2)^2} = 3t^2\sqrt{\alpha^2+\beta^2}$.

54. During an adiabatic process, the pressure of a gas is found to be proportional to the cube of its absolute temperature. The ratio $\frac{C_p}{C_v}$ for the gas is

(A) 4/3 (B) 2

(C) 5/3 (D) 3/2

54. A.

$\frac{C_P}{C_v} = \frac{4}{3}$.

55. Which of the following parameters does not characterize the thermodynamic state of matter?

(A) temperature (B) pressure

(C) work (D) volume

55. C.

The work done does not characterize a thermodynamic state of matter. It gives only a relationship between two different thermodynamic state.

56. A carnot engine takes 3×10^6 cal of heat from a reservoir at 627°C, and gives it to a sink at 27°C. The work done by the engine is

(A) 4.2×10^6 J (B) 8.4×10^6 J

(C) 16.8×10^6 J (D) zero.

56. B.

Work done by the engine while taking heat

$Q = 3 \times 10^6$ cal is $W = 2 \times 10^6 \times 4.2 = 8.4 \times 10^6$ J.

57. A spring of spring constant 5×10^3 N/m is stretched initially by 5 cm from the unstretched position. Then the work required to stretch is further by another 5 cm is

(A) 12.50 N-m (B) 18.75 N-m

(C) 25.00 N-m (D) 6.25 N-m

57. B.

Required work done = 25 – 6.25 = 18.75 N–m.

58. A metal wire of linear mass density of 9.8 gm is stretched with a tension of 10 kg-wt between two rigid supports 1 metre apart. The wire passes at its middle point between the poles of a per magnet and it vibrates in resonance when carrying an alternating current of frequency n. The frequency n of the alternating source is

(A) 50 Hz (B) 100 Hz

(C) 200 Hz (D) 25 Hz

58. A.

Frequency of oscillation $n = \frac{1}{2L}\sqrt{\frac{T}{m}}$

$= \frac{1}{2L}\sqrt{\frac{10 \times 9.8}{9.8 \times 10^{-3}}} = \frac{1}{2L} \times 10^2 = \frac{1}{2 \times 1} \times 10^2 = 50\text{Hz}$

59. A tuning fork of known frequency 256 Hz makes 5 beats per second with the vibrating string of a piano. The beat frequency decreases to 2 beats per second when the tension in the piano string is slightly increased. The frequency of the piano string before increasing the tension was
(A) (256 + 2) Hz (B) (256 – 2) Hz
(C) (256 – 5) Hz (D) (256 + 5) Hz

59. C.

60. A body executes simple harmonic motion. The potential energy (P.E.), the kinetic energy (K.E.) and total energy (T.E.) are measured as function of displacement x. Which of the following statement is true?
(A) K.E. is maximum when x = 0 (B) T.E. is zero when x = 0
(C) K.E. is maximum when x is maximum (D) P.E. is maximum when x = 0.

60. A.
Since at x = 0, the potential energy is minimum, the kinetic energy is maximum.

61. In the nuclear fusion reaction,
${}_1^2H + {}_1^3H \rightarrow {}_2^4He + n$ given that the repulsive potential energy between the two nuclei is -7.7×10^{-14} J, the temperature at which the gases must be heated to initiate the reaction is nearly [Boltzmann's constant k = 1.38×10^{-23} J/K]
(A) 10^7K (B) 10^5 K
(C) 10^3 K (D) 10^9 K

61. D.

$$T = \frac{7.7 \times 10^{-14} \times 2}{3 \times 1.38 \times 10^{-23}} = 3.7 \times 10^{-9} \text{ K.}$$

62. Which of the following atoms has the lowest ionization potential?
(A) ${}_7^{14}N$ (B) ${}_{55}^{133}Cs$
(C) ${}_{18}^{40}Ar$ (D) ${}_8^{16}O$

62. B.
Since ${}_{55}^{133}Cs$ has larger size among the four atoms given, thus the electrons present in the outermost orbit will be away from the nucleus and the electrostatic force experienced by electrons due to nucleus will be minimum. Therefore the energy required to liberate electron from outer orbit will be minimum in the case of ${}_{55}^{133}Cs$.

63. The wavelengths involved in the spectrum of deuterium (${}_1^2D$) are slightly different from that of hydrogen spectrum, because
(A) size of the two nuclei are different
(B) nuclear forces are different in the two cases
(C) masses of the two nuclei are different
(D) attraction between the electron and the nucleus is different in the two cases.

63. C.

64. In the middle of the depletion layer of a reverse-biased p-n junction, the
(A) electric field is zero (B) potential is maximum
(C) electric field is maximum (D) potential is zero

64. A.

65. If the binding energy of the electron in a hydrogen atom is 13.6 eV, the energy required to remove the electron from the first excited state of Li^{++} is
(A) 30.6 eV (B) 13.6 eV
(C) 3.4 eV (D) 122.4 eV.

65. A.

The energy of the first excited state of Li^{++} is

$$E_2 = -\frac{Z^2E_0}{n^2} = \frac{3^2 \times 13.6}{2^2} = -30.6 \text{ eV}.$$

66. A body is moved along a straight line by a machine delivering a constant power. The distance moved by the body in time t is proportional to

(A) $t^{3/4}$ (B) $t^{3/2}$

(C) $t^{1/4}$ (D) $t^{1/2}$

66. B.

Distance goes as $t^{3/2}$

67. A rocket with a lift-off mass 3.5×10^4 kg is blasted upwards with an initial acceleration of 10 m/s^2. Then the initial thrust of the blast is

(A) 3.5×10^5 N (B) 7.0×10^5 N

(C) 14.0×10^5 N (D) 1.75×10^5 N

67. A.

68. To demonstrate the phenomenon of interference we require two soruces which emit radiation of

(A) nearly the same frequency

(B) the same frequency

(C) different wavelength

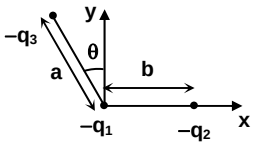
(D) the same frequency and having a definite phase relationship.

68. A.

Initial thrust of the blast = $m \times a = 3.5 \times 10^4 \times 10$

$= 3.5 \times 10^5$ N

69. Three charges $-q_1$, $+q_2$ and $-q_3$ are placed as shown in the figure. The x-component of the force on $-q_1$ is proportional to



(A) $\frac{q_2}{b^2} - \frac{q_3}{a^2}\cos\theta$ (B) $\frac{q_2}{b^2} + \frac{q_3}{a^2}\sin\theta$

(C) $\frac{q_2}{b^2} + \frac{q_3}{a^2}\cos\theta$ (D) $\frac{q_2}{b^2} - \frac{q_3}{a^2}\sin\theta$

69. B.

$$F_x \propto \frac{q_2}{b^2} + \frac{q_3}{a^2}\sin\theta$$

70. A 220 volt, 1000 watt bulb is connected across a 110 volt mains supply. The power consumed will be

(A) 750 watt (B) 500 watt

(C) 250 watt (D) 1000 watt

70. C.

$$P_{consumed} = \frac{V^2}{R} = \frac{(110)^2}{(220)^2/1000} = 250 \text{ watt}.$$

71. The image formed by an objective of a compound microscope is

(A) virtual and diminished (B) real and diminished

(C) real and enlarged (D) virtual and enlarged

71. C.

The objective of compound microscope is a convex lens. We know that a convex lens forms real and enlarged image when an object is placed between its focus and lens.

72. The earth radiates in the infra-red region of the spectrum. The spectrum is correctly given by

(A) Rayleigh Jeans law (B) Planck's law of radiation

(C) Stefan's law of radiation (D) Wien's law

72. D.

73. To get three images of a single object, one should have two plane mirrors at an angle of

(A) 60° (B) 90°

(C) 120° (D) 30°

73. B.

The number of images formed of two plane mirrors are placed at an angle θ is n = $\frac{360°}{\theta} - 1$

Here n = 3

$\therefore \quad 3 = \frac{360°}{\theta} - 1$

$\Rightarrow \quad \theta = \frac{360°}{4} = 90°$

74. According to Newton's law of cooling, the rate of cooling of a body is proportional to $(\Delta\theta)^n$, where $\Delta\theta$ is the difference of the temperature of the body and the surroundings, and n is equal to

(A) two (B) three

(C) four (D) one

74. D.

According to Newton's law of cooling.

