MATHEMATICS

1. Let L_1 be a tangent to the parabola $y^2 = 4(x+1)$ and L_2 be a tangent to the parabola $y^2 = 8(x+2)$ such that L_1 and L_2 intersect at right angles. Then L_1 and L_2 the straight line :
(a) x + 3 = 0 (b) 2x + 1 = 0
(c) x + 2y = 0 (d) x + 2 = 0

2. If f(x + y) = f(x) f(y) and $\sum_{x=1}^{\infty} f(x) = 2, x, y \in N$, where N is the set of all natural numbers, then the value of $\frac{f(4)}{f(2)}$ is :
(a) $\frac{2}{3}$ (b) $\frac{1}{3}$
(c) $\frac{1}{9}$ (d) $\frac{4}{9}$

3. If $\sum_{i=1}^{n}(x_i - a) = n$ and $\sum_{i=1}^{n}(x_i - a)^2 = na$, $(n, a > 1)$ then the standard deviation of n observations $x_1, x_2, \ldots\ldots x_n$ is
(a) a – 1 (b) $\sqrt{n(a-1)}$
(c) $n\sqrt{(a-1)}$ (d) $\sqrt{(a-1)}$

4. A $\lim_{x \to 1}\left[\frac{\int_0^{(x-1)^2} t\cos(t^2)dt}{(x-1)\sin(x-1)}\right]$
(a) is equal to 1 (b) does not exist
(c) is equal to $\frac{1}{2}$ (d) is equal to $-\frac{1}{2}$

5. Which of the following points lies on the locus of he foot of perpendicular drawn upon any tangent to the ellipse, $\frac{x^2}{4} + \frac{y^2}{2} = 1$ from any of its foci ?
(a) $(-2, \sqrt{3})$ (b) $(-1, \sqrt{2})$
(c) (1, 2) (d) $(-1, \sqrt{3})$

6. Out of 11 consecutive natural number if three numbers are selected at random (without repetition), then the probability that they are in A.P. with positive common difference is :
(a) $\frac{10}{99}$ (b) $\frac{15}{101}$ (c) $\frac{5}{33}$ (d) $\frac{5}{101}$

7. The area (in sq. units) of the region $= \{(x, y): |x| + |y| \leq 1, 2y^2 \geq |x|\}$:
(a) $\frac{5}{6}$ (b) $\frac{7}{6}$ (c) $\frac{1}{3}$ (d) $\frac{1}{6}$

8. Let m and M be respectively the minimum and maximum value values of $\begin{vmatrix} cos^2x & 1+sin^2x & sin2x \\ 1+cos^2x & sin^2x & sin2x \\ cos^2x & sin^2x & 1+sin2x \end{vmatrix}$ Then the ordered pair (m, M) is equal to :
(a) (–3, 3) (b) (1, 3) (c) (–3, –1) (d) (–4, –1)

9. Let a, b, c, d and ay non zero distinct real numbers such that
$(a^2+b^2+c^2)p^2 - 2(ab+bc+cd)p + (b^2+c^2+d^2) = 0$
Then :
(a) a, b, c, d are in A.P.
(b) a, c, p are in G.P.
(c) a, b, c, d are in G.P.
(d) a, c, p, are in A.P.

10. If {p} denotes the fractional part of the number p, then $\left\{\frac{3^{200}}{8}\right\}$, is equal to :
(a) $\frac{1}{8}$ (b) $\frac{3}{8}$ (c) $\frac{7}{8}$ (d) $\frac{5}{8}$

11. The values of λ and μ for which the system of linear equations
x + y + z = 2
x + 2y + 3z = 5
x + 3y + λz = μ
has infinitely many solutions are respectively :
(a) 5 and 8 (b) 4 and 9
(c) 6 and 8 (d) 5 and 7

12. The region represented by $\{z = x + iy \in C: |z| - Re(z) \leq 1\}$ is also given by the inequality
(a) $y^2 \geq 2(x+1)$
(b) $y^2 \leq x + \frac{1}{2}$
(c) $y^2 \leq \left(x + \frac{1}{2}\right)$
(d) $y^2 \geq x + 1$

13. The general solution of the differential equation $\sqrt{1+x^2+y^2+x^2y^2} + xy\frac{dy}{dx} = 0$ (where C is a constant of integration)
(a) $\sqrt{1+y^2} + \sqrt{1+x^2} = \frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right) + C$
(b) $\sqrt{1+y^2} + \sqrt{1+x^2} = \frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}+1}\right) + C$
(c) $\sqrt{1+y^2} - \sqrt{1+x^2} = \frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right) + C$
(d) $\sqrt{1+y^2} - \sqrt{1+x^2} = \frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right) + C$

