MATHEMATICS

1. Let y =y(x) be the solution of the differential equation, $xy' - y = x^2(xcosx + sinx), x > 0$. If $y(\pi) = \pi$, then $y''\left(\frac{\pi}{2}\right) + y\left(\frac{\pi}{2}\right)$ is equal to :

(a) $1 + \frac{\pi}{2}$ (b) $1 + \frac{\pi}{2} + \frac{\pi^2}{4}$

(c) $2 + \frac{\pi}{2} + \frac{\pi^2}{4}$ (d) $2 + \frac{\pi}{2}$

2. A triangle ABC lying in the first quadrant has two vertices as A(1, 2) and B(3, 1). If $\angle BAC = 90°$, and ar$(\Delta ABC) = 5\sqrt{5}$ sq. units, then the abscissa of the vertex C is :

(a) $1 + \sqrt{5}$ (b) $2 + \sqrt{5}$

(c) $1 + 2\sqrt{5}$ (d) $2\sqrt{5} - 1$

3. Let f(x) = |x – 2| and g(x) = f(f(x)), x ∈ [0, 4]. Then $\int_0^3 (g(x) - f(x))dx$ is equal to :

(a) 0 (b) $\frac{1}{2}$

(c) $\frac{3}{2}$ (d) 1

4. The integral $\int \left(\frac{x}{xsinx+cosx}\right)^2 dx$ is equal to (where C is a constant of integration) :

(a) $tanx + \frac{xsecx}{xsinx+cosx} + C$ (b) $tanx - \frac{xsecx}{xsinx+cosx} + C$

(c) $secx + \frac{xtanx}{xsinx+cosx} + C$ (d) $secx - \frac{xtanx}{xsinx+cosx} + C$

5. If $1 + (1 - 2^2.1) + (1 - 4^2.3) + (1 - 6^2.5) + \dots\dots + (1 - 20^2.19) = \alpha - 220\beta$ then an ordered pair (α, β) is equal to :

(a) (11, 97) (b) (10, 103)

(c) (11, 103) (d) (10, 97)

6. A survey shows that 63% of the people in a city read newspaper A whereas 76% read news paper B. If x% of the people read both the newspapers, then a possible value of x can be :

(a) 65 (b) 55 (c) 37 (d) 29

7. Two vertical poles AB = 15 m and CD = 10 m are standing apart on a horizontal ground with points A and C on the ground. If P is the point of intersection of BC and AD, then the height of P (in m) above the line AC is :

(a) 10/3 (b) 6 (c) 5 (d) 20/3

8. Let P(3, 3) be a point on the hyperbola, $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If the normal to it at P intersects the x-axis at (9, 0) and e is its eccentricity, then the ordered pair (a^2, e^2) is equal to :

(a) $\left(\frac{9}{2}, 3\right)$ (b) $\left(\frac{3}{2}, 2\right)$ (c) (9, 3) (d) $\left(\frac{9}{2}, 2\right)$

9. Let [t] denote the greatest integer $\leq t$. Then the equation in x, $[x]^2 + 2[x + 2] - 7 = 0$ has :

(a) no integral solution

(b) infinitely many solutions

(c) exactly two solutions

(d) exactly four integral solutions

10. If $\left(a + \sqrt{2}\, b \cos x\right)\left(a - \sqrt{2}\, b \cos x\right) = a^2 - b^2$, where a > b > 0, then $\frac{dx}{dy}$ at $\left(\frac{\pi}{4}, \frac{\pi}{4}\right)$ is :

(a) $\frac{2a+b}{2a-b}$ (b) $\frac{a+b}{a-b}$ (c) $\frac{a-b}{a+b}$ (d) $\frac{a-2b}{a+2b}$

11. Let x_0 be the point of local maxima of $f(x) = \vec{a}.(\vec{b} \times \vec{c})$, where $\vec{a} = x\hat{\imath} - 2\hat{\jmath} + 3\hat{k}$, $\vec{b} = -2\hat{\imath} + x\hat{\jmath} - \hat{k}$ and $\vec{c} = 7\hat{\imath} - 2\hat{\jmath} + x\hat{k}$. Then the value of $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$ at x = x_0 is

(a) –22 (b) 14

(c) –4 (d) –30

12. Let $f(x) = \int \frac{\sqrt{x}}{(1+x)^2} dx (x \geq 0)$. Then f(3) – f(1) is equal to :

(a) $-\frac{\pi}{6} + \frac{1}{2} + \frac{\sqrt{3}}{4}$

(b) $-\frac{\pi}{12} + \frac{1}{2} + \frac{\sqrt{3}}{4}$

(c) $\frac{\pi}{12} + \frac{1}{2} - \frac{\sqrt{3}}{4}$

(d) $\frac{\pi}{6} + \frac{1}{2} - \frac{\sqrt{3}}{4}$

13. The mean and variance of 8 observations are 10 and 13.5, respectively. If 6 of these observations are 5, 7, 10, 12, 14, 15, then the absolute difference of the remaining two observations is :