Rate of cooling $\frac{d\theta}{dt} \propto \Delta\theta$

Therefore n = 1.

75. The length of a given cylindrical wire is increased by 100%. Due to the consequent decrease in diameter the change in the resistance of the wire will be

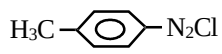
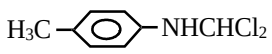
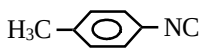
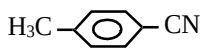
(A) 200% (B) 100%

(C) 50% (D) 300%

75. D.

%change = $\frac{3R}{R} \times 100\% = 300\%$.

76. Which of the following could act as apropellant for rockets?
(a) Liquid oxygen + liquid argon (b) Liquid hydrogen + liquid oxygen
(c) Liquid nitrogen + liquid oxygen (d) Liquid hydrogen + liquid nitrogen

77. The reaction of chloroform with alcoholic KOH and p-toluidine forms
(a)  (b)  (c)  (d) 

78. Nylon threads are made of
(a) polyester polymer (b) polyamide polymer (c) polyethylene polymer (d) polyvinyl polymer

79. The correct order of increasing basic nature for the bases NH_3, CH_3NH_2 and $(CH_3)_2$ NH is
(a) $(CH_3)_2NH < NH_3 < CH_3NH_2$ (b) $NH_3 < CH_3NH_2 < (CH_3)_2NH$
(c) $CH_3NH_2 < (CH_3)_2NH < NH_3$ (d) $CH_3NH_2 < NH_3 < (CH_3)_2NH$

80. Bottles containing C_6H_5I and $C_6H_5CH_2I$ lost their original labels. They were labelled A and B for testing A and B were separately taken in test tubes and boiled with NaOH solution. The end solution in each tube was made acidic with dilute HNO_3 and then some $AgNO_3$ solution was added. Substance B gave a yellow precipitate. Which one of the following statements is true for this experiment?
(a) A and $C_6H_5CH_2I$ (b) B and C_6H_5I
(c) Addition of HNO_3 was unnecessary (d) A was C_6H_5I

81. The internal energy change when a system goes from state A to B is 40 kJ/mole. If the system goes from A to B by a reversible path and returns to state A by an irreversible path what would be the net change in internal energy?
(a)> 40 kJ (b) < 40kJ (c) Zero (d) 40 kJ

82. If at 298 K the bond energies of C-H, C-C, C = C and H-H bonds are respectively 414, 347, 615 and 435 kJ mol^{-1}, the value of enthalpy change for the reaction $H_2C = CH_2(g) + H_2(g) \rightarrow H_3C - CH_3(g)$ at 298 K will be
(a) -250 kJ (b) + 125 kJ (c) -125 kJ (d) + 250 kJ

83. The radionucleide $^{234}_{90}Th$ undergoes two successive β-decays followed by one α-decay. The atomic number and the mass number respectively of the resulting radionucleide are
(a) 94 and 230 (b) 90 and 230 (c) 92 and 230 (d) 92 and 234

84. The half-life of a radioactive isotope is three hours. If the initial mass of the isotope were 256 g, the mass of it remaining undecayed after 18 hours would be
(a) 8.0 g (b) 12.0 g (c) 16.0 g (d) 4.0 g

85. If liquids A and B form an ideal solution
(a) the entropy of mixing is zero (b) the free energy of mixing is zero
(c) the free energy as well as the entropy of mixing are each zero (d) the enthalpy of mixing is zero

86. The radius of La^{3+} (Atomic number of La = 57) is 1.06Å. Which one of the following given values will be closest to the radius of Lu^{3+} (Atomic number of Lu = 71) ?
(a) 1.40Å (b) 1.06Å (c) 0.85Å (d) 1.60Å

87. Ammonia forms the complex ion $[Cu(NH_3)_4]^{2+}$ with copper ions in alkaline solutions but not in acidic solutions. What is the reason for it?
(a) In acidic solutions protons coordinate with ammonia molecules forming NH^+_4 ions and NH_3 molecules are not available
(b) In alkaline solutions insoluble $Cu(OH)_2$ is precipitated which is soluble in excess of any alkali
(c) Copper hydroxide is an amphoteric substance
(d) In acidic solutions hydration protects copper ions.

88. One mole of the complex compound $Co(NH_3)_5Cl_3$, gives 3 moles of ions on dissolution in water. One mole of the same complex reacts with two moles of $AgNO_3$ solution to yield two moles of AgCl (s). The structure of the complex is

(a) $[Co(NH_3)_3Cl_3].2NH_3$ (b) $[Co(NH_3)_4Cl_2]Cl.NH_3$ (c) $[Co(NH_3)_4Cl]Cl_2 . NH_3$ (d) $[Co(NH_3)_5Cl]Cl_2$

89 In the coordination compound, $K_4[Ni(CN)_4]$, the oxidation state of nickel is

(a) 0 (b) +1 (c) +2 (d) -1

90. In curing cement plasters water is sprinkled from time to time. This helps in

(a) developing interlocking needle-like crystals of hydrated silicates

(b) hydrating sand and gravel mixed with cement

(c) converting sand into silicic acid (d) keeping it cool

91. Which one of the following statements is not true?

(a) pH + pOH = 14 for all aqueous solutions (b) The pH of 1×10^{-8} M HCI is 8

(c) 96,500 coulombs of electricity when passed through a $CuSO_4$ solution deposits 1 gram equivalent of copper at the cathode

(d) The conjugate base of $H_2PO_4^-$ is HPO^{2-}_4

92. On mixing a certain alkane with chlorine and irradiating it with ultravioletlight, it forms only one monochloroalkane. This alkane could be

(a) pentane (b) isopentane (c) neopentane (d) propane

93. Butene-1 may be converted to butane by reaction with

(a) Sn - HCI (b) Zn - Hg (c) Pd/H_2 (d) Zn - HCI

94. What may be expected to happen when phosphine gas is mixed with chlorine gas?

(a) PCI_3 and HCI are formed and the mixture warms up

(b) PCI_5 and HCI are formed and the mixture cools down

(c) $PH_3.Cl_2$ is formed with warming up (d) The mixture only cools down

95. The number of d-electrons retained in Fe^{2+} (At.no.of Fe = 26) ion is

(a) 4 (b) 5 (c) 6 (d) 3

96. Concentrated hydrochloric acid when kept in open air sometimes produces a cloud of white fumes. The explanation for it is that

(a) oxygen in air reacts with the emitted HCI gas to form a cloud of chlorine gas

(b) strong affinity of HCI gas for miosture in air results in forming of droplets of liquid solution which appears like a cloudy smoke.

(c) due to strong affinity for water, concentrated hydrochloric acid pulls moisture of air towards it self. This moisture forms droplets of water and hence the cloud.

(d) concentrated hydrochloric acid emits strongly smelling HCI gas all the time.

97. An ether is more volatile than an alcohol having the same molecular formula. This is due to

(a) alcohols having resonance structures (b) inter-molecular hydrogen bonding in ethers

(c) inter-molecular hydrogen bonding in alcohols (d) dipolar character of ethers

98. Graphite is a soft solid lubricant extremely difficult to melt. The reason for this anomalous behaviour is that graphite

(a) is an allotropic form of diamond (b) has molecules of variable molecular masses like polymers

(c) has carbon atoms arranged in large plates of rings of strongly bound carbon atoms with weak interplate bonds

(d) is a non-crystalline substance

99. According to the Periodic Law of elements, the variation in properties of elements is related to their

(a) nuclear masses (b) atomic numbers (c) nuclear neutron-proton number ratios (d) atomic masses

100. Which one of the following statements is correct?
(a) From a mixed precipitate of AgCl and AgI, ammonia solution dissolves only AgCl
(b) Ferric ions give a deep green precipitate on adding potassium ferrocyanide solution
(c) On boiling a solution having K^+, Ca^{2+} and HCO^-_3 ions we get a precipitate of $K_2Ca(CO_3)_2$.
(d) Manganese salts give a violet borax bead test in the reducing flame

101. Glass is a
(a) super-cooled liquid (b) gel (c) polymeric mixture (d) micro-crystalline solid

102. The orbital angular momentum for an electron revolving in an orbit is given by $\sqrt{l(l+1)}.\frac{h}{2\pi}$. This momentum for an s-electron will be given by
(a) zero (b) $\frac{h}{2\pi}$ (c) $\sqrt{2}.\frac{h}{2\pi}$ (d) $+\frac{1}{2}.\frac{h}{2\pi}$

