MATHEMATICS

1. Let A be a 3 × 3 matrix such that

$adj\ A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 0 & 2 \\ 1 & -2 & -1 \end{bmatrix}$ and B = adj (adj A).

If $|A| = \lambda$ and $|(B^{-1})^T| = \mu$, then the ordered pair, $(|\lambda|, \mu)$ is equal to

(a) $\left(3, \frac{1}{81}\right)$ (b) $\left(9, \frac{1}{9}\right)$

(c) (3, 81) (d) $\left(9, \frac{1}{81}\right)$

2. If $x^3 dy + xy.dx = x^2 dy + 2y\,dx\,; y(2) = e$ and x > 1, then y(4) is equal to :

(a) $\frac{\sqrt{e}}{2}$ (b) $\frac{1}{2} + \sqrt{e}$

(c) $\frac{3}{2}\sqrt{e}$ (d) $\frac{3}{2} + \sqrt{e}$

3. If the sum of the series $20 + 19\frac{3}{5} + 19.\frac{1}{5} + 18\frac{4}{5} + \ldots\ldots$ upto nth term is 488 and the nth term is negative, then :

(a) nth term is $-4\frac{2}{5}$ (b) n = 41

(c) nth term is -4 (d) n = 60

4. The set of all real values of λ for which the quadratic equation $(\lambda^2 + 1)x^2 - 4\lambda x + 2 = 0$ always have exactly one root in the interval (0, 1) is :

(a) (–3, –1) (b) (0, 2)

(c) (1, 3] (d) (2, 4]

5. Let R_1 and R_2 be two relations defined as follows :

$R_1 = \{(a, b) \in R^2 : a^2 + b^2 \in Q\}$ and

$R_2 = \{(a, b) \in R^2 : a^2 + b^2 \notin Q\}$,

Where Q is the set of all rational numbers, then

(a) R_1 is transitive but R_2 is not transitive.

(b) R_2 is transitive but R_1 is not transitive.

(c) Neither R_1 nor R_2 is transitive.

(d) R_1 and R_2 are both transitive.

6. The Plane which bisects the line joining the points (4, –2, 3) and (2, 4, –1) at right angles also passes through the point :

(a) (0, –1, 1) (b) (4, 0, –1)

(c) (4, 0, 1) (d) (0, 1, –1)

7. Let p, q, r be three statements such that the truth value of $(p \wedge q) \to (\sim q \vee r)$ is F. Then the truth values of p, q, r are respectively

(a) T, T, F (b) T, T, T

(c) T, F, T (d) F, T, F

8. If a ΔABC has vertices A(–1, 7), B (–7, 1) and C (5, –5), then its orthocenter has coordinates :

(a) (–3, 3) (b) (3, –3)

(c) $\left(-\frac{3}{5}, \frac{3}{5}\right)$ (d) $\left(\frac{3}{5}, -\frac{3}{5}\right)$

9. Suppose f(x) is a polynomial of degree four, having critical points at –1, 0, 1. If T = {x ∈ R| f (x) = f(0)}, then the sum of squares of all the elements of T is :

(a) 4 (b) 6

(c) 2 (d) 8

10. If the term independent of x in the expansion of $\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$ is k, then 18k is equal to

(a) 11 (b) 5

(c) 9 (d) 7

11. If z_1, z_2 are complex numbers such that Re(z_1) = |z_1 = 1| and Re(z_2) = |z_2 = 1| and arg $(z_1 - z_2) = \frac{\pi}{6}$, then Im($z_1 + z_2$) is equal to :

(a) $2\sqrt{3}$ (b) $\frac{\sqrt{3}}{2}$

(c) $\frac{1}{\sqrt{3}}$ (d) $\frac{2}{\sqrt{3}}$

12. Let $x_i (1 \le i \le 10)$ be ten observation of a random variable X. If $\sum_{i=1}^{10}(x_i - p) = 3$ and $\sum_{i=1}^{10}(x_i - p)^2 = 9$ where $0 \ne p \in R$, then the standard deviation of these observations is :

(a) $\frac{4}{5}$ (b) $\sqrt{\frac{3}{5}}$

(c) $\frac{9}{10}$ (d) $\frac{7}{10}$

13. The probability that a randomly chosen 5-digit number is made from exactly two digits is :

(a) $\frac{135}{10^4}$ (b) $\frac{150}{10^4}$

(c) $\frac{134}{10^4}$ (d) $\frac{121}{10^4}$

14. Let a, b, c ∈ R be such that $a^2 + b^2 + c^2 = 1$. If a $cos\theta = bcos\left(\theta + \frac{2\pi}{3}\right) = c\cos\left(\theta + \frac{4\pi}{3}\right)$, where $\theta = \frac{\pi}{9}$, then the angle between the vectors $a\hat{i} + b\hat{j} + c\hat{k}$ and $b\hat{i} + c\hat{j} + a\hat{k}$ is :