(a) 9 (b) 5

(c) 3 (d) 7

14. Let $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 (a > b)$ be a given ellipse, length of whose latus rectum is 10. If its eccentricity is the maximum value of the function $\phi(t) = \frac{5}{12} + t - t^2$ then $a^2 + b^2$ is equal to :

(a) 145 (b) 126

(c) 116 (d) 135

15. Let f be a twice differentiable function on (1, 6). If f(2) = 8. F'(2) = 5, f'(x) ≥ 4, for all x ∈ (1, 6), then :

(a) f(5) + f'(5) ≥ 28 (b) f'(5) + f''(5) ≤ 20

(c) f(5) ≤ 10 (d) f(5) + f'(5) ≤ 26

16. Let $u = \frac{2z+i}{z-ki}$, z = x + iy and k > 0. If the curve represented by Re (u) + Im(u) = 1 intersects the y-axis at points P and Q where PQ = 5 then the value of k is

(a) $\frac{3}{2}$ (b) $\frac{1}{2}$

(c) 4 (d) 2

17. If $A = \begin{bmatrix} cos\theta & isin\theta \\ isin\theta & cos\theta \end{bmatrix}, \left(\theta = \frac{\pi}{24}\right)$ and $A^5 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, where $i = \sqrt{-1}$, then which one of the following is not true ?

(a) $0 \leq a^2 + b^2 \leq 1$ (b) $a^2 - d^2 = 0$

(c) $a^2 - b^2 = \frac{1}{2}$ (d) $a^2 - c^2 = 1$

18. Let α and β be the roots of $x^2 - 3x + p = 0$ and γ and δ be the roots $x^2 - 6x + q = 0$. If $\alpha, \beta, \gamma, \delta$ from a geometric progression. Then ratio (2q + p) : (2q – p) is

(a) 3 : 1 (b) 5 : 3

(c) 9 : 7 (d) 33 : 31

19. The value of $\sum_{r=0}^{20} C_6^{50-*r}$ is equal to :
(a) ${}^{51}C_7 - {}^{30}C_7$ (b) ${}^{51}C_7 + {}^{30}C_7$
(c) ${}^{50}C_7 - {}^{30}C_7$ (d) ${}^{50}C_6 - {}^{30}C_6$

20. Given the following two statements :
$(S_1) : (q \vee p) \rightarrow (p \leftrightarrow \sim q)$ is a tautology.
$(S_2) : \sim q \wedge (\sim p \leftrightarrow q)$ is a fallacy. Then :
(a) only (S_2) is correct
(b) both (S_1) and (S_2) are not correct
(c) both (S_1) and (S_2) are correct
(d) only (S_1) is correct

21. If the equation of a plane P, passing through the intersection of the planes, x + 4y – z + 7 = 0 and 3x + y + 5z = 8 is ax + by + 6z = 15 for some a, b ∈ R, then the distance of the point (3, 2, –1) from the plane P is ……

22. Suppose a differentiable function f(x) satisfies the identity $f(x+y) = f(x) + f(y) + xy^2 + x^2y$, for all real x and y. If $\lim_{x \to 0} \frac{f(x)}{x} = 1$, then f'(3) is equal to :

23. The probability of a man hitting a target is $\frac{1}{10}$. The least number of shots required, so that the probability of his hitting the target at least once is greater than $\frac{1}{4}$, is ……

24. Let $(2x^2 + 3x + 4)^{10} = \sum_{r=0}^{20} a_r x^r$. Then $\frac{a_7}{a_{13}}$ is equal to …….

25. If the system of equations
x – 2y + 3z = 9
2x + y + z = b
x – 7y + az = 24, has infinitely many solutions, then a – b is equal to :

ANSWER

1. (d)	**2.** (c)	**3.** (d)	**4.** (b)	**5.** (c)	**6.** (b)	**7.** (b)	**8.** (a)	**9.** (b)	**10.** (b)
11. (a)	**12.** (c)	**13.** (d)	**14.** (b)	**15.** (a)	**16.** (d)	**17.** (c)	**18.** (c)	**19.** (a)	**20.** (b)
21. (3)	**22.** (10)	**23.** (3)	**24.** (8)	**25.** (5)					