103. How many unit cells are present in a cubeshaped ideal crystal of NaCl of mass 1.00 g?
[Atomic masses: Na = 23, Cl = 35.5]
(a) 5.14×10^{21} unit cells (b) 1.28×10^{21} unit cells
(c) 1.71×10^{21} unit cells (d) 2.57×10^{21} unit cells

104. In the anion $HCOO^-$ the two carbon-oxygen bonds are found to be of equal length. What is the reason for it?
(a) The C = O bond is weaker than the C-O bond
(b) The anion $HCOO^-$ has two resonating structures
(c) The anion is obtained by removal of a proton from the acid molecule
(d) Electronic orbitals of carbon atom are hybridised

105. Which one of the following characteristics is not correct for physical adsorption?
(a) Adsorption increases with incresae in temperature
(b) Adsorption is spontaneous (c) Both enthalpy and entropy of adsorption are negative
(d) Adsorption on solids is reversible

106. For a cell reaction involving a two-electron change, the standard e.m.f. of the cell is found to be 0.295 V at 25^0C. The equilibrium constant of the reaction at 25^0C will be
(a) 29.5×10^{-2} (b) 10 (c) 1×10^{10} (d) 1×10^{-10}

107. In an irreversible process taking place at constant T and P and in which only pressure-volume work is being done, the change in Gibbs free energy (dG) and change in entropy (dS), satisfy the criteria
(a) $(dS)_{V,E} > 0, (dG)_{T,P} < 0$ (b) $(dS)_{V,E} = 0, (dG)_{T,P} = 0$ (c) $(dS)_{V,E} = 0, (dG)_{T,P} > 0$ (d) $(dS)_{V,E} < 0, (dG)_{T,P} < 0$

108. The solubility in water of a sparingly soluble salt AB_2 is 1.0×10^{-5} mol L^{-1}. Its solubility product number will be
(a) 4×10^{-10} (b) 1×10^{-15} (c) 1×10^{-10} (d) 4×10^{-15}

109. What volume of hydrogen gas, at 273 K and 1 atm, pressure will be consumed in obtaining 21.6 g of elemental boron (atomic mass = 10.8) from the reducti on of boron trichloride by hydrogen?
(a) 67.2 L (b) 44.8 L (c) 22.4 L (d) 89.6 L

110. For the reaction equilibrium $N_2O_4(g) \rightleftharpoons 2NO_2(g)$ the concentrations of N_2O_4 and NO_2 at equilibrium are 4.8×10^{-2} and 1.2×10^{-2} mol L^{-1} respectively. The value of K_c for the reaction is
(a) 3×10^{-1} mol L^{-1} (b) 3×10^{-3} mol L^{-1} (c) 3×10^{3} mol L^{-1} (d) 3.3×10^{2} mol L^{-1}

111. Consider the reaction equilibrium $2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g)$; $\Delta H^0 = -198$ kJ. On the basis of Le Chatelier's principle, the condition favourable for the forward reaction is
(a) increasing temperature as well as pressure (b) lowering the temperature and increasing the pressure
(c) any value of temperature and pressure (d) lowering of temperature as well as pressure

112. Which one of the following is an amphoteric oxide?

(a) Na_2O (b) SO_2 (c) B_2O_3 (d) ZnO

113. A red solid is insoluble in water. However it becomes soluble if some KI is added to water. Heating the red solid in a test tube results in liberation of some violet coloured fumes and droplets of a metal appear on the cooler parts of the test tube. The red solid is

(a) HgI_2 (b) HgO (c) Pb_3O_4 (d) $(NH_4)_2Cr_2O_7$

114. Standard reduction electrode potentials of three metals A,B&C are respectively +0.5 V, -3.0 V & -1.2 V. The reducing, powers of these metals are

(a) A > B > C (c) C > B > A (c) A > C > B (d) B > C > A

115. Which one of the following substances has the highest proton affinity?

(a) H_2S (b) NH_3 (c) PH_3 (d) H_2O

116. In a 0.2 molal aqueous solution of a weak acid HX the degree of ionization is 0.3. Taking k_f for water as 1.85, the freezing point of the solution will be nearest to

(a) -0.360^0C (b) -0.260^0C (c) $+0.480^0C$ (d) -0.480^0C

117. When during electrolysis of a solution of $AgNO_3$ 9650 coulombs of charge pass through the electroplating bath, the mass of silver deposited on the cathode will be

(a) 10.8 g (b) 21.6 g (c) 108 g (d) 1.08 g

118. For the redox reaction $Zn(s) + Cu^{2+}(0.1\ M) \rightarrow Zn^{2+}(1M) + Cu(s)$ taking place in a cell, E^0_{cell} is 1.10 volt. E_{cell} for the cell will be $\left(2.303\frac{RT}{F} = 0.0591\right)$

(a) 1.80 volt (b) 1.07 volt (c) 0.82 volt (d) 2.14 volt

119. In respect of the equation $k = Ae^{-E_a/RT}$ in chemical kinetics, which one of the following statements is correct?

(a) A is adsorption factor (b) E_a is energy of activation

(c) R is Rydberg's constant (d) k is equilibrium constant

120. A reduction in atomic size with increase in atomic number is a characteristic of element of

(A) d-block (b) f-block (c) radioactive series (d) high atomic masses

121. The IUPAC name of $CH_3COCH(CH_3)_2$ is

(a) 2-methyl-3-butanone (b) 4-methylisopropyl ketone (c) 3-methyl-2-butanone (d) Isopropylmethyl ketone

122. When $CH_2 = CH - COOH$ is reduced with $LiAlH_4$, the compound obtained will be

(a) $CH_2 = CH - CH_2OH$ (b) $CH_3 - CH_2 - CH_2OH$

(c) $CH_3 - CH_2 - CHO$ (d) $CH_3 - CH_2 - COOH$

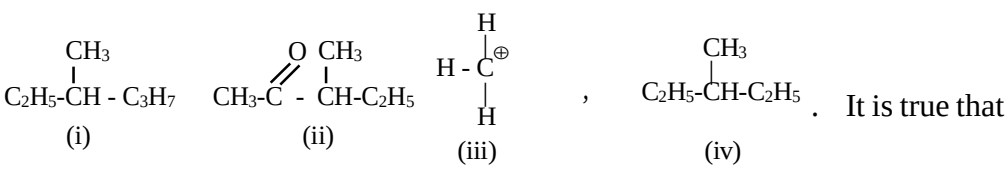
123. According to the kinetic theory of gases, in an ideal gas, between two successive collisions a gas molecule travels

(a) in a wavy path (b) in a straight line path (c) with an accelerated velocity (d) in a circular path

124. The general formula $C_nH_{2n}O_2$ could be for open chain

(a) carboxylic acids (b) diols (c) dialdehydes (d) deketones

125. Among the following four structures I to IV.



(i) (ii) (iii) (iv)

. It is true that

(a) only I and II are chiral compounds (b) only III i a chiral compound

(c) only II and IV are chiral compounds (d) all four are chiral compounds

126. What would happen when a solution of potassium chromate is treated with an excess of dilute nitric acid?
(a) $Cr_2O^{2-}_7$ and H_2O are formed (b) CrO^{2-}_4 is reduced to +3 state of Cr
(c) CrO^{2-}_4 is oxidized to +7 state of Cr (d) Cr^{3+} and $Cr_2O^{2-}_7$ are formed

127. For making good quality mirrors, plates of float glass are used. These are obtained by floating molten glass over a liquid metal which does not solidify before glass. The metal used can be
(a) tin (b) sodium (c) magnesium (d) mercury

128. The substance not likely to contain $CaCO_3$ is
(a) calcined gypsum (b) sea shells (c) dolomite (d) a marble statue

129. Complete hydrolysis of cellulose gives
(a) D-ribose (b) D-glucose (c) L-glucose (d) D-fructose

130. Which one of the following nitrates will leave behind a metal on strong heating?
(a) Copper nitrate (b) Manganese nitrate (c) Silver nitrate (d) Ferric nitrate

131. During dehydration of alcohols to alkenes by heating with conc. H_2SO_4 the initiation step is
(a) formation of carbocation (b) elimination of water
(c) formation of an ester (d) protonation of alcohol molecule