(a) 0 (b) $\frac{2\pi}{3}$

(c) $\frac{\pi}{2}$ (d) $\frac{\pi}{9}$

15. If the surface area of a cube is increasing at a rate of 3.6 cm^2/sec, retaining its shape; then the rate of change of its volume (in cm^3/sec), when the length of a side of the cube is 10 cm, is

(a) 20 (b) 10

(c) 18 (d) 9

16. $\lim_{x \to 0} \frac{(a+2x)^{\frac{1}{3}} - (3x)^{\frac{1}{3}}}{(3a+x)^{\frac{1}{3}} - (4x)^{\frac{1}{3}}} (a \ne 0)$ is equal to :

(a) $\left(\frac{2}{9}\right)\left(\frac{2}{3}\right)^{1/3}$ (b) $\left(\frac{2}{3}\right)^{4/3}$

(c) $\left(\frac{2}{9}\right)^{4/3}$ (d) $\left(\frac{2}{3}\right)\left(\frac{2}{9}\right)^{1/3}$

17. Let e_1 and e_2 be the eccentricities of the ellipse, $\frac{x^2}{25}+\frac{y^2}{b^2}=1 (b<5)$ and the hyperbola, $\frac{x^2}{16}-\frac{y^2}{b^2}=1$ respectively satisfying $e_1e_2 = 1$. If α and β are the distances between the foci of the ellipse and the foci of the hyperbola respectively, then the ordered pair (α, β) is equal to :

(a) (8, 10) (b) $\left(\frac{20}{3}, 12\right)$

(c) (8, 12) (d) $\left(\frac{24}{5}, 10\right)$

18. If $\int sin^{-1}\sqrt{\frac{x}{1+x}}dx = A(x)tan^{-1}(\sqrt{x}) + B(x) + C$, where C is a constant of integration, then the ordered pair (A(x), B(x)) can be :

(a) $(x-1, \sqrt{x})$ (b) $(x-1, -\sqrt{x})$

(c) $(x+1, \sqrt{x})$ (d) $(x+1, -\sqrt{x})$

19. If the value of the integral $\int_0^{1/2}\frac{x^2}{(1-x^2)^{3/2}}dx$ is $\frac{k}{6}$, then k is equal to :

(a) $2\sqrt{3}+\pi$ (b) $2\sqrt{3}-\pi$

(c) $3\sqrt{2}+\pi$ (d) $3\sqrt{2}-\pi$

20. Let the latus rectum of the parabola $y^2 = 4x$ be the common chord to the circles C_1 and C_2 each of them having radius $2\sqrt{5}$. Then, the distance between the centres of the circles C_1 and C_2 is

(a) 12 (b) 8

(c) $8\sqrt{5}$ (d) $4\sqrt{5}$

21. If the tangent to the curve, $y = e^x$ at a point (c, e^c) and the normal to the parabola, $y^2 = 4x$ at the point (1, 2) intersect at the same point on the x-axis, then the value of c is …………

22. Let a plane P contain two lines

$\vec{r} = \hat{\imath} + \lambda(\hat{\imath}+\hat{\jmath}), \lambda \varepsilon R$ and

$\vec{r} = -\hat{\jmath} + \mu(\hat{\jmath}-\hat{k}), \mu \varepsilon R$

If $Q(\alpha, \beta, \gamma)$ is the foot of the perpendicular drawn from the point M(1, 0, 1) to P, then $3(\alpha+\beta+\gamma)$ equals …..

23. If m arithmetic means (A.Ms) and three geometric means (G.Ms) are inserted between 3 and 243 such that 4^{th} A.M. is equal to 2^{nd} G.M., then m is equal to :

24. The total number of 3-digit numbers, whose sum of digits is 10, is ……

25. Let S be the set of all integer solutions, (x, y, z) of the system of equations

x – 2y + 5z = 0

–2x + 4y + z = 0

–7x + 14y + 9z = 0

Such that $15 \le x^2+y^2+z^2 \le 150$. Then, the number of elements in the set s is equal to …..

ANSWER

1.	(a)	**2.**	(c)	**3.**	(c)	**4.**	(c)	**5.**	(c)	**6.**	(b)	**7.**	(a)	**8.**	(a)	**9.**	(a)	**10.**	(d)
11.	(a)	**12.**	(c)	**13.**	(a)	**14.**	(c)	**15.**	(d)	**16.**	(d)	**17.**	(a)	**18.**	(d)	**19.**	(b)	**20.**	(b)
21.	(04.00)	**22.**	(05.00)	**23.**	(39.00)	**24.**	(54.00)	**25.**	(08.00)										