MATHEMATICS

1. Let $f:(0,\infty)\to(0,\infty)$ be a differentiable function such that f(1) = e and $\lim_{t\to x}\frac{t^2f^2(x)-x^2f^2(t)}{t-x}=0$. If f(x) = 1, then x is equal to :

(a) $\frac{1}{e}$ (b) $2e$

(c) $\frac{1}{2e}$ (d) e

2. Contrapositive o the statement :
'If a function f is differentiable at a, then it is also continuous at a', is :

(a) If a function f is not continuous at a, then it is not differentiable at a.

(b) If a function f is continuous at a, then it is differentiable at a.

(c) If a function f is not continuous at a. then it is differentiable at a.

(d) If a function f is continuous at a, then it is not differentiable at a.

3. The solution of the differential equation $\frac{dy}{dx}-\frac{y+3x}{log_e(y+3x)}+3=0$ is (where C is a constant of integration)

(a) $x-2log_e(y+3x)=C$

(b) $x-log_e(y+3x)=C$

(c) $y+3x-\frac{1}{2}(log_e x)^2=C$

(d) $y-\frac{1}{2}(log_e(y+3x)^2=C$

4. If for some positive integer n, the coefficients of three consecutive terms in the binomial expansion of $(1+x)^{n+5}$ are in the ratio 5 : 10 : 14, then the largest coefficient in the expansion is :

(a) 330 (b) 252

(c) 792 (d) 462

5. The circle passing through the intersection of the circles, $x^2+y^2-6x=0$ and $x^2+y^2-4y=0$, having its centre on the line, 2x – 3y + 12 = 0, also passes through the point :

(a) (1, –3) (b) (–1, 3)

(c) (–3, 6) (d) (–3, 1)

6. In a game two players A and B take turns in throwing a pair of fair dice starting with player A and total of scores on the two dice, in each throw is noted. A wins the game if he throws a total of 6 before B throws a total of 7 and B wins the game if he throws a total of 7 before A throws a total of six. The game stops as soon as either of the players wins. The probability of A winning the game is :

(a) $\frac{30}{61}$ (b) $\frac{5}{6}$ (c) $\frac{5}{31}$ (d) $\frac{31}{61}$

7. The angle of elevation of a cloud C from a point P, 200 m above a still take is 30°. If the angle of depression of the image of C in the lake from the point P is 60°, then PC (in m) is equal to

(a) 100 (b) $400\sqrt{3}$ (c) $200\sqrt{3}$ (d) 400

8. The function $f(x)=\begin{cases}\frac{\pi}{4}+tan^{-1}x, & |x|\le 1\\ \frac{1}{2}(|x|-1), & |x|>1\end{cases}$ is :

(a) continuous on R – {–1} and differentiable on R –{–1, 1}

(b) both continuous and differentiable on R –{–1}

(c) both continuous and differentiable on R –{1}

(d) continuous on R – {1} and differentiable on R –{–1, 1}

9. Suppose the vectors x_1, x_2 and x_3 are the solutions of the system of linear equations, Ax = b when the vector b on the right side is equal to b_1, b_2 and b_3 respectively. If

$x_1=\begin{bmatrix}1\\1\\1\end{bmatrix}$, $x_2=\begin{bmatrix}0\\2\\1\end{bmatrix}$, $x_3=\begin{bmatrix}0\\0\\1\end{bmatrix}$, $b_1=\begin{bmatrix}1\\0\\0\end{bmatrix}$, $b_2=\begin{bmatrix}0\\2\\0\end{bmatrix}$

and $b_3=\begin{bmatrix}0\\0\\2\end{bmatrix}$, then the determinant of A is equal of A is equal to

(a) $\frac{3}{2}$ (b) 4

(c) $\frac{1}{2}$ (d) 2

10. Let $a_1, a_2, \ldots\ldots a_n$ be a given A.P. whose common difference is an integer and $S_n=a_1+a_2+\ldots.+a_n$. If $a_1=1$, $a_1=300$ and $15\le n\le 50$, then the ordered pair (S_{n-4}, a_{n-4}) is equal to :

(a) (2490, 248) (b) (2490, 249)

(c) (2480, 249) (d) (2480, 248)

11. Let $\cup_{i=1}^{50}X=\cup_{i=1}^{n}Y_i=T$, where each X_i contains 10 elements and each Y_i contains 5 elements. If each element of the set T is an element of exactly 20 of sets $X_i's$ and exactly 6 of sets $Y_i's$ then n is equal to :