132. The solubilities of carbonates decrease down the magnesium group due to a decrease in
(a) hydration energies of cations (b) inter-ionic attraction
(c) entropy of solution formation (d) lattice energies of solids

133. When rain is accompanied by a thunderstorm, the collected rain water will have a pH value
(a) slightly higher than that when the thunderstorm is not there
(b) uninfluenced by occurence of thunderstorm
(c) which depends on the amount of dust in air
(d) slightly lower than that of rain water without thunderstorm

134. The reason for double helical structure of DNA is operation of
(a) dipole-dipole interaction (b) hydrogen bonding (c) electrostatic attractions (d) van der Waals' forces

135. 25 ml of a solution of barrium hydroxide on titration with a 0.1 molar solution of hydrochloric acid gave a litre value of 35 ml. The molarity of barium hydroxide solution was
(a) 0.14 (b) 0.28 (c) 0.35 (d) 0.07

136. The correct relationship between free energy change in a reaction and the corresponding equilibrium constant K_c is
(a) $-\Delta G = RT\ ln\ K_c$ (b) $\Delta G^0 = RT\ ln\ K_c$ (c) $-\Delta G^0 = RT\ In\ K_c$ (d) $\Delta G = RT\ ln\ K_c$

137. The rate law for a reaction between the substances A and B is given by Rate = $k[A]^n\ [B]^m$ On doubling the concentration of A and halving the concentration of B, the ratio of the new rate to the earlier rate of the reaction will be as
(a) (m + n) (b) (n - m) (c) $2^{(n-m)}$ (d) $\frac{1}{2^{(m+n)}}$

138. Ethyl isocyanide on hydrolysis in acidic medium generates
(a) propanoic acid and ammonium salt (b) ethanoic acid and ammonium salt
(c) methylamine salt and ethanoic acid (d) ethylamine salt and methanoic acid

139. The enthalpy change for a reaction does not depend upon
(a) use of different reactants for the same product (b) the nature of intermediate reaction steps
(c) the differences in initial or final temperatures of involved substances
(d) the physical states of reactants and products

140. A pressure cooker reduces cooking time for food because
(a) boiling point of water involved in cooking is increased
(b) the higher pressure inside the cooker crushes the food material
(c) cooking involves chemical changes helped by a rise in temperature
(d) heat is more evenly distributed in the cooking space

141. For the reaction system: $2NO(g) + O_2(g) \rightarrow 2NO_2(g)$ volume is suddenly reduce to half its value by increasing the pressure on it. If the reaction is of first order with respect to O_2 and second order with respect to NO, the rate of reaction will
(a) diminish to one-eighth of its initial value
(b) increase to eight times of its initial value
(c) increase to four times of its initial value
(d) diminish to one-fourth of its initial value

142. Several blocks of magnesium are fixed to the bottom of a ship to
(a) make the ship lighter
(b) prevent action of water and salt
(c) prevent puncturing by under-sea rocks
(d) keep away the sharks

143. Which one of the following pairs of molecules will have permanent dipole moments for both members?
(a) NO_2 and CO_2 (b) NO_2 and O_3 (c) SiF_4 and CO_2 (d) SiF_4 and NO_2

144. Which one of the following groupings represents a collection of isoelectronic species? (At. nos,: 55, Br:35)
(a) N^{3-}, F^-, Na^+ (b) Be, Al^{3+}, Cl^- (c) Ca^{2+}, Cs^+, Br (d) Na^+, Ca^{2+}, Mg^{2+}

145. Which one of the following processes will produce hard water?
(a) Saturation of water with $MgCO_3$
(b) Saturation of water with $CaSO_4$
(c) Addition of Na_2SO_4 to water
(d) Saturation of water with $CaCO_3$

146. Which one of the following compounds has the smallest bond angle in its molecule?
(a) OH_2 (b) SH_2 (c) NH_3 (d) SO_2

147. The pair of species having identical shapes for molecules of both species is
(a) XeF_2, CO_2 (b) BF_3, PCl_3
(c) PF_5, IF_5 (d) CF_4, SF_4

148. The atomic numbers of vanadium (V), Chromium (Cr), manganese (Mn) and iron (Fe) are respectively 23, 24, 25 and 26. Which one of these may be expected to have the highest second ionization enthalpy?
(a) Cr (b) Mn (c) Fe (d) V

149. In Bohr series of lines of hydrogen spectrum, the third line from the red end corresponds to which one of the following inter-orbit jumps of the electron for Bohr orbits in an atom of hydrogen
(a) $5 \rightarrow 2$ (b) $4 \rightarrow 1$ (c) $2 \rightarrow 5$ (d) $3 \rightarrow 2$

150. The de Broglie wavelength of a tennis ball of mass 60 g moving with a velocity of 10 metres per second is approximately
(a) 10^{-31} metres
(b) 10^{-16} metres
(c) 10^{-25} metres
(d) 10^{-33} metres Planck's constant, $h = 6.63 \times 10^{-34}$ Js.

AIEEE–CBSE–ENG–03

1. A function f from the set of natural numbers to integers defined by

$$f(n) = \begin{cases} \frac{n-1}{2}, & \text{when is odd} \\ -\frac{n}{2}, & \text{when n is even} \end{cases}$$ is

(A) one–one but not onto
(B) onto but not one–one
(C) one–one and onto both
(D) neither one–one nor onto

2. Let z_1 and z_2 be two roots of the equation $z^2 + az + b = 0$, z being complex. Further, assume that the origin, z_1 and z_2 form an equilateral triangle, then
(A) $a^2 = b$
(B) $a^2 = 2b$
(C) $a^2 = 3b$
(D) $a^2 = 4b$

3. If z and ω are two non–zero complex numbers such that $|z\omega| = 1$, and Arg (z) – Arg (ω) = $\frac{\pi}{2}$, then $\bar{z}\omega$ is equal to
(A) 1
(B) – 1
(C) i
(D) – i

4. If $\left(\frac{1+i}{1-i}\right)^x = 1$, then
(A) x = 4n, where n is any positive integer
(B) x = 2n, where n is any positive integer
(C) x = 4n + 1, where n is any positive integer
(D) x = 2n + 1, where n is any positive integer

5. If $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$ and vectors (1, a, a^2) (1, b, b^2) and (1, c, c^2) are non–coplanar, then the product abc equals
(A) 2
(B) – 1
(C) 1
(D) 0

6. If the system of linear equations
x + 2ay + az = 0
x + 3by + bz = 0
x + 4cy + cz = 0
has a non–zero solution, then a, b, c
(A) are in A. P.
(B) are in G.P.
(C) are in H.P.
(D) satisfy a + 2b + 3c = 0

7. If the sum of the roots of the quadratic equation $ax^2 + bx + c = 0$ is equal to the sum of the squares of their reciprocals, then $\frac{a}{c}, \frac{b}{a}$ and $\frac{c}{b}$ are in
(A) arithmetic progression
(B) geometric progression
(C) harmonic progression
(D) arithmetic–geometric–progression

8. The number of real solutions of the equation $x^2 - 3|x| + 2 = 0$ is
(A) 2
(B) 4
(C) 1
(D) 3

9. The value of 'a' for which one root of the quadratic equation $(a^2 - 5a + 3)\,x^2 + (3a - 1)\,x + 2 = 0$ is twice as large as the other, is

(A) $\frac{2}{3}$ (B) $-\frac{2}{3}$

(C) $\frac{1}{3}$ (D) $-\frac{1}{3}$

I10. If $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$ and $A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$, then

(A) $\alpha = a^2 + b^2$, $\beta = ab$ (B) $\alpha = a^2 + b^2$, $\beta = 2ab$

(C) $\alpha = a^2 + b^2$, $\beta = a^2 - b^2$ (D) $\alpha = 2ab$, $\beta = a^2 + b^2$

11. A student is to answer 10 out of 13 questions in an examination such that he must choose at least 4 from the first five questions. The number of choices available to him is

(A) 140 (B) 196

(C) 280 (D) 346

12. The number of ways in which 6 men and 5 women can dine at a round table if no two women are to sit together is given by

(A) $6! \times 5!$ (B) 30

(C) $5! \times 4!$ (D) $7! \times 5!$

13. If 1, ω, ω^2 are the cube roots of unity, then

$\Delta = \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^n & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^n \end{vmatrix}$ is equal to

(A) 0 (B) 1

(C) ω (D) ω^2

14. If ${}^nC_r$ denotes the number of combinations of n things taken r at a time, then the expression ${}^nC_{r+1} + {}^nC_{r-1} + 2 \times {}^nC_r$ equals

(A) ${}^{n+2}C_r$ (B) ${}^{n+2}C_{r+1}$

(C) ${}^{n+1}C_r$ (D) ${}^{n+1}C_{r+1}$

15. The number of integral terms in the expansion of $(\sqrt{3} + \sqrt[8]{5})^{256}$ is

(A) 32 (B) 33

(C) 34 (D) 35

16. If x is positive, the first negative term in the expansion of $(1 + x)^{27/5}$ is

(A) 7^{th} term (B) 5^{th} term

(C) 8^{th} term (D) 6^{th} term

17. The sum of the series $\frac{1}{1\cdot 2} - \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} - \ldots\ldots$ upto ∞ is equal to

(A) $2 \log_e 2$ (B) $\log_2 2 - 1$

(C) $\log_e 2$ (D) $\log_e\left(\frac{4}{e}\right)$

18. Let f (x) be a polynomial function of second degree. If f (1) = f (– 1) and a, b, c are in A. P., then f′ (a), f′ (b) and f′ (c) are in

(A) A.P. (B) G.P.