14. The position of moving car at time tis given by $f(t) = at^2 + bt + c, t > 0$, where a, b and c are real numbes greater than 1. Then the average speed of the car over the time interval $[t_1, t_2]$ is attained at the point :
(a) $(t_1 + t_3)/2$ (b) $(t_2 - t_1)/2$
(c) $2a(t_1 + t_2) + b$ (d) $a(t_2 - t_1) + b$

15. The negation of the Boolean expression p ∨ (~ p ∧ q) is equivalent to :
(a) p∧ ~ q (b) ~ p ∨ q
(c) ~p∧ ~ q (d) ~ p ∨ ~ q

16. Two families with three members each and one family with four members are to be seated in a row. In how many ways can they be seated so that the same family members are not separated ?
(a) 2! 3! 4! (b) 3! $(4!)^3$
(c) (3!)2.(4!) (d) $(3!)^3$.(4!)

17. The shortest distance between the lines $\frac{x-1}{0} = \frac{y+1}{-1} = \frac{z}{1}$ and x + y + z + 1 = 0, 2x – y + z + 3 = 0 is
(a) $\frac{1}{2}$ (b) 1
(c) $\frac{1}{\sqrt{3}}$ (d) $\frac{1}{\sqrt{2}}$

18. A ray of light coming from the point $(2, 2\sqrt{3})$ is incident at an angle 30° on the line x = 1 at the point A. The ray gets reflected on the line x = 1 and meets x-axis at the point B. Then, the line AB passes through the point ;

(a) $\left(4, -\frac{\sqrt{3}}{2}\right)$ (b) $\left(3, -\sqrt{3}\right)$

(c) $\left(4, -\sqrt{3}\right)$ (d) $\left(3, -\frac{1}{\sqrt{3}}\right)$

19. If $I_1 = \int_0^1 (1 - x^{50})^{100} dx$ and $I_2 = \int_0^1 (1 - x^{50})^{101} dx$ $I_2 = \alpha I_1$ then α equals to :

(a) $\frac{5049}{5050}$ (b) $\frac{5051}{5050}$ (c) $\frac{5050}{5051}$ (d) $\frac{5050}{5049}$

20. If α and β be two roots of the equation $x^2 - 64x + 256 = 0$. Then the value of $\left(\frac{\alpha^3}{\beta^5}\right)^{\frac{1}{8}} + \left(\frac{\beta^3}{\alpha^5}\right)^{\frac{1}{8}}$ is :

(a) 2 (b) 4 (c) 3 (d) 1

21. If $\vec{a}$ and $\vec{b}$ are unit vectors, then the greatest value of $\sqrt{3}|\vec{a} + \vec{b}| + |\vec{a} - \vec{b}|$ is

22. Let AD and BC be two vertical poles at A and B respectively on a horizontal ground. If AD = 8 m, BC = 11 m and AB = 10 m; then the distance (in meters) of a point M on AB from the point A such that $MD^2 + MC^2$ is minimums is ______

23. The angle of elevation of the top of a hill from a point on the horizontal plane passing through the foot of the hill is found to be 45°. After waling a distance of 80 meters towards the top, up a slope inclined at angle of 30° to the horizontal plane the angle of elevation of the top of the hill becomes 75°. Then the height of the hill (in meters) is ____

24. Set A has m elements and Set B has n elements. If the total number of subsets of A is 112 more than the total number of subsets of B, then the value of m-n is _______

25. Let $f: R \to R$ be defined as

$$f(x) = \begin{cases} x^5 sin\left(\frac{1}{x}\right) + 5x^2, & x < 0 \\ 0 & x = 0 \\ x^5 cos\left(\frac{1}{x}\right) + \lambda x^2, & \end{cases}$$

The value of λ for which f''(0) exists, is _____

ANSWER

1. (a)	**2.** (d)	**3.** (d)	**4.** (bonus)	**5.** (d)	**6.** (c)	**7.** (a)	**8.** (c)	**9.** (c)	**10.** (a)
11. (a)	**12.** (c)	**13.** (b)	**14.** (a)	**15.** (c)	**16.** (d)	**17.** (c)	**18.** (b)	**19.** (c)	**20.** (a)
21. (4)	**22.** (5)	**23.** (80)	**24.** (28)	**25.** (05)					