MATHEMATICS

1. If α and β are the roots of the equation, $7x^2 - 3x - 2 = 0$, then the value of $\frac{\alpha}{1-\alpha^2} + \frac{\beta}{1-\beta^2}$ is equal to

(a) $\frac{1}{24}$ (b) $\frac{27}{32}$

(c) $\frac{27}{16}$ (d) $\frac{3}{8}$

2. The statement

$(p \to (q \to p)) \to (p \to (p \vee q))$ is :

(a) equivalent to $(p \vee q) \wedge (\sim p)$

(b) a contradiction

(c) a tautology

(d) equivalent to $(p \wedge q) \vee (\sim q)$

3. If the line y = mx + c is a common tangent to the hyperbola $\frac{x^2}{100} - \frac{y^2}{64} = 1$ and the circle $x^2 + y^2 = 36$, then which one of the following is true ?

(a) $4c^2 = 369$ (b) 5m = 4

(c) $c^2 = 369$ (d) 8m + 5 = 0

4. If the length of the chord of the circle $x^2 + y^2 = r^2 (r > 0)$ along the line $y - 2x = 3$ is r, then r^2 is equal to :

(a) $\frac{9}{5}$ (b) 12

(c) $\frac{12}{5}$ (d) $\frac{24}{5}$

5. If x = 1 is a critical point of the function $f(x) = (3x^2 + ax - 2 - a)e^x$, then :

(a) x = 1 and $x = -\frac{2}{3}$ are local minima of f

(b) x = 1 is a local maxima and $x = -\frac{2}{3}$ is a local minima of f.

(c) x = 1 is a local manima and $x = -\frac{2}{3}$ is a local maxima of f.

(d) x = 1 and $x = -\frac{2}{3}$ are local maxima of f.

6. The derivative of $tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$ at $x = \frac{1}{2}$ is :

(a) $\frac{\sqrt{3}}{10}$ (b) $\frac{\sqrt{3}}{12}$ (c) $\frac{2\sqrt{3}}{5}$ (d) $\frac{2\sqrt{3}}{3}$

7. If the sum of the second, third and fourth terms of a positive term G.P. is 3 and the sum of its sixth, seventh and eighth terms is 243, then the sum of the first 50 terms of this G.P. is

(a) $\frac{2}{13}(3^{50} - 1)$ (b) $\frac{1}{26}(3^{49} - 1)$

(c) $\frac{1}{13}(3^{50} - 1)$ (d) $\frac{1}{26}(3^{50} - 1)$

8. If the mean and the standard deviation of the data 3, 5, 7, a, b are 5 and 2 respectively, then a and b are the roots of the equation :

(a) $x^2 - 20x + 18 = 0$ (b) $2x^2 - 20x + 19 = 0$

(c) $x^2 - 10x + 19 = 0$ (d) $x^2 - 10x + 18 = 0$

9. If $\int \frac{cos\theta}{5+7sin\theta-2cos^{2\theta}} d\theta = Alog_e|B(\theta)| + C$, where C is a constant of integration, then $\frac{B(\theta)}{A}$ can be :

(a) $\frac{2sin\theta+1}{5(sin\theta+3)}$ (b) $\frac{5(2sin\theta+1)}{sin\theta+3}$

(c) $\frac{2sin\theta+1}{sin\theta+3}$ (d) $\frac{5(sin\theta+3)}{2sin\theta+1}$

10. The value of $\left(\frac{-1+i\sqrt{3}}{1-i}\right)^{30}$ is

(a) 6^5 (b) $2^{15} i$ (c) -2^{15} (d) $-2^{15} i$

11. $\lim_{x \to 0} \frac{xe^{\sqrt{1+x^2+x^4}-1)/x}-1}{\sqrt{1+x^2+x^4}-1}$

(a) does not exist (b) is equal to 1

(c) is equal to $\sqrt{e}$ (d) is equal to 0

12. The area (in sq. units) of the region $a = \{x, y): (x - 1)[x] \le y \le 2\sqrt{x}, 0 \le x \le 2\}$, where [t] denotes the greatest integer function, is

(a) $\frac{4}{3}\sqrt{2} - \frac{1}{2}$ (b) $\frac{8}{3}\sqrt{2} - 1$

(c) $\frac{4}{3}\sqrt{2} + 1$ (d) $\frac{8}{3}\sqrt{2} - \frac{1}{2}$

13. If the system of linear equations

x + y + 3z = 0

$x + 3y + k^2z = 0$

3x + y + 3z = 0

has a non – zero solution (x, y, z) for some k ∈ R, then $x + \left(\frac{y}{z}\right)$ is equal to :