(C) H. P. (D) arithmetic–geometric progression

19. If x_1, x_2, x_3 and y_1, y_2, y_3 are both in G.P. with the same common ratio, then the points (x_1, y_1) (x_2, y_2) and (x_3, y_3)
(A) lie on a straight line
(B) lie on an ellipse
(C) lie on a circle
(D) are vertices of a triangle

20. The sum of the radii of inscribed and circumscribed circles for an n sided regular polygon of side a, is
(A) $a \cot\left(\frac{\pi}{n}\right)$
(B) $\frac{a}{2} \cot\left(\frac{\pi}{2n}\right)$
(C) $a \cot\left(\frac{\pi}{2n}\right)$
(D) $\frac{a}{4} \cot\left(\frac{\pi}{2n}\right)$

21. If in a triangle ABC $a \cos^2\left(\frac{C}{2}\right) + c \cos^2\left(\frac{A}{2}\right) = \frac{3b}{2}$, then the sides a, b and c
(A) are in A.P.
(B) are in G.P.
(C) are in H.P.
(D) satisfy $a + b = c$

22. In a triangle ABC, medians AD and BE are drawn. If AD = 4, $\angle$ DAB = $\frac{\pi}{6}$ and $\angle$ ABE = $\frac{\pi}{3}$, then the area of the Δ ABC is
(A) $\frac{8}{3}$
(B) $\frac{16}{3}$
(C) $\frac{32}{3}$
(D) $\frac{64}{3}$

23. The trigonometric equation $\sin^{-1} x = 2 \sin^{-1} a$, has a solution for
(A) $\frac{1}{2} < |a| < \frac{1}{\sqrt{2}}$
(B) all real values of a
(C) $|a| < \frac{1}{2}$
(D) $|a| \geq \frac{1}{\sqrt{2}}$

24. The upper $\frac{3}{4}$th portion of a vertical pole subtends an angle $\tan^{-1} \frac{3}{5}$ at point in the horizontal plane through its foot and at a distance 40 m from the foot. A possible height of the vertical pole is
(A) 20 m
(B) 40 m
(C) 60 m
(D) 80 m

25. The real number x when added to its inverse gives the minimum value of the sum at x equal to
(A) 2
(B) 1
(C) -1
(D) -2

26. If $f : R \to R$ satisfies $f(x + y) = f(x) + f(y)$, for all $x, y \in R$ and $f(1) = 7$, then $\sum_{r=1}^{n} f(r)$ is
(A) $\frac{7n}{2}$
(B) $\frac{7(n+1)}{2}$
(C) $7n(n + 1)$
(D) $\frac{7n(n+1)}{2}$

27. If f (x) = x^n, then the value of $f(1) - \frac{f'(1)}{1!} + \frac{f''(1)}{2!} - \frac{f'''(1)}{3!} + \ldots + \frac{(-1)^n f^n(1)}{n!}$ is

(A) 2^n (B) 2^{n-1}
(C) 0 (D) 1

28. Domain of definition of the function $f(x) = \frac{3}{4 - x^2} + \log_{10}(x^3 - x)$, is

(A) (1, 2) (B) $(-1, 0) \cup (1, 2)$
(C) $(1, 2) \cup (2, \infty)$ (D) $(-1, 0) \cup (1, 2) \cup (2, \infty)$

29. $\lim_{x \to \pi/2} \frac{\left[1 - \tan\left(\frac{x}{2}\right)\right][1 - \sin x]}{\left[1 + \tan\left(\frac{x}{2}\right)\right][\pi - 2x]^3}$ is

(A) $\frac{1}{8}$ (B) 0
(C) $\frac{1}{32}$ (D) ∞

30. If $\lim_{x \to 0} \frac{\log(3 + x) - \log(3 - x)}{x}$ = k, the value of k is

(A) 0 (B) $-\frac{1}{3}$
(C) $\frac{2}{3}$ (D) $-\frac{2}{3}$

31. Let f (a) = g (a) = k and their n^{th} derivatives f^n (a), g^n (a) exist and are not equal for some n. Further if $\lim_{x \to a} \frac{f(a)g(x) - f(a) - g(a)f(x) + g(a)}{g(x) - f(x)}$ = 4, then the value of k is

(A) 4 (B) 2
(C) 1 (D) 0

32. The function f (x) = log $(x + \sqrt{x^2 + 1})$, is

(A) an even function (B) an odd function
(C) a periodic function (D) neither an even nor an odd function

33. If $f(x) = \begin{cases} xe^{-\left(\frac{1}{|x|} + \frac{1}{x}\right)}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ then f (x) is

(A) continuous as well as differentiable for all x
(B) continuous for all x but not differentiable at x = 0
(C) neither differentiable nor continuous at x = 0
(D) discontinuous everywhere

34. If the function $f(x) = 2x^3 - 9ax^2 + 12a^2 x + 1$, where a > 0, attains its maximum and minimum at p and q respectively such that $p^2 = q$, then a equals

(A) 3 (B) 1
(C) 2 (D) $\frac{1}{2}$

35. If $f(y) = e^y$, $g(y) = y$; $y > 0$ and $F(t) = \int_0^t f(t-y)\, g(y)\, dy$, then

(A) $F(t) = 1 - e^{-t}(1 + t)$ (B) $F(t) = e^t - (1 + t)$
(C) $F(t) = t\, e^t$ (D) $F(t) = t\, e^{-t}$

36. If $f(a + b - x) = f(x)$, then $\int_a^b x\ f(x)\, dx$ is equal to

(A) $\frac{a+b}{2}\int_a^b f(b-x)dx$ (B) $\frac{a+b}{2}\int_a^b f(x)dx$
(C) $\frac{b-a}{2}\int_a^b f(x)dx$ (D) $\frac{a+b}{2}\int_a^b f(a+b-x)dx$

37. The value of $\lim_{x\to 0} \frac{\int_0^{x^2} \sec^2 t\, dt}{x \sin x}$ is

(A) 3 (B) 2
(C) 1 (D) 0

38. The value of the integral $I = \int_0^1 x\,(1-x)^n\, dx$ is

(A) $\frac{1}{n+1}$ (B) $\frac{1}{n+2}$
(C) $\frac{1}{n+1} - \frac{1}{n+2}$ (D) $\frac{1}{n+1} + \frac{1}{n+2}$

39. $\lim_{n\to\infty} \frac{1+2^4+3^4+.....+n^4}{n^5} - \lim_{n\to\infty} \frac{1+2^3+3^3+.....+n^3}{n^5}$ is

(A) $\frac{1}{30}$ (B) zero
(C) $\frac{1}{4}$ (D) $\frac{1}{5}$

40. Let $\frac{d}{dx}F(x) = \left(\frac{e^{\sin x}}{x}\right)$, $x > 0$. If $\int_1^4 \frac{3}{x} e^{\sin x^3}\, dx = F(k) - F(1)$, then one of the possible values of k, is

(A) 15 (B) 16
(C) 63 (D) 64

41. The area of the region bounded by the curves $y = |x - 1|$ and $y = 3 - |x|$ is

(A) 2 sq units (B) 3 sq units
(C) 4 sq units (D) 6 sq units

42. Let f (x) be a function satisfying $f'(x) = f(x)$ with $f(0) = 1$ and g (x) be a function that satisfies $f(x) + g(x) = x^2$. Then the value of the integral $\int_0^1 f(x)\, g(x)\, dx$, is