(a) 45 (b) 15

(c) 30 (d) 50

12. The area (in sq. units) of the largest rectangle ABCD whose vertices A and B lie on the x-axis and vertices C and D lie on the parabola, $y=x^2-1$ below the x-axis is :

(a) $\frac{1}{3\sqrt{3}}$ (b) $\frac{4}{3}$

(c) $\frac{4}{3\sqrt{3}}$ (d) $\frac{2}{3\sqrt{3}}$

13. Let x = 4 be a directrix to an ellipse whose centre is at the origin and its eccentricity is $\frac{1}{2}$. If $P(1,\beta), \beta>0$ is a point on this ellipse. Then the equation of the normal to it at P is

(a) 7x – 4 y = 1 (b) 4x – 2 y = 1

(c) 8x – 2 y = 5 (d) 4x – 3 y = 2

14. The distance of the point (1,–2, 3) from the plane x – y + z = 5 measured parallel to the line $\frac{x}{2}=\frac{y}{3}=\frac{z}{-6}$ is :

(a) $\frac{1}{7}$ (b) 7

(c) 1 (d) $\frac{7}{5}$

15. If the perpendicular bisector of the line segment joining the points P(1, 4) and Q(k, 3) has y-intercept equal to –4, then a value of k is

(a) $\sqrt{15}$ (b) –4

(c) –2 (d) $\sqrt{14}$

16. The integral
$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} tan^3x.sin^2 3x(2sec^2xsin^2 3x + 3tanx\sin 6x)\, dx$ is equal to :

(a) $-\frac{1}{9}$ (b) $\frac{9}{2}$

(c) $-\frac{1}{18}$ (d) $\frac{7}{18}$

17. The minimum value of $2^{sinx} + 2^{cosx}$ is :

(a) $2^{-1+\frac{1}{\sqrt{2}}}$ (b) $2^{-1+\sqrt{2}}$

(c) $2^{1-\sqrt{2}}$ (d) $2^{1-\frac{1}{\sqrt{2}}}$

18. If a and b are real numbers such that $(2+\alpha)^4 = a + b\alpha$, where $\alpha = \frac{-1+i\sqrt{3}}{2}$ then a + b is equal to :

(a) 33 (b) 24

(c) 9 (d) 57

19. Let $\lambda \neq 0$ be in R. If α and β are the roots of the equation, $x^2 - x + 2\lambda = 0$ and α and γ are the roots the equation, $3x^2 - 10x + 27\lambda = 0$, then $\frac{\beta\gamma}{\lambda}$ is equal to :

(a) 27 (b) 36

(c) 9 (d) 18

20. If the system of equations

x + y + z = 2

2x + 4y – z = 6

3x + 2y + $\lambda z = \mu$

(a) $\lambda + 2\mu = 14$ (b) $2\lambda - \mu = 5$

(c) $2\lambda + \mu = 14$ (d) $\lambda - 2\mu = -5$

21. Let PQ be a diameter of the circle $x^2 + y^2 = 9$. If α and β are the lengths of the perpendiculars from P and Q on the straight line, x + y = 2 respectively, then the maximum value of $\alpha\beta$ is ……..

22. If the variance of the following frequency distribution :

Class :	10–20	20–30	30–40
Frequency :	2	x	2

Is 50, then x is equal to ……

23. A test consists of 6 multiple choice questions, each having 4 alternative answers of which only one is correct. The number of ways, in which a candidate answers all six questions such that exactly four of the answers are correct, is ………

24. Let {x} and [x] denote the fractional part of x and the greatest integer $\leq x$ respectively of a real number x. if $\int_0^n \{x\}dx, \int_0^n [x]dx$ and $10(n^2 - n), (n \in N, n > 1)$ are three consecutive terms of a G.P. then n is equal to …….

25. If $\vec{a} = 2\hat{\imath} + \hat{\jmath} + 2\hat{k}$, then the value of $|\hat{\imath} \times (\vec{a} \times \hat{\imath})|^2 + |\hat{\jmath}(\vec{a} \times \hat{\jmath})|^2 + \left|\hat{k} \times (\vec{a} \times \hat{k})\right|^2$ is equal to :

ANSWER

1. (a)	**2.** (a)	**3.** (d)	**4.** (d)	**5.** (c)	**6.** (a)	**7.** (d)	**8.** (d)	**9.** (d)	**10.** (a)
11. (c)	**12.** (c)	**13.** (b)	**14.** (c)	**15.** (b)	**16.** (c)	**17.** (d)	**18.** (c)	**19.** (d)	**20.** (c)
21. (7)	**22.** (4)	**23.** (135)	**24.** (21)	**25.** (18)					