(a) –9 (b) –3

(c) 9 (d) 3

14. If $L = sin^2\left(\frac{\pi}{16}\right) - sin^2\left(\frac{\pi}{8}\right)$ and $M = cos^2\left(\frac{\pi}{16}\right) - sin^2\left(\frac{\pi}{8}\right)$

(a) $M = \frac{1}{4\sqrt{2}} + \frac{1}{4}cos\frac{\pi}{8}$ (b) $M = \frac{1}{2\sqrt{2}} + \frac{1}{2}cos\frac{\pi}{8}$

(c) $L = \frac{1}{2\sqrt{2}} + \frac{1}{2}cos\frac{\pi}{8}$ (d) $L = \frac{1}{4\sqrt{2}} - \frac{1}{4}cos\frac{\pi}{8}$

15. If for some $\alpha \in R$, the lines

$L_1 : \frac{x+1}{2} = \frac{y-2}{-1} = \frac{z-1}{1}$ and $L_2 : \frac{x+2}{\alpha} = \frac{y+1}{5-\alpha} = \frac{z+1}{1}$ are coplanar, then the line L_2 passes through the point :

(a) (2, –10, –2) (b) (10, 2, 2)

(c) (–2, 10, 2) (d) (10, –2, –2)

16. If a + x = b + y = c + z + 1, where a, b, c x, y, z are non-zero distinct real numbers, then $\begin{vmatrix} x & a+y & x+a \\ y & b+y & y+b \\ z & c+y & z+c \end{vmatrix}$ is equal to :

(a) y(a – b) (b) 0

(c) y(b – a) (d) y(a – c)

17. Let y = y(x) be the solution of the differential equation

$cosx\frac{dy}{dx} + 2y\sin x = sin2x, x \in \left(0, \frac{\pi}{2}\right)$

If $y(\pi/3) = 0$, then $y(\pi/4)$ is equal to :

(a) $\frac{1}{\sqrt{2}} - 1$ (b) $2 - \sqrt{2}$

(c) $\sqrt{2} - 2$ (d) $2 + \sqrt{2}$

18. There are 3 sections in a question paper and each section contains 5 questions. A candidate has to answer a total of 5 questions, choosing at least one question from each section. Then the number of ways, in which the candidate can choose the questions, is

(a) 3000 (b) 2250
(c) 2255 (d) 1500

19. If the sum of the first 20 terms of the series $\log log_{(7^{1/2})} x + log_{(7^{1/3})} x + log_{(7^{1/4})} x + \cdots$ is 460, then x is equal to :

(a) e^2 (b) $7^{46/21}$ (c) 7^2 (d) $7^{1/2}$

20. Which of the following points lies on the tangent to the curve $x^4 e^y + 2\sqrt{y+1} = 3$ at the point (1, 0) ?

(a) (–2, 4) (b) (2, 2) (c) (–2, 6) (d) (2, 6)

21. In a bombing attack, there is 50% chance that a bomb will hit the target. At least two independent hits are required to destroy the target completely. Then the minimum number of bombs, that must be dropped to ensure that there is at least 99% chance of completely destroying the target is …..

22. Let the vectors $\vec{a}, \vec{b}, \vec{c}$ be such that $|\vec{a}| = 2$, $|\vec{b}| = 4$ and $|\vec{c}| = 4$. If the projection of $\vec{b}$, on $\vec{a}$ is equal to the projection of $\vec{c}$ on $\vec{a}$ and $\vec{b}$ is perpendicular to $\vec{c}$, then the value of $|\vec{a} + \vec{b} + \vec{c}|$ is …….

23. Let A = {a, b, c} and B = {1, 2, 3, 4}. Then the number of elements in the set C = {f : A → B | 2 ∈ f(A) and f is not one-one} is …….

24. If the lines x + y = a and x – y = b touch the curve $y = x^2 - 3x + 2$ at the points where the curve intersects the x-axis, then $\frac{a}{b}$ is equal to ……

25. The coefficient of x^4 in the expansion of $(1 + x + x^2 + x^3)^6$ in powers of x, is ……

ANSWER

1.	(c)	**2.**	(c)	**3.**	(a)	**4.**	(c)	**5.**	(c)	**6.**	(a)	**7.**	(d)	**8.**	(c)	**9.**	(b)	**10.**	(d)
11.	(b)	**12.**	(d)	**13.**	(b)	**14.**	(b)	**15.**	(a)	**16.**	(a)	**17.**	(c)	**18.**	(b)	**19.**	(c)	**20.**	(c)
21.	(11)	**22.**	(6)	**23.**	(19)	**24.**	(0.5)	**25.**	(49)										