(A) $e - \frac{e^2}{2} - \frac{5}{2}$ (B) $e + \frac{e^2}{2} - \frac{3}{2}$

(C) $e - \frac{e^2}{2} - \frac{3}{2}$ (D) $e + \frac{e^2}{2} + \frac{5}{2}$

43. The degree and order of the differential equation of the family of all parabolas whose axis is x–axis, are respectively
(A) 2, 1 (B) 1, 2
(C) 3, 2 (D) 2, 3

44. The solution of the differential equation $(1 + y^2) + (x - e^{\tan^{-1} y}) \frac{dy}{dx} = 0$, is
(A) $(x - 2) = k\ e^{-\tan^{-1} y}$ (B) $2x e^{2\tan^{-1} y} + k$
(C) $x\ e^{\tan^{-1} y} = \tan^{-1} y + k$ (D) $x e^{2\tan^{-1} y} = e^{\tan^{-1} y} + k$

45. If the equation of the locus of a point equidistant from the points (a_1, b_1) and (a_2, b_2) is $(a_1 - a_2)\ x + (b_1 - b_2)\ y + c = 0$, then the value of 'c' is
(A) $\frac{1}{2}(a_2^2 + b_2^2 - a_1^2 - b_1^2)$ (B) $a_1^2 + a_2^2 + b_1^2 - b_2^2$
(C) $\frac{1}{2}(a_1^2 + a_2^2 - b_1^2 - b_2^2)$ (D) $\sqrt{a_1^2 + b_1^2 - a_2^2 - b_2^2}$

46. Locus of centroid of the triangle whose vertices are (a cos t, a sin t), (b sin t, – b cos t) and (1, 0), where t is a parameter, is
(A) $(3x - 1)^2 + (3y)^2 = a^2 - b^2$ (B) $(3x - 1)^2 + (3y)^2 = a^2 + b^2$
(C) $(3x + 1)^2 + (3y)^2 = a^2 + b^2$ (D) $(3x + 1)^2 + (3y)^2 = a^2 - b^2$

47. If the pair of straight lines $x^2 - 2pxy - y^2 = 0$ and $x^2 - 2qxy - y^2 = 0$ be such that each pair bisects the angle between the other pair, then
(A) p = q (B) p = – q
(C) pq = 1 (D) pq = – 1

48. a square of side a lies above the x–axis and has one vertex at the origin. The side passing through the origin makes an angle α $(0 < \alpha < \frac{\pi}{4})$ with the positive direction of x–axis. The equation of its diagonal not passing through the origin is
(A) $y(\cos\alpha - \sin\alpha) - x(\sin\alpha - \cos\alpha) = a$
(B) $y(\cos\alpha + \sin\alpha) + x(\sin\alpha - \cos\alpha) = a$
(C) $y(\cos\alpha + \sin\alpha) + x(\sin\alpha + \cos\alpha) = a$
(D) $y(\cos\alpha + \sin\alpha) + x(\cos\alpha - \sin\alpha) = a$

49. If the two circles $(x - 1)^2 + (y - 3)^2 = r^2$ and $x^2 + y^2 - 8x + 2y + 8 = 0$ intersect in two distinct points, then
(A) $2 < r < 8$ (B) $r < 2$
(C) r = 2 (D) $r > 2$

50. The lines $2x - 3y = 5$ and $3x - 4y = 7$ are diameters of a circle having area as 154 sq units. Then the equation of the circle is
(A) $x^2 + y^2 + 2x - 2y = 62$ (B) $x^2 + y^2 + 2x - 2y = 47$
(C) $x^2 + y^2 - 2x + 2y = 47$ (D) $x^2 + y^2 - 2x + 2y = 62$

51. The normal at the point $(bt_1^2, 2bt_1)$ on a parabola meets the parabola again in the point $(bt_2^2, 2bt_2)$, then

(A) $t_2 = -t_1 - \frac{2}{t_1}$ (B) $t_2 = -t_1 + \frac{2}{t_1}$

(D) $t_2 = t_1 - \frac{2}{t_1}$ (D) $t_2 = t_1 + \frac{2}{t_1}$

52. The foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ and the hyperbola $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$ coincide. Then the value of b^2 is

(A) 1 (B) 5

(C) 7 (D) 9

53. A tetrahedron has vertices at O (0, 0, 0), A (1, 2, 1), B (2, 1, 3) and C (– 1, 1, 2). Then the angle between the faces OAB and ABC will be

(A) $\cos^{-1}\left(\frac{19}{35}\right)$ (B) $\cos^{-1}\left(\frac{17}{31}\right)$

(C) 30^0 (D) 90^0

54. The radius of the circle in which the sphere $x^2 + y^2 + z^2 + 2x - 2y - 4z - 19 = 0$ is cut by the plane $x + 2y + 2z + 7 = 0$ is

(A) 1 (B) 2

(C) 3 (D) 4

55. The lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$ and $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$ are coplanar if

(A) k = 0 or – 1 (B) k = 1 or – 1

(C) k = 0 or – 3 (D) k = 3 or – 3

56. The two lines x = ay + b, z = cy + d and x = a′y + b′, z = c′y + d′ will be perpendicular, if and only if

(A) aa′ + bb′ + cc′ + 1 = 0 (B) aa′ + bb′ + cc′ = 0

(C) (a + a′) (b + b′) + (c + c′) = 0 (D) aa′ + cc′ + 1 = 0

57. The shortest distance from the plane 12x + 4y + 3z = 327 to the sphere $x^2 + y^2 + z^2 + 4x - 2y - 6z = 155$ is

(A) 26 (B) $11\frac{4}{13}$

(C) 13 (D) 39

58. Two systems of rectangular axes have the same origin. If a plane cuts them at distances a, b, c and a′, b′, c′ from the origin, then

(A) $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} + \frac{1}{a'^2} + \frac{1}{b'^2} + \frac{1}{c'^2} = 0$ (B) $\frac{1}{a^2} + \frac{1}{b^2} - \frac{1}{c^2} + \frac{1}{a'^2} + \frac{1}{b'^2} - \frac{1}{c'^2} = 0$

(C) $\frac{1}{a^2} - \frac{1}{b^2} - \frac{1}{c^2} + \frac{1}{a'^2} - \frac{1}{b'^2} - \frac{1}{c'^2} = 0$ (D) $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} - \frac{1}{a'^2} - \frac{1}{b'^2} - \frac{1}{c'^2} = 0$

59. $\vec{a}, \vec{b}, \vec{c}$ are 3 vectors, such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, $|\vec{a}| = 1$, $|\vec{b}| = 2$, $|\vec{c}| = 3$, then $\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}$ is equal to

(A) 0 (B) – 7

(C) 7 (D) 1

60. If $\vec{u}, \vec{v}$ and $\vec{w}$ are three non–coplanar vectors, then $(\vec{u} + \vec{v} - \vec{w})\cdot(\vec{u} - \vec{v})\times(\vec{v} - \vec{w})$ equals

(A) 0 (B) $\vec{u}\cdot\vec{v}\times\vec{w}$

(C) $\vec{u} \cdot \vec{w} \times \vec{v}$ (D) $3\vec{u} \cdot \vec{v} \times \vec{w}$

61. Consider points A, B, C and D with position vectors $7\hat{i} - 4\hat{j} + 7\hat{k}$, $\hat{i} - 6\hat{j} + 10\hat{k}$, $-\hat{i} - 3\hat{j} + 4\hat{k}$ and $5\hat{i} - \hat{j} + 5\hat{k}$ respectively. Then ABCD is a
(A) square (B) rhombus
(C) rectangle (D) parallelogram but not a rhombus

62. The vectors $\overrightarrow{AB} = 3\hat{i} + 4\hat{k}$, and $\overrightarrow{AC} = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are the sides of a triangle ABC. The length of the median through A is
(A) $\sqrt{18}$ (B) $\sqrt{72}$
(C) $\sqrt{33}$ (D) $\sqrt{288}$

63. A particle acted on by constant forces $4\hat{i} + \hat{j} - 3\hat{k}$ and $3\hat{i} + \hat{j} - \hat{k}$ is displaced from the point $\hat{i} + 2\hat{j} + 3\hat{k}$ to the point $5\hat{i} + 4\hat{j} + \hat{k}$. The total work done by the forces is
(A) 20 units (B) 30 units
(C) 40 units (D) 50 units

64. Let $\vec{u} = \hat{i} + \hat{j}$, $\vec{v} = \hat{i} - \hat{j}$ and $\vec{w} = \hat{i} + 2\hat{j} + 3\hat{k}$. If $\hat{n}$ is unit vector such that $\vec{u} \cdot \hat{n} = 0$ and $\vec{v} \cdot \hat{n} = 0$, then $|\vec{w} \cdot \hat{n}|$ is equal to
(A) 0 (B) 1
(C) 2 (D) 3

65. The median of a set of 9 distinct observations is 20.5. If each of the largest 4 observations of the set is increased by 2, then the median of the new set
(A) is increased by 2 (B) is decreased by 2
(C) is two times the original median (D) remains the same as that of the original set

66. In an experiment with 15 observations on x, then following results were available:
$\sum x^2 = 2830$, $\sum x = 170$
One observation that was 20 was found to be wrong and was replaced by the correct value 30. Then the corrected variance is
(A) 78.00 (B) 188.66
(C) 177.33 (D) 8.33

67. Five horses are in a race. Mr. A selects two of the horses at random and bets on them. The probability that Mr. A selected the winning horse is
(A) $\frac{4}{5}$ (B) $\frac{3}{5}$
(C) $\frac{1}{5}$ (D) $\frac{2}{5}$

68. Events A, B, C are mutually exclusive events such that P (A) = $\frac{3x+1}{3}$, P (B) = $\frac{1-x}{4}$ and P (C) = $\frac{1-2x}{2}$. The set of possible values of x are in the interval
(A) $\left[\frac{1}{3}, \frac{1}{2}\right]$ (B) $\left[\frac{1}{3}, \frac{2}{3}\right]$
(C) $\left[\frac{1}{3}, \frac{13}{3}\right]$ (D) [0, 1]

69. The mean and variance of a random variable having a binomial distribution are 4 and 2 respectively, then P (X = 1) is

(A) $\frac{1}{32}$ (B) $\frac{1}{16}$

(C) $\frac{1}{8}$ (D) $\frac{1}{4}$

70. The resultant of forces $\vec{P}$ and $\vec{Q}$ is $\vec{R}$. If $\vec{Q}$ is doubled then $\vec{R}$ is doubled. If the direction of $\vec{Q}$ is reversed, then $\vec{R}$ is again doubled. Then $P^2 : Q^2 : R^2$ is

(A) 3 : 1 : 1 (B) 2 : 3 : 2

(C) 1 : 2 : 3 (D) 2 : 3 : 1

71. Let R_1 and R_2 respectively be the maximum ranges up and down an inclined plane and R be the maximum range on the horizontal plane. Then R_1, R, R_2 are in

(A) arithmetic–geometric progression (B) A.P.

(C) G.P. (D) H.P.

72. A couple is of moment $\vec{G}$ and the force forming the couple is $\vec{P}$. If $\vec{P}$ is turned through a right angle, the moment of the couple thus formed is $\vec{H}$. If instead, the forces $\vec{P}$ are turned through an angle α, then the moment of couple becomes

(A) $\vec{G} \sin\alpha - \vec{H} \cos\alpha$ (B) $\vec{H} \cos\alpha + \vec{G} \sin\alpha$

(C) $\vec{G} \cos\alpha - \vec{H} \sin\alpha$ (D) $\vec{H} \sin\alpha - \vec{G} \cos\alpha$

73. Two particles start simultaneously from the same point and move along two straight lines, one with uniform velocity $\vec{u}$ and the other from rest with uniform acceleration $\vec{f}$. Let α be the angle between their directions of motion. The relative velocity of the second particle with respect to the first is least after a time

(A) $\frac{u\sin\alpha}{f}$ (B) $\frac{f\cos\alpha}{u}$

(C) $u \sin\alpha$ (D) $\frac{u\cos\alpha}{f}$

74. Two stones are projected from the top of a cliff h meters high, with the same speed u so as to hit the ground at the same spot. If one of the stones is projected horizontally and the other is projected at an angle θ to the horizontal then $\tan\theta$ equals

(A) $\sqrt{\frac{2u}{gh}}$ (B) $2g\sqrt{\frac{u}{h}}$

(C) $2h\sqrt{\frac{u}{g}}$ (D) $u\sqrt{\frac{2}{gh}}$

75. A body travels a distances s in t seconds. It starts from rest and ends at rest. In the first part of the journey, it moves with constant acceleration f and in the second part with constant retardation r. The value of t is given by

(A) $2s\left(\frac{1}{f}+\frac{1}{r}\right)$ (B) $\frac{2s}{\frac{1}{f}+\frac{1}{r}}$

(C) $\sqrt{2s(f+r)}$ (D) $\sqrt{2s\left(\frac{1}{f}+\frac{1}{r}\right)}$

Solutions

1. Clearly both one – one and onto
Because if n is odd, values are set of all non–negative integers and if n is an even, values are set of all negative integers.
Hence, (C) is the correct answer.

2. $z_1^2 + z_2^2 - z_1z_2 = 0$
$(z_1 + z_2)^2 - 3z_1z_2 = 0$
$a^2 = 3b$.
Hence, (C) is the correct answer.

3. $\text{Arg}\left(\frac{z}{\omega}\right) = \frac{\pi}{2}$
$|z\omega| = 1$
$\bar{z}\,\omega = -i$ or $+i$.

4. $\frac{1+i}{1-i} = \frac{(1+i)^2}{2} = i$
$\left(\frac{1+i}{1-i}\right)^x = i^x$
$\Rightarrow x = 4n$.
Hence, (A) is the correct answer.

5. $\begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} + \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0$
$(1 + abc)\begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} = 0$
$\Rightarrow abc = -1$.
Hence, (B) is the correct answer

6. Coefficient determinant = $\begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0$
$\Rightarrow b = \frac{2ac}{a+c}$.
Hence, (C) is the correct answer

7. Let α, β be the roots
$\alpha + \beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2}$
$\alpha + \beta = \frac{\alpha^2 + \beta^2 - 2\alpha\beta}{(\alpha + \beta)}$
$\left(\frac{b}{a}\right) = \frac{b^2}{c^2} \quad \frac{2ac}{}$
$\Rightarrow 2a^2c = b(a^2 + bc)$
$\Rightarrow \frac{a}{c}, \frac{b}{a}, \frac{c}{b}$ are in H.P.
Hence, (C) is the correct answer

8. $x^2 - 3|x| + 2 = 0$
$(|x| - 1)(|x| - 2) = 0$
$\Rightarrow x = \pm 1, \pm 2$.
Hence, (B) is the correct answer

9. $\beta = 2\alpha$
$3\alpha = \frac{3a \quad 1}{a^2 - 5a + 3}$
$2\alpha^2 = \frac{2}{a^2 - 5a + 6}$
$\frac{(3a-1)^2}{a(a^2 - 5a + 3)^2} = \frac{1}{a^2 + 5a + 6}$
$\Rightarrow a = \frac{2}{3}$.
Hence, (A) is the correct answer

10. $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$

$A^2 = \begin{bmatrix} a & b \\ b & a \end{bmatrix} \begin{bmatrix} a & b \\ b & a \end{bmatrix}$

$= \begin{bmatrix} a^2+b^2 & 2ab \\ 2ab & a^2+b^2 \end{bmatrix}$

$\Rightarrow \alpha = a^2 + b^2, \beta = 2ab.$

Hence, (B) is the correct answer.

11. Number of choices = ${}^5C_4 \times {}^8C_6 + {}^5C_5 \times {}^8C_5$

= 140 + 56.

Hence, (B) is the correct answer

12. Clearly $5! \times 6!$

(A) is the correct answer

13. $\Delta = \begin{vmatrix} 1+\omega^n+\omega^{2n} & \omega^n & \omega^{2n} \\ 1+\omega^n+\omega^{2n} & \omega^{2n} & 1 \\ 1+\omega^n+\omega^{2n} & 1 & \omega^n \end{vmatrix}$

= 0

Since, $1 + \omega^n + \omega^{2n} = 0$, if n is not a multiple of 3

Therefore, the roots are identical.

Hence, (A) is the correct answer

14. ${}^nC_{r+1} + {}^nC_{r\ 1} + {}^nC_r + {}^nC_r$

$= {}^{n+1}C_{r+1} + {}^{n+1}C_r$

$= {}^{n+2}C_{r+1}.$

Hence, (B) is the correct answer

15. General term = ${}^{256}C_r (\sqrt{3})^{256\ r} [(5)^{1/8}]^r$

From integral terms, or should be 8k

$\Rightarrow$ k = 0 to 32.

Hence, (B) is the correct answer.

17. $\frac{1}{1\cdot 2} - \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} - \ldots\ldots\ldots$

$= 1 - \frac{1}{2} - \frac{1}{2} + \frac{1}{3} + \frac{1}{3} - \frac{1}{4} - \ldots\ldots\ldots$

$= 1 - 2\left(\frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \ldots\ldots\ldots\right)$

$= 2\left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots\ldots\ldots\right) - 1$

$= 2 \log 2 - \log e$

$= \log \left(\frac{4}{e}\right).$

Hence, (D) is the correct answer.

18. $f(x) = ax^2 + bx + c$

$f(1) = a + b + c$

$f(-1) = a - b + c$

$\Rightarrow a + b + c = a - b + c$ also $2b = a + c$

$f'(x) = 2ax + b = 2ax$

$f'(a) = 2a^2$

$f'(b) = 2ab$

$f'(c) = 2ac$

$\Rightarrow$ AP.

Hence, (A) is the correct answer.

19. Result (A) is correct answer.

20. (B)

21. $a\left(\frac{1+\cos C}{2}\right) + c\left(\frac{1+\cos A}{2}\right) = \frac{3b}{2}$

$\Rightarrow a + c + b = 3b$

$a + c = 2b.$

Hence, (A) is the correct answer

23. $-\frac{\pi}{4} \le \frac{\sin^2 x}{2} \le \frac{\pi}{4}$

$-\frac{\pi}{4} \le \sin^{-1}(a) \le \frac{\pi}{4}$

$\frac{1}{2} \le |a| \le \frac{1}{\sqrt{2}}.$

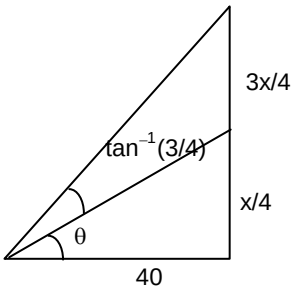
Hence, (D) is the correct answer

24. $\sin(\theta + \alpha) = \frac{x}{40}$

$\sin a = \frac{x}{140}$

$\Rightarrow x = 40.$

Hence, (B) is the correct answer



25. (B)

26. $f(1) = 7$

$f(1 + 1) = f(1) + f(1)$

$f(2) = 2 \times 7$

only $f(3) = 3 \times 7$

$\sum_{r=1}^{n} f(r) = 7(1 + 2 + \ldots\ldots + n)$

$= 7 \frac{n(n+1)}{2}.$

27. $LHS = 1 - \frac{n}{1!} + \frac{n(n-1)}{2!} - \frac{n(n-1)(n-2)}{3!} + \ldots\ldots$

$= 1 - {}^{n}C_1 + {}^{n}C_2 - \ldots\ldots$

$= 0.$

Hence, (C) is the correct answer

28. $4 - x^2 \neq 0$

$\Rightarrow x \neq \pm 2$

$x^3 - x > 0$

$\Rightarrow x(x + 1)(x - 1) > 0.$

Hence (D) is the correct answer.

29. $\lim_{x \to \pi/2} \frac{\tan\left(\frac{\pi}{4} - \frac{x}{2}\right)(1 - \sin x)}{4\left(\frac{\pi}{4} - \frac{x}{2}\right)^3 (\pi - 2x)^2}$

$= \frac{1}{32}.$

Hence, (C) is the correct answer.

30. $\lim_{x \to 0} \frac{(3+x)^{\frac{1}{3}} - 3^{\frac{1}{3}}}{x} = \frac{2}{3}.$

Hence, (C) is the correct answer.

31. Applying L. Hospital's Rule

$\lim_{x \to 2a} \frac{f(a)g'(a) - g(a)f'(a)}{g'(a) - f'(a)} = 4$

$\frac{k(g'(a) - ff'(a))}{(g'(a) - f'(a))} = 4$

$k = 4.$

Hence, (A) is the correct answer.

32. $f(-x) = -f(x)$

Hence, (B) is the correct answer.

33. $f'(0)$

$f'(0 - h) = 1$

$f'(0 + h) = 0$

LHD $\neq$ RHD.

Hence, (B) is the correct answer.

34. $f(x) = 0$ at $x = p, q$

$6p^2 + 18ap + 12a^2 = 0$

$6q^2 + 18aq + 12a^2 = 0$

$f''(x) < 0$ at $x = p$

and $f''(x) > 0$ at $x = q$.

Clearly $f''(x) > 0$ for $x = 2a$

$\Rightarrow q = 2a < 0$ for $x = a \Rightarrow p = a$

or $p^2 = q \Rightarrow a = 2.$

Hence, (C) is the correct answer.

35. $F(t) = \int_0^t f(t - y) f(y)\, dy$

$= \int_0^t f(y) f(t - y)\, dy$

$= \int_0^t e^y (t - y)\, dy$

$= x^t - (1 + t).$

Hence, (B) is the correct answer.

36. $\int_a^b x f(x)\, dx$

$= \int_a^b (a + b - x) f(a + b - x)\, dx.$

Hence, (B) is the correct answer.

37. $\lim_{x\to 0} \frac{\tan(x^2)}{x\sin x}$

$= \lim_{x\to 0} \frac{\tan(x^2)}{x^2\left(\frac{\sin x}{x}\right)}$

= 1.

Hence (C) is the correct answer.

38. $\int_0^1 x(1-x)^n\,dx = \int_0^1 x^n(1-x)$

$= \int_0^1 (x^n - x^{n+1}) = \frac{1}{n+1} - \frac{1}{n+2}$.

Hence, (C) is the correct answer.

40. $F'(x) = \frac{e^{\sin x}}{3^x}$

$= \int_x^3 e^{\sin x}\,dx = F(k) - F(1)$

$= \int_1^{64} \frac{e^{\sin x}}{x}\,dx = F(k) - F(1)$

$= \int_1^{64} F'(x)\,dx = F(k) - F(1)$

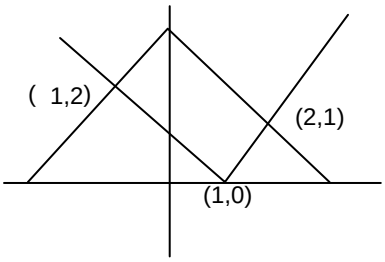
$F(64) - F(1) = F(k) - F(1)$

$\Rightarrow k = 64$.

Hence, (D) is the correct answer.

41. Clearly area = $2\sqrt{2}\times\sqrt{2}$

= sq units



42. $\int_0^1 f(x)[k^2 - f(x)]\,dx$

solving this by putting $f'(x) = f(x)$.

Hence, (B) is the correct answer.

43. Equation $y^2 = 4a(9x - h)$

$2yy_1 = 4a \Rightarrow yy_1 = 2a$

$yy_2 + y_1^2 = 0$.

Hence (B) is the correct answer.

45. Let p (x, y)

$(x - a_1)^2 + (y - b_1)^2 = (x - a_2)^2 + (y - b_2)^2$

$(a_1 - a_2)x + (b_1 - b_2)y + \frac{1}{2}(b_2^2 - b_1^2 + a_2^2 - a_1^2) = 0$.

Hence, (A) is the correct answer.

47. (D)

46. $x = \frac{a\cos t + b\sin t + 1}{3}$, $y = \frac{a\sin t - b\cos t + 1}{3}$

$\left(x - \frac{1}{3}\right)^2 + y^2 = \frac{a^2 + b^2}{9}$.

Hence, (B) is the correct answer.

49. $\frac{dx}{dy}(1 + y^2) = (e^{\sin^{-1} y} - x)$

$\frac{k(g'(a) - ff'(a))}{(g'(a) - f(a))} = 4$

$k = 4$.

Hence, (A) is the correct answer.

50. Intersection of diameter is the point (1, 1)

$\pi s^2 = 154$

$\Rightarrow s^2 = 49$

$(x-1)^2 + (y+1)^2 = 49$

Hence, (C) is the correct